THE GROVND OF ARTS:

TEACHING THE PERFECT worke and practiſe of Arithmeticke, both in whole Numbers and Fractions, after a more eaſie and exact forme then in former time hath beene ſet forth: Made by Mr. ROBERT RECORD Dr. *in Phyſick.*

Afterward, augmented by Mr. Iohn Dee.

And ſince enlarged with a Third part of Rules of Practiſe, abridged into a briefer method then hitherto hath beene publiſhed, with diuers neceſſary Rules incident to the Trade of Merchandiſe: with Tables of the valuation of all Coynes, as they are currant at this preſent time:

By IOHN MELLIS.

And now diligently peruſed, corrected, illuſtrated and enlarged; with an *Appendix* of Figurate Numbers, and the Extraction of their Roots, according to the Method of *Chriſtian Vrſtitius*: with Tables of Board and Timber meaſure; and new Tables of *Intereſt* vpon *Intereſt*, after 10 and 8 *per* 100; with the true value of Annuities to be bought or ſold preſent, Reſpited, or in Reuerſion: the firſt calculated by *R. C.* but corrected, and the latter diligently calculated by *Robert Hartwell, Philomathemat.*

Scientia non habet inimicum niſi ignorantem.

Fide ——— ſed ——— Vide.

LONDON:
Printed by THO. HARPER, for *Iohn Hariſon*, and are to be ſold by *Richard More*, in S. *Dunſtans* Church-yard in *Fleet-ſtreet*. 1631.

THat which my friend hath well begun,
For very loue to Common weale,
Need not all whole to be new done,
But new encreaſe I doe reueale.

Something herein I once redreſt,
And now againe for thy behooſe,
Of zeale I doe, and at requeſt,
Both mend and adde, fit for all prooſe.

Of numbers vſe, the endleſſe might,
No wit nor language can expreſſe,
Apply and try both day and night,
And then this truth thou wilt confeſſe.

I. Dec.

The Bookes Verdict.

TO pleaſe or diſpleaſe ſure I am,
But not of one ſort to euery man:
To pleaſe the beſt ſort would I faine,
The froward diſpleaſe ſhall I certaine.
Yet wiſh I will, though not with hope,
All eares or mouthes to pleaſe or ſtoppe.

TO THE MOST

mighty Prince *Edward* the sixt,

by the grace of God, King of *England*, *France*, and *Ireland*, &c.

He Excellency of mans nature being such, as it is by Gods diuine fauor (most mighty Prince) not onely created in highnes of degree farre aboue all other corporall things, but by perfection, reason and search of wit, much approaching toward the Image of God, as not onely the holy Scriptures doe testifie, but also those naturall Philosophers, which exactly did consider the nature of man, and namely the far reach and infinite compasse of the works of the mind, were inforced to confesse, that man scarsely was able to know himselfe. And if he would duely ponder the nature of himselfe, hee would find it so strange, that it might seeme vnto him a very miracle. And thereof sprang that saying: *Magnum miraculum est homo: maximum miraculum sapiens homo.* For vndoubtedly, as man is one of the greatest miracles that euer God wrought, so a wise man is plainely the greatest.

And therefore was it that some did account the head of a man the greatest miracle in the world, because not onely of the strange workmanship that is in it, but much more of the efficacie of reason, witte, memory, imagination, and such other powers, and works of the mind, which can more easily conceiue any thing in a manner, then vnderstand it selfe. Amongst all the creatures of God, it findeth none more difficult to be perceiued then these same powers of it selfe; whereby it doth conceiue & iudge: as it may be well coniectured by the diuersity of opinions, that the wisest Philosophers did vtter touching the spirit of man, & the substance of it: whereof, I now intend to make no rehearsall; but who so listeth to

read thereof, may find it largely ſet forth, not onely in *Ariſtotle* his bookes *De anima*, but alſo in *Galene*, his book called *Hiſtoria Philoſophica*, and againe in *Plutarch* his worke, *De Philoſophorum placitis*, whoſe words are alſo repeated of *Euſebius* in the xv booke τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whom I remit them that haue deſired to vnderſtand intricate difficulty of knowing our owne ſelues, as touching our beſt part, and that part whereby we deſerue to beare the name of men.

This matter ſeemed ſo obſcure and difficult in knowledge, that *Galen*, who for his excellent wiſdome and iudgement in naturall works, is called of many men a Miracle in nature, yet in ſearching the nature and ſubſtance of the ſpirit of man, he not onely confeſſeth himſelfe ignorant, but counteth it plaine temerity to attempt to finde it. So farre aboue the hope of mans knowledge is that part, whereby man doth know and iudge of things. And although the ignorant ſort (which hate all things that they know not) doe little eſteeme the profoundneſſe of mans ſpirit and of reaſon, the chiefe power and faculty of it: yet as there is a kind of feare and obedience of vnreaſonable Beaſts vnto man, by the working power of God, ſo is there in thoſe ſmall reaſoned perſons, a certaine kind of reuerence toward wiſdome and reaſon, which they doe ſhew oftentimes, and by power of perſwaſion, are inforced to obey reaſon, will they nill they.

And hereby came it to paſſe, that the rudeneſſe of the firſt age of Man was brought vnto ſome more ciuill trade, as it is well declared by *Cicero*, in the beginning of his firſt booke *De inventione Rhetorica*, where he ſaith thus: *Nam fuit quoddam tempus quum in agris homines paſſim beſtiarum more vagabantur, & ſibi victu ferino vitam propagabant, nec ratione animi quicquam, ſed pleraq́, viribus corporis adminiſtrabant. Nondum diuinæ religionis, non humani ratio colebatur. Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos; non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atq́ inſcitiam cæca ac temeraria dominatrix animi cupiditas, ad ſe explendam viribus corporis abutebatur, pernicioſiſſimis ſatellitibus. Quo tem-*

pore

porequidam, magnus videlicet, vir & sapiens, cognouit quæ materia esset, & quanta ad maximas res opportunitas in animis inesset hominum, si quis eam posset elicere, & præcipienda meliorem reddere. Qui dispersos homines in Agris, & in tectis syluestribus abditos, ratione quadam compulit in vnum locum, & congregauit: & eos in vnamquamq; rem inducens vtilem atq; honestam, primò propter insolentiam reclamantes, deinde propter rationem atq; orationem studiosiùs audientes, ex feris & immanibus, mites reddidit, & mansuétos.

This long repetition of *Tullyes* words will seeme tedious to them that loue but little, and care much lesse for the knowledge of reason, but vnto your Maiesty (I dare say) it is a delectable remembrance, and vnto me it seemed so pleasant, that I could scarce stay my pen from writing all that mine eyes did so greedily read.

This sentence of *Cicero* am I loth to translate into English, partly for that vnto your Maiesty it needeth no translation, but especially knowing how far the grace of *Tullyes* Eloquence doth excell any Englishmans tongue, and much more exceedeth the baseneſſe of my barbarous style: yet for the fruite of the sentence, I had rather vnto my meere English countrimen vtter the rudeneſſe of my translation, then to defraud them the benefit of so good a lesson; trusting they will so learne to loue reason, that they will also gladly and greedily embrace all good Sciences, that may helpe to the iust furniture, of the same, when they consider that informed reason was the onely instrument, or at least the chiefest meanes to bring men vnto ciuill regiment, from barbarous manners, and beastly conditions.

" For the time was (saith *Tully*) that men wandred a-
" broad in the fields vp and down like beasts, and vsed
" no better order in feeding then they; so that by reasons
" rule they wrought nothing, but most of their do-
" ings did they atchieue by force of strength. At this
" time there was no iust regard of religion toward God,
" nor of duty toward man. No man had seen right vse of
" mariage, neither did any man know their owne chil-
" dren from other; nor no man had felt the commodi-
" ty of iust Lawes: so that through error and ignorance,
" wilfull lust, like a blind and heady ruler, abused bodi-

"ly ſtrength as a moſt mortall miniſter for the ſatisfy-
"ing of his deſire. At that time was there one which
"not onely in power, but alſo in wiſdome was great,
"and hee conſidered how that in the mindes of men
"was both apt inſtruments, and great occaſion to the
"due accompliſhment of moſt weighty affaires, if a man
"could apply them to vſe, & by teaching of rules frame
"them to better trade. This man with perſwaſion of
"reaſon gathered into one place the people that were
"wandring about the fields, and lay lurking in wilde
"cottages and woods, & bringing them into one com-
"mon ſociety, did trade them to all ſuch things, as either
"were profitable or honeſt, although not without re-
"pining at the firſt, by reaſon that they had not been ſo
"accuſtomed before. Yet at length through reaſon and
"perſwaſion of words they obeyed him more diligent-
"ly, and ſo of a wilde and cruell people, he made them
"curteous and gentle.

Thus hath *Tullie* ſet forth the efficacie of reaſon and perſwaſion, how it was able to conuert wild people to a mildneſſe, and to change their furious cruelneſſe into gentle curteſie: were it not now a great reproach in this our time (when knowledge raigneth ſo large) that men ſhould ſhew themſelues leſſe obſequious to reaſon? Vnleſſe it may be thought, that now euery man hauing ſufficient knowledge of himſelfe, needeth not to hearken to the perſwaſion of others.

Indeed he that thinketh himſelfe wiſe, will not eſteem the reaſon of any other, be hee neuer ſo wiſe; ſo that of ſuch a one it may well be ſaid: Hee that thinketh himſelfe wiſer then he is, may iuſtly be counted a double foole. Wherefore ſuch men are not to be permitted in open audience to talke, but muſt bee put to ſilence, & be made to giue eare to reaſon; which reaſon conſiſteth not in a multitude of words heaped raſhly together, & applied for one purpoſe, but reaſon is the expreſſing of a iuſt matter with witty perſwaſions furniſhed with learned knowledge: ſuch knowledge had *Moſes*, being expert in all learning of the Egyptians, as the Scriptures declare, & therefore was able to perſwade the ſtubborne

people of the Iewes, although not without paine. Such knowledge and such reasons did *Druys* shew, which was the first Law-maker of all the West part of Europe. Like reason and wisedome did *Xamolxis* vse amongst the Gothes: *Lycurgus* vnto the Lacedemonians, *Zelcucus* to the Locrians, *Solon* to the Athenienses, & *Dunwallo Molmutius* two thousand yeeres past amongst the old Britains of this Realme. And hereby came it to passe, that their Lawes continued long, til more perfect reason altered many of them, and willfull power oppressed most of them.

Druis was sonne to King Sarron, and succeded him in his kingdome.

At the beginning when these wise men perceiued how hard it was to bring the rude people to vnderstand reason, they iudged the best meanes to attaine this honest purpose to depend of learning in euery kind: for by learning (as *Ouid* saith:) *Pectora mollescunt, asperitásque fugit*: Stout stomackes doe waxe milde, and sharpe fiercenesse is exilde. Therefore as *Berosus* doth testifie, *Saron* that was the third King ouer all this West part of Europe, for to bring the people from beastly rage to manly reason, did erect Schooles of liberall Arts, which tooke so good successe, that his name continued in that sort famous aboue two thousand yeeres, after: for *Diodorus Siculus* which was in the time of *Iulius Cæsar*, maketh mention of the learned men or Gothes of Celtes, and nameth them *Sarronides*, that is to say, *Sarron his Scholers and followers*.

Among these Arts that then were taught, some did informe the tongue, and make them able both to vtter aptly their mind, and also to perswade; as Grammer, Logicke, and Rhetoricke, although not so curiously as in this time: some other did appertaine to the iust order of partition of lands, the trueysing of weights, measures & reckonings in al sorts of bargaines, and for order of building and sundry other vses; thole were Arithmeticke & Geometry Againe, to incourage men to the honor of God, they taught Astronomy, whereby the wonderfull workes of God were so manifestly set forth, that no mans tongue nor penne can in like sort expresse his infinite power, his vnspeakeable wisedome, and his exceeding goodnesse toward man, whereby he doth bountifully

prouide for man all neceſſaries, not only to liue, but alſo to liue pleaſantly. And ſo was their confidence in Gods prouidence ſtrongly ſtayed, knowing his goodneſſe to be ſuch, that he would helpe man as he could, & his power to be ſo great, that he could doe what hee would: and thirdly his wiſedome to be ſo pure, that hee would doe nothing, but that that was beſt. Beſide theſe Sciences they taught alſo Muſicke, which moſt commonly they did apply partly to religious Seruices, to draw men to delight therein, & partly to ſongs made of the manners of men, in praiſe of Vertue, and diſcommendation of, Vice, whereby it came to paſſe, that no man would diſpleaſe them, nor doe any thing euill that might come to their hearing: for their ſongs did make euil men more abhorred in that time than any excommunication doth in this time. The poſterity of theſe Muſitians continue yet both in Wales and Ireland, called *Bardes* vnto this day, by the ancient name of *Bardus*, their firſt founder.

This Bardus Druidius the 5 King of the Celtes, reigned 69 yeeres, and died 1832 yeeres before Chriſt.

And as theſe Sciences did encreaſe, ſo did Vertue encreaſe thereby. Againe, as thoſe Sciences did decay, ſo vertue loſt her eſtimation, and conſequently was little in vſe: whereof to make a full declaration were a thing meete for a Prince to heare, but it would require a peculiar Treatiſe. Wherefore at this preſent I count it ſufficient lightly to haue touched this matter in generall words, and to ſay no more of the particularity thereof, but onely touching one of thoſe Sciences, that is, *Arithmeticke*, by which not onely iuſt partition of lands was made, but alſo touching buying and ſelling, all Aſſiſes, Weights, and Meaſures were deuiſed, and all reckonings and accounts driuen; yea by proportion of it were the true orders of Iuſtice limited, as *Ariſtotle* in his Ethicks doth declare, and the degrees of eſtates in the commonwealth eſtabliſhed. Although that proportion be called Geometrical, and not Arithmeticall, yet doth that proportion appertaine to the Art of Arithmeticke, and in Arithmeticke is taught the progreſſion of ſuch proportions, & all things thereto belonging. Wherefore I may well ſay, that ſeeing Arithmeticke is ſo many waies needefull vnto the firſt planting of a Common-

wealth,

wealth, it muſt needs be as much required to the preſervation of it alſo, for by the ſame meanes is any Cõmonwealth continued, by which it was erected & ſtabliſhed. And if I ſhal in ſmall matters in appearance, but indeed very waighty, put one example or two. What ſhall we ſay for the Statutes of this Realme, which be the only ſtay of good order in manner now? As touching the meaſuring of ground by length and breadth, there is a good & an ancient Statute made by Act of Arithmeticke; & now it ſhall be to little vſe, if by the ſame Art it be not practiſed and tried. For the aſſiſe of Bread and Drinke, the two moſt common and moſt neceſſary things for ſuſtentation of man, there was a goodly ordinance in the Law made, which by ignorance hath ſo growne out of knowledge and vſe, that few men do vnderſtand it, & therefore the Statute bookes wonderfully corrupted, & the Commons cruelly oppreſſed: notwithſtanding ſome men haue written, that it is too doubtfull a matter to execute thoſe aſſiſes by thoſe Statutes, by reaſon they depend of the ſtanderd of the coyne, which is much changed from the ſtate of that time, when thoſe Statutes, were made. Thus ſhall euery man read (that liſteth) in the Abridgement of the Statutes, in the title of Weights and Meaſures, in the ſeuenth number of the Engliſh Booke, where hee ſhould haue tranſlated a good ordinance which is ſet forth in the French booke: but no maruell if the Abridgement doth omit it, ſeeing the great Booke of Statutes doth omit the ſame Statute as it hath done diuers other very good Lawes. And this is the fruit of ignorance, to reiect & condemne all that it vnderſtandeth not, although they vſe ſome clokes for it: but ſuch cloakes as being allowed, might ſerue to repell all good Lawes, which God forbid.

Againe, there is an ancient order for Aſſiſe of fire, Wood and Coales, which was renued not many yeeres paſt; and now how Auarice and Ignorance doth canuaſe that Statute, it is too pittifull to talke of, and more miſerable to feele.

Furthermore, for the Statute of Coynage, and the ſtanderd thereof, if the people vnderſtood rightly the

Statute,

Statute, they ſhould not, nor would not (as they often doe) gather an excuſe for their folly thereby : but as I ſaid, theſe Statutes by wiſdome and good knowledge of Arithmeticke were made, and by the ſame muſt they be continued. And let ignorance no more meddle with the vſe of them, than it did with the making of them. Oh in how miſerable caſe is that Realme, where the miniſters & interpreters of the Law are deſtitute of all good Sciences, which be the Keyes of the Lawes. How can they either make good Lawes, or maintaine them, that lacke that true knowledge whereby to iudge them ? And happy may that Realme be accounted, where the Prince himſelfe is ſtudious of learning, and deſireth to vnderſtand equity in all Lawes. Therefore moſt happy are we the louing Subiects of your Maieſty, which may ſee in your Highneſſe not only ſuch towardneſſe, but alſo ſuch knowledge of diuers Arts, as ſeldome hath beene ſeene in any Prince of ſuch yeeres, whereby we are enforced to conceiue this hope certainly, that hee which in thoſe yeeres ſeeketh know'edge when knowledge is leaſt eſteemed, and of ſuch an age can diſcerne them to bee enemies both to his Royall perſon, and to his Realmes, which labour to withdraw him from knowledge, to exceſſiue paſtime, and from reaſonable ſtudy, to idle or noyſome pleaſures, he muſt needs, when hee commeth to more mature yeeres, bee a moſt prudent Prince, a moſt iuſt Gouernour, and a right Iudge, not onely of his Subiects commonly, but alſo of the miniſters of his Lawes, yea, and of the Lawes themſelues : and to bee able to conceiue the true equity and exact vnderſtanding of all his Lawes and Statutes, to the comfort of his good Subiects, and the confuſion and reproach of them which labour to obſcure or peruert the equity of the ſame Lawes and Statutes. How ſome of theſe Statutes may be applyed to vſe, as well in this our time, as in any other time, I haue peculiarly declared in this Booke, and ſome other I haue omitted for iuſt conſiderations, till I may offer them firſt vnto your Maieſty, to wey them, as to your Highneſſe ſhall ſeeme good : for many things in them are not to be publiſhed without your Highneſſe

knowledge

knowledge and approbation, namely, becauſe in them is declared all the rates of alloyes for all ſtandards from one ounce vpward, with other myſteries of mint matters, and alſo moſt part of the varieties of coynes that haue beene currant in this your Maieſties Realme by the ſpace almoſt of ſixe hundred yeeres laſt paſt, and many of them that were currant in the time that the Romanes ruled heere.

All which with the ancient deſcription of England and Ireland, and my ſimple cenſure of the ſame, I haue almoſt completed to be exhibited to your Highneſſe. In the meane ſeaſon moſt humbly beſeeching your Maieſty to accept this ſimple Treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your Maieſty is, that this amongſt other of my bookes may paſſe vnder the protection of your Highneſſe, whom I beſeech God moſt earneſtly and daily, according to my duty, to aduance in all honour, and Princely Regalty, and to encreaſe in all knowledge, iuſtice, and godly policy. Amen.

Your Maieſties moſt obe-

dient Subiect and ſeruant

ROBERT RECORD.

TO

TO THE LOVING Readers : the Preface of Mr. Robert Record.

SOre oft times haue I lamented with my ſelfe the infortunate condition of England, ſeeing ſo many great Clerks to ariſe in ſundry other parts of the world, & ſo few to appeare in this our Natiõ: where as for pregnancy of natural wit (I thinke) few Nations doe excell Engliſhmen. But I cannot impute the cauſe to any other thing, than to the contempt or miſregard of learning. For as Engliſhmen are inferiour to no men in mother wit, ſo they paſſe all men in vaine pleaſures, to which they may attaine with great paine and labour: and are as ſlacke to any neuer ſo great commodity, if there hang of it any painfull ſtudy or trauelſome labour.

Howbeit, yet all men are not of that ſort, though the moſt part be, the more pity it is: but of thẽ that are ſo glad, not only with painfull ſtudy, & ſtudious paine to attaine learning, bnt alſo with as great ſtudy and paine to communicate their learning to other, and make all England (if it might be) partakers of the ſame; the moſt part are ſuch, that vneth they can ſupport their owne neceſſary charges, ſo that they are not able to beare any charges in doing of that good, that elſe they deſire to doe.

But a greater cauſe of lamentation is this, that when learned men haue takẽ pains to do things for the

the aid of the vnlearned, ſcarce they ſhal be allowed for their wel doing, but derided and ſcorned, and ſo vtterly diſcouraged to take in hand any like enterpriſe againe. So that if any be found (as there are ſome) that doe fauour learning, & learned wits, and can be content to further knowledge, yea only with their word; ſuch perſons though they be rare, yet ſhal they encourage learned men to enterpriſe ſome thing at the leaſt that England may reioyce of. And I haue good hope that England wil (after ſhe hath taken ſome ſure taſte of learning) not only bring forth more fauourers of it, but alſo ſuch learned men that ſhe ſhall be able to compare with any Realme in the world. But in the meane ſeaſon, where ſo few regarders of learning are, how greatly they are to bee eſteemed that doe fauour and further it, my pen will not ſuffice at full to declare.

Therefore gentle Reader, whereas I do vpon moſt inſt occaſion iudge, yea and know aſſuredly, that there be ſome men in this Realme, which both loue and alſo much deſire to further good learning, and yet am not wel able to write their condigne praiſe for the ſame, I thinke it better with ſilence to ouerpaſſe it, then either to ſay too little of it, or to prouoke againſt them the malice of ſuch other, which do nothing themſelues that is praiſe-worthy, and therefore cannot abide to heare the praiſe of any other mans good deed.

And conſidering their great fauour vnto learning, though I my ſelfe be not worthy to be reckoned in the number of great learned men, yet am I bold to put my ſelfe in Preſſe, with ſuch ability as God hath

hath lent me, though not with ſo great cunning as many men, yet with as great affection as any man, to helpe my Country-men, and will not ceaſe daily, as much as my ſmall ability will ſuffer me) to endite ſome ſuch thing, that ſhall be to the inſtruction, though not of learned men, yet at the leaſt of the vulgar ſort, whoſe argument alwaies ſhall be ſuch as it ſhall delight all learned wits; though they doe not learne any great things out of it.

But to ſpeake of this preſent book of Arithmetick, I dare not, nor will not ſet it forth with any words, but remit it to the iudgement of all gentle Readers, and namely, ſuch as loue good learning, beſeeching them ſo to eſteeme it, as it doth ſeeme worthy. And ſo either to accept the thing for it ſelfe, either at the leaſt to allow my good endeauour. But I perceiue I need not vſe any perſwaſions vnto them, whoſe gentle nature and fauourable mind is ready to receiue thankfully, and interpret to the beſt all ſuch enterpriſes attempted for ſo good an end, though the thing do not alwayes ſatisfy mens expectation. This conſidered, did bolden mee to publiſh abroad this little booke of the Art of numbring, which if you ſhall receiue fauourably, you ſhall encourage mee to gratifie you hereafter with ſome greater thing.

And as I iudge ſome men of ſo louing a minde to their natiue Country, that they would much reioyce to ſee it proſper in good learning, and witty Arts; ſo I hope well of all the reſt of Engliſhmen, that they will not be vnmindfull of his due praiſe, by whoſe meanes they are helped and furthered in any thing. Neither ought they to eſteeme this thing

of

of ſo little value, as many men of little diſcretion oftentimes doe. For who ſo ſetteth ſmall price by the witty deuice and knowledge of numbring, he little conſidereth it to be the chiefe point, (in manner) whereby men differ from all brute beaſts: for as in all other things (almoſt) Beaſts are partakers with vs, ſo in numbring we differ cleane from them, and in manner peculiarly, ſith that in many things they excell vs againe.

The Foxe in crafty wit exceedeth moſt men,
A Dogge in ſmelling hath no man his peere
To foreſight of weather if you looke then,
Many beaſts excell men; this is cleere.
The wittineſſe of Elephants doth letters attaine,
But what cunning doth there in the Bee remaine?
The Emmet foreſeeing the hardneſſe of Winter,
Prouideth victualls in time of Summer.
The Nightingale, the Linet, the Thruſh, the Larke,
In Muſicall harmony paſſe many a Clarke.
The Hedghogge of Aſtronomy ſeemeth to know,
And ſtoppeth his Caue where the wind will blow.
The Spider in weauing ſuch Art doth ſhow,
No man can him mend, nor follow I trow.
When a houſe will fall, the Myſe right quicke,
Flee thence before; can man doe the like?

Many things elſe of the wittineſſe of Beaſts and Birds might I here ſay, ſaue that another time of them I intend to write wherein they excell in manner al men, as it is daily ſeen: but in nũber was there neuer beaſt found ſo cũning, that could know or diſcerne one thing from many, as by daily experience you may well conſider, when a Bitch hath many whelps, or a Hen many chickens: & likwiſe of other whatſoeuer they be, take from them all their young

ſaning

ſauing onely one, and you ſhall perceiue plainly, that they miſſe none, though they will reſiſt you in taking them away, and will ſeeke them againe, if they may know where they be, but elſe they will neuer miſſe them truly; but take away that one that is left, and then will they cry & complaine; and reſtore to them that one, then are they pleaſed againe. So that of number, this may I iuſtly ſay, it is the onely thing almoſt that ſeparateth man from beaſts. He therefore that ſhall contemne number, declareth himſelfe as brutiſh as a beaſt, & vnworthy to be counted in the fellowſhip of men. But I truſt there is no man ſo foule ouer-ſeene, though many right ſmally doe it regard.

Why the Author wrot in Dialogue wiſe.

Therefore will I now ſtay to write againſt ſuch, and returne againe to this my Booke, which I haue written in the forme of a Dialogue, becauſe I iudge that to be the eaſieſt way of inſtruction, when the Schollar may aske euery doubt orderly, and the Maſter may anſwer to his queſtion plainly.

Howbeit I thinke not the contrary, but as it is eaſier to blame another mans worke, then to make the like; ſo there will be ſome that will find fault, becauſe I write in a Dialogue: but as I coniecture, thoſe ſhall be ſuch as doe not, cannot, or will not perceiue the reaſon of right teaching, and therefore are vnmeet to be anſwered vnto, for ſuch men with no reaſon will be ſatisfied.

And if any man obiect, that other Bookes haue bin written of Arithmeticke *already ſo ſufficiently, that I needed not now to put Pen to the Booke, except I will condemne other mens writings: to them*

I an-

I anſwer: That as I condemne no mans diligence, ſo I know that no one man can ſatisfie euery man: and therefore like as many doe eſteeme greatly other Bookes, ſo I doubt not but ſome will like this my Booke aboue any other Engliſh Arithmeticke hitherto written, and namely ſuch as ſhall lacke inſtructers, for whoſe ſake I haue ſo plainly ſet forth the Examples, as no Booke that I haue ſeene hath done hitherto: which thing ſhall be great eaſe to the rude Readers.

Therefore (gentle Reader) though this Booke can be but ſmall aid to the learned ſort, yet vnto the ſimple ignorant (which needeth moſt help) it may be a good furtherance and meane vnto knowledge.

And though vnto the King his Maieſty priuately I do it dedicate, yet I doubt not (ſuch is his clemency) but that he can be content, yea, and much deſirous, that all his louing Subiects ſhall take the vſe of it, and imploy the ſame to their moſt profit: Which thing if I perceiue that they thankefully doe, and receiue with as good will as it was written, then will I ſhortly with no leſſe kindneſſe ſet forth ſuch introductions into Geometry and Coſmography, as I haue at times promiſed, and as hitherto in Engliſh hath not beene interpriſed, wherewith I dare ſay all honeſt hearts will be pleaſed, & all ſtudious wits greatly delighted.

I will ſay no more, but let euery man iudge as he ſhall ſee cauſe. And thus for this time I will ſtay my Pen, committing you all to that true fountaine of perfect number, which wrought the whole world by number & meaſure: he is Trinity in Vnity, and Vnity in Trinity: To whom bee all praiſe, honor, and glory. Amen.

Heere followeth a Table of the whole Contens of this Booke.

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Before

Before the Introduction of Arithmeticke, it were very good to haue some vnderstanding and knowledge of these Figures and Notes.

i	1	*one.*	xx.	20	*twenty.*
ii	2	*two.*	xl.	40	*forty.*
iii	3	*three.*	l.	50	*fifty.*
iiii	4	*foure.*	lx.	60	*sixty.*
v	5	*fiue.*	lxx.	70	*seuenty.*
vi	6	*sixe.*	xc.	90	*ninety.*
vii	7	*seuen.*	C.	100	*a hundred.*
viii	8	*eight.*	CC.	200	*2 hundred.*
ix	9	*nine.*	D.	500	*5 hundred.*
x	10	*ten.*	DC.	600	*6 hundred.*
xi	11	*eleuen.*	M.	1000	*a thousand.*
xii	12	*twelue.*	MD	1500	*a thou. 5. hund.*
&c.	&c.	&c.	&c.	&c.	&c.

A Dialogue

A Dialogue betweene the Maſter and the Scholar: teaching the ART and vſe of Arithmeticke with Pen.

The Scholar ſpeaketh.

SIR, ſuch is your authority in mine eſtimation, that I am content to conſent to your ſaying, and to receiue it as truth, though I ſee none other reaſon that doth leade me thereunto: whereas elſe in mine owne conceit appeareth but vaine, to beſtow any time priuately in learning of that thing, that euery child may, and doth learne at all times and houres, when hee doth any thing himſelfe alone, and much more when he talketh or reaſoneth with others.

Maſter. Loe, this is the faſhion and chance of all them that ſéeke to defend their blinde ignorance, that when they thinke they haue made ſtrong reaſon for themſelues, then haue they proued quite contrary. For if numbring be ſo common (as you grant it to bée) that no man can doe any thing alone, and much leſſe talke or bargaine with other, but he

he ſhall ſtill haue to doe with number: this proueth not number to be contemptible and vile, but rather right excellent and of high reputation, ſith it is the ground of all mens affaires, ſo that without it no tale can be told, no communication without it can be continued, no bargaining without it can duely be ended, or no buſineſſe that man hath, iuſtly completed. Theſe commodities, if there were none other, are ſufficient to approue the worthineſſe of number. But there are other innumerable, farre paſſing all theſe, which declare number to excéd all praiſe. Wherefore in all great workes are Clerkes ſo much deſired: Wherefore are Auditors ſo richly fed: what cauſeth Geometricians ſo highly to be enhaunſed: why are Aſtronomers ſo greatly aduanced: becauſe that by number ſuch things they finde, which elſe would farre excell mans minde.

Scholar. Verily ſir, if it be ſo, that theſe men by numbring, their cunning do attaine, at whoſe great workes moſt men doe wonder, then I ſée well I was much deceiued, and numbring is a more cunning thing then I toke it to be.

Maſter. If number were ſo vile a thing as you did eſtéeme it, then néed it not to be vſed ſo much in mens communication. Exclude Number, and anſwer to this queſtion: how many yeares old are you.

Scholar.

Scholar. Mum.

Maſter. How many dayes in a weeke? How many weekes in a yeare? What lands hath your Father? How many men doth hée keepe? How long is it ſince you came from him to me?

Scholar. Mum.

Maſter. So that if number want, you anſwer all by Mummes: How many miles to London?

Scholar. A poake full of Plummes.

Maſter. Why, thus you may ſée, what rule number beareth, and that if number bée lacking, it maketh men dumbe, ſo that to moſt queſtions they muſt anſwer Mum.

Scholar. This is the cauſe (ſir) that I indged it ſo vile, becauſe it is ſo common in talking euery whyle: For plenty is not daintie, as the common ſaying is.

Maſter. No, nor ſtore is no ſore: perceiue you this? The moze common that the thing is, being nédfully required, the better is the thing, and the moze to be deſired. But in numbring, as ſome of it is light and plaine, ſo the moſt part is difficult, and not eaſie to attaine. The eaſier part ſerueth all men in common, and the other part requireth ſome learning. Wherefoze as without numbring a man can doe almoſt nothing, ſo with the helpe of it yau may attaine to all things.

Scholar. Yea ſir? why then it were beſt to learne

learne the Art of numbring, first of all other learning, and then a man need learne no more, if all other come with it.

Master. Nay, not so: but if it be first learned, then shall a man be able (I meane) to learne, perceiue, and attaine to other Sciences; which without it he could neuer get.

Scholar. I perceiue by your former words that Astronomy and Geometry depend much on the helpe of numbring: but that other sciences, as Musicke, Physicke, Law, Grammar, and such like, haue any helpe of Arithmeticke, I perceiue not.

Master. I may perceiue your great Clerkliness by the couering of your Sciences: but I will let that passe now, because it toucheth not the matter that I intend and I will shew you how Arithmeticke, doth profit in all these somewhat grosly, according to your small vnderstanding, omitting other reasons more substantiall.

First (as you reckon them) Musicke hath not only great helpe of Arithmeticke, but is made, and hath his perfectnesse of it: for all
Musick. Musick standeth by number and proportion. And in Physicke, beside the calculation of
Physik. criticall dayes, with other things, which I omit, how can any man iudge the pulse rightly, that is ignorant of the proportion of Numbers.
Law. And as for the Law, it is plaine, that the man

man that is ignorant of Arithmetick, is neither meet to be a Iudge, neither an Aduocate, nor yet a Proctor. For how can hee well vnderstand another mans cause, appertaining to distribution of goods, or other debts, or of sums of money, if hee bee ignorant of Arithmetick? This oftentimes causeth right to be hindred, when the Iudge either delighteth not to heare of a matter that hee perceiueth not, or cannot iudge for lacke of vnderstanding: this commeth by ignorance of Arithmetick.

Now, as for Grammer, me thinketh you should not doubt in what it needeth number sith you haue learned that Nounes of all sorts, Pronounes, Verbs, and Participles are distinct diuersly by numbers: besides the variety of Nounes of Number, and Aduerbs. And if you take away number from Grammar, then is all the quantity of Syllables lost. And many other wayes doth number help Grammar. Whereby were all kinds of Meeters found and made? Was it not by Number? Grammar

But how needfull Arithmeticke is to all parts of Philosophy, they may soone see, that doe read either Aristotle, Plato, or any other Philosophers writings. For all their examples almost, and their probations, depend of Arithmeticke. It is the saying of Aristotle, that he that is ignorant of Arithmeticke, is meet for no Science. And Plato his Master wrote Philosophy.

wrote a little sentence ouer his Schoolhouse doore, Let none enter in hither (quoth he) that is ignorant of Geometry. Seeing hee would haue all his Scholars expert in Geometry, much rather he would the same in Arithmeticke, without which Geometry cannot stand.

Diuinity. And how needfull Arithmeticke is to Diuinity, it appeareth, seeing so many Doctors gather so great misteries out of number, and so much doe write of it. And if I should goe about to write all the commodities of Arithmeticke in ciuil acts, as in gouernance of common weales in time of peace, and in due prouision, and order of armies, in time of warre, Armies. for numbring of the host, summing of their wages; prouisions of victuals, viewing of artillery, with other Armour; beside the cunningest point of all, for casting of ground, for encamping of men, with such other like: And how many waies also Arithmeticke is conducible for all priuate weales, of Lords and all possessioners, of Merchants and all other occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receiuers, Stewards, Bailiffes, and such like, whose offices without Arithmetick, are nothing: if I should (I say) particularly repeat all such commodities of the noble Science of Arithmeticke, it were enough to make a very great booke.

Scholar. No, no sir, you shall not need: For I doubt not, but this, that you haue said, were

enough

enough to perſwade any man to thinke this Art to be right excellent and good, and ſo neceſſary for man, that (as I thinke now) ſo much as a man lacketh of it, ſo much he lacketh of his ſenſe and wit.

Maſter. What are you ſo farre changed ſince, by hearing theſe few commodities in generall? by likelyhood you would bée farre changed if you knew all the particular commodities.

Scholar. I beſéech you Sir, reſerue thoſe commodities that reſt yet behind vnto their place more conuenient: and if yée will bée ſo good as to vtter at this time this excellent treaſure, ſo that I may bée somewhat inriched thereby, if euer I ſhall be able, I will requite your paine.

Maſter. I am very glad of your requeſt, and will doe it ſpéedily, ſith that to learne it you be ſo ready.

Scholar. And I to your authority my wit doe ſubdue, whatſoeuer you ſay, I take it for true. The duty of a Scholar.

Maſter. That is too much, and méete for no man to be beléeued in all things, without ſhewing of reaſon. Though I might of my Scholar ſome credence require, yet except I ſhew reaſon, I doe it not deſire. But now ſith you are ſo earneſtly ſet this Art to attaine, beſt it is to giue no time, leſt ſome other paſſion coole this great heate, and then Perſeuerance in ſtudy.

you leaue off before you sée the end.

Scholar. Though many there bée so vnconstant of minde, that flitter and turne with euery winde, which often begin, and neuer come to the end, I am none of this sort, as I trust you partly know. For by my good will what I once begin, till I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto indéed, and I trust you will increase rather then goe backe. For better it were neuer to assay, then to shrinke and fly in the mid-way: But I trust you will not so doe, therfore tell me briefly: What call you the Science that you desire so greatly?

Scholar. Why Sir, you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to haue studied a Science, and yet cannot tell how it is named.

Scholar. Some call it Arsemetrike, and some Augrime.

Master. And what doe these names betoken?

Scholar. That, if it please you, of you would I learne:

Master. Both names are corruptly written: Arsmetricke for Arithmeticke as the Greekes call it, and Augrim for Algorisme, as the Arabians sound it: which both betoken

Αριθμητικὴ.

ken the Science of Numbring: for Arithmos in Greeke is called Number; and of it commeth Arithmeticke, the Art of Numbring. So that Arithmeticke is a Science or Art teaching the manner and vse of Numbring. This Art may bee wrought diuerſly, with Penne or with Counters But I will firſt ſhew you the working with the penne, and then the other in order. Ἀριθμὸς.

Scholar. This I will remember. But how many things are to bee learned to attaine this Art fully?

Maſter. There are reckoned commonly ſeauen parts or workes of it.

Numeration, Addition, Subſtraction, Multiplication, Diuiſion, Progreſſion, and Extraction of rootes: to theſe ſome men adde Duplation, Triplation, and Mediation. But as for theſe three laſt, they are contained vnder the other ſeuen. For Duplation and Triplation are contained vnder Multiplication, as it ſhall appeare in their place. And Mediation is contained vnder Diuiſion, as I will declare in his place alſo.

Scholar. Yet then there remaine the firſt ſeauen kinds of Numbring.

Maſter. So there doth: Howbeit if I ſhal ſpeake exactly of the parts of numbring, I muſt make but fiue of them: for Progreſſion is a compound operatiõ of Additiõ, Multiplication, and Diuiſion. And ſo is the Extraction

traction of votes. But it is no harme to name them as kindes seuerall, seeing they appeare to haue some seuerall working. For it forceth not so much to contend for the number of them, as for the due knowledge and practising of them.

Scholar. Then you will that I shall name them as seauen kinds distinct. But now I desire you to instruct me in the vse of each of them.

Master. So I will, but it must be done in order: for you may not learne the last as soone as the first, but you must learne them in that order, as I did rehearse them, if you will learne them spéedily and well.

Scholar. Euen as you please. Then to begin; Numeration is the first in order: what shall I doe with it?

Master. First, you must know what the thing is, and then after learne the vse of the same.

Numeration.

Numeration is that Arithmeticall skill, whereby wee may duly value, expresse, and read any *number* or *summe* propounded: or else in apt *figures* and *places* set downe any *number* knowne or named.

Scholar.

Scholar. Why: then me thinketh you put a difference betweene the value and the figures.

Maſter. Yea ſo doe I: For the value is one thing, and the Figures are another thing: and that commeth partly by the diuerſity of Figures, but chiefly of the places wherein they be ſet.

Scholar. Then I muſt know here three things: the value, the figure, and the place.

Maſter. Euen ſo: but yet adde Order to them as the fourth. And firſt marke, that there are but ten figures that are vſed in Arithmeticke; and of thoſe ten, one doth ſignifie nothing, which is made like an o, and is priuately called a Cipher, though all the other ſometime be likewiſe named. The other nine are called ſignifying Figures, and be thus figured. A Cypher. Figures.

1. 2. 3. 4. 5. 6. 7. 8. 9.

And this is their value.

i. ij. iij. iiij. v. vj. vij. viij. ix.

But here you muſt marke, that euery Figure hath two values: One alwaies certaine, that it ſignifieth properly, which it hath of his forme; and the other vncertaine, which he taketh of his place.

A place is called the ſeate or roome that a Figure ſtandeth in. And loke how many A place.

figures are written in one ſumme, ſo many places hath that whole number. And that muſt bée called the firſt place that is next to the right hand, and ſo reckoning by order towards the left hand, ſo that that place is laſt that is next to the left hand. As for example: If there ſtood before you ſixe men in a rowe, ſide by ſide, and you ſhould tell them as they ſtand in order, beginning with the man that were next to your right hand: then he that were next him ſhould bée called the ſecond, and ſo forth to the furtheſt from your right hand, which is the ſixt and the laſt,

Scholar. Sir, I perceiue you well: ſo might I reckon Letters or any other thing. As if I ſhould write eight letters after this order, a, b, c, d, e, f, g, h, then muſt I ſay, h, is the firſt, g, the ſecond, f, the third, e, the fourth, d, the fift, c, the ſixt, b, the ſeuenth, and a, the eight.

Maſter. That is well done. And after the ſame ſort vſe hereafter, that what I declare by one example, doe you expreſſe by another: and ſo ſhall I perceiue whether you vnderſtand it or no. And ſo paſſe ouer nothing, till you perceiue it well, and be expert therein.

Scholar. Sir, I pray you how many of theſe places be there in all?

Maſter. There is no certaine number of them, but they are ſometimes more & ſometimes fewer, according to the ſumme that is expreſ-

expressed. For so many as the figures are, so many are the places: and the last place is so called, not because it is last of all other; but it is the last of that present summe, and it may be the middle place in another summe.

Scholar. Me seemeth I perceiue this very well, as touching the order of reckoning of the places; but as for the number of them, you say there is no certainty. Now there resteth to declare the value of the figures by the diuersity of places, which you called the value vncertaine. Value vncertaine.

Master. But first let mée heare whether you know perfectly the certaine value. Value certaine.

Scholar. Yes Sir, as you wrote them, so I marked them.

Master. How write you then fiue?

Scholar. By this figure 5.

Master. And how sixe?

Scholar. Thus 6.

Master. Write these three numbers, each by it selfe, as I spake them, vij. iiij. iij.

Scholar. 7. 4. 3.

Master. How write you these foure other, ij. i. ir. viij?

Scholar. Thus (I trow) 2. 1. 6. 8.

Master. Nay, there you misse; looke on mine example againe.

Scholar. Sir, true it is, I was to blame, I tooke 6. for 9. but I will beware hereafter.

Master. Now then take héede, those cer-

taine values euery Figure repꝛesenteth when it is alone wꝛitten without other Figures ioyned to him. And also when it is in the first place, though many other doe follow: as foꝛ example; This Figure 9. is ix. standing now alone.

Scholar. How is he alone, and standeth in the middle of so many Letters?

Master. The Letters are none of his fellowes. Foꝛ if you were in France in the middle of a thousand Frenchmen, if there were no Englishman with you, you would reckon your selfe to be alone.

Scholar. So it is. Then 9. without moe figures of Arithmetick, betokeneth ix. whatsoeuer other Letters be about it.

Master. Euen so, and so doth it, if it bée in the first place ioyned with other, how many soeuer doe follow, as in this example, 3 6 79. You sée 9. in the first place, and doth betoken nine, as it were alone.

Scholar. I perceiue that, and doth not 7. that standeth in the second place, betoken lxx. and 6. in the third place, betoken vj. and so 3. in the fourth place betoken three?

Master. Their figures be as you haue said, but their values are not so. Foꝛ as in the first place euery figure betokeneth his own value certain only, so in the second place euery figure betokeneth his own value certaine, ten times: as in the example 7. in the second

place

place is ſeuen times ten, & is, lxx. And in the third place euery figure betokeneth his owne value an hundred times, ſo the 6. in that place betokeneth vi, C. & in the fourth place euery figure betokeneth his owne value a M times, as in the aforeſaid number, 3. in the fourth place ſtandeth for 3. M. & in the fift place, euery figure ſtandeth for his own value x M times, & in the ſixt place CM times, & in the ſeuenth place a MM times, and in the eight place, x MM. ſo that euery place excédeth the former ten times.

Scholar. As thus: if I make this number at all aduentures, 9 1 3 5 9 6 8 4, here are eight places. In the firſt place is 4. & betokeneth but foure: in the ſecond place is 8. & betokeneth ten times 8. that is 80. in the third place is 6, & betokeneth ſix hundred: in the fourth place 9. is nine thouſand: and 5. in the fifth place is x M times 5. that is fifty M. So 3. in the ſixth place is a CM times 3. that is, CCCM. Then 1. in the ſeuenth place, one MM. and 9. in the eight place, ten thouſand thouſand times 9. that is, xcMM. But now I cannot eaſily nor quickly read it in order, A generall rule.

Maſter. That ſhall you practice by this meanes. Firſt, put a pricke ouer the fourth figure, and ſo ouer the ſeuenth. And (if you haue ſo many) ouer the tenth, thirteenth, ſixteenth, and ſo forth, ſtill leauing two figures betwéene each two prickes. And theſe two

Ternaries roomes betwéene the prickes are called Ternaries.

Then begin at the last pricke, and sée how many figures are betwéene him and the end, which cannot passe thrée, reckoning himselfe for one: then pronounce them as if they were written alone from the rest, and at the end of their value, so many times thousands, as your nnmbers hath prickes.

After that come to the next thrée figures, and sound them as if they were apart from the rest, & adde to their value so many times thousands, as there are prickes betwéene them & the first place of your whole number. And so doe by euery other thrée Figures following, if you haue moe. As in example 9 1 3 5 9 6 8 4. this was your number.

Put a pricke ouer 9. in the fourth place, and ouer 1. in the seauenth place, and then no moze (for your places come not to ten) as thus: 9 1 3 5 9 6 8 4.

Now goe to the last prick ouer 1. and take it and the Figure 9. that followeth it, and value them alone.

Scholar. 91. that is xci.

Master. So it is, then adde for the number of your prickes twice M.

Scholar. That is, xci. thousand thousand.

Master. So it is. Then take the thrée other Figures from one to the next prick, and value them.

Scholar.

Scholar. 3 5 9. that is, CCC.lix.

Master. Now adde for the one pricke, that is betweén them and the first place, Y.

Scholar. CCC.lix.thousand.

Master. Then come to the other three Figures that remaine.

Scholar. 6 8 4. that is, vi Clxxxiiij.

Master. Now haue you valued all. And at the end of the last number you shall adde nothing, because there remaineth no prick nor number after it; yet proue it in another number, as thus, 2 3 0 8 6 4 0 8 9 1 0 5 3 4 0.

Schol. 2 3 0 8 6 4 0 8 9 1 0 5 3 4 0. I haue pricked them as you taught me; but I am in doubt whether I haue done well or no, because of the Ciphers; for I remember you told me that they doe signifie nothing, and therefore I doubt whether I should reckon them for a figure in setting of the pricks; and againe, I know not wherefore they serue.

Master. That will I tell you now. Indéed they are of no value themselues, but they serue to make vp the number of places, & so make the Figure following them to be in a further place, & therefore to signifie the more value, as in this example 90. the cipher is of no value, but yet hée occupieth the first place and causeth 9. to bée in the second place, and so to signifie ten times 9. that is rc. so that two ciphers thrusteth the Figure following them into the third place, and so forth.

The vse of Cyphers.

Scholar.

Scholar. Then I perceiue in the example aboue, I haue pricked well enough: for though that Cypher that is pricked signifie nothing, yet must he haue the prick, because he came in the thirtēnth place. Then will I proue to nūber that summe. First, there is 2 3 0, Y, Y, Y, Y, and then followeth 8 6 4, Y, Y, Y. And what shal I now doe? There is a Cypher in the third place and no figure after him, but they that I haue reckoned.

Master. Yée did serue for them that you haue already reckoned, to make them in a place further then they should be, if he were away: and therefore now yée shall let him goe. And so doe alwaies when he occupieth that place next before any pricke, which is the last of that Ternary, and a Cipher in the last place doth nothing.

Scholar. Then shall I say but 8 9. Y. Y.

Master. So, but goe forth.

Scholar. 1 0 5. thousand. Now are all my prickes spent, and yet remaine 3 4 0. so that I must value them, CCC. xl. onely.

Master. Now can you reckon after this sort: and remember, that euery such rome so parted, is called a Ternary or Trinity: for you haue numbred or valued the summe most truely, and by the aide of the prickes each Denomination is distinct most plainly.

Trinity.

Denominations.

Scholar. What call you Denomination?

Master.

Maſter. It is the laſt value or name added to any ſumme. As when I ſay, a hundred two and twenty pounds. Pounds is the Denomination. And likewiſe in ſaying 25. men: Men is the Denomination, and ſo of other. But in this place (that I ſpake of before) the laſt number of every Ternary is the Denomination of it. As for the firſt ternary, the Denomination is vnites, and of the ſecond Ternary, the Denomination is thouſands: And of the third Ternary, thouſand thouſands, or Millions: of the fourth, thouſand thouſand thouſands, or thouſand Millions: and ſo forth.

Scholar, And what ſhall I call the value of the three Figures that may bee pronounced before the Denomination? as in ſaying: 203000000. that is, two hundred three millions. I perceiue by your words, that millions is the Denomination: but what ſhall I call the CCIij. ioyned before the millions?

Maſter. That is called the Numerator or valuer, and the whole ſumme that reſulteth of them both, is called the ſumme, value, or number. Numerator. Summe or value.

Scholar. Now is there any thing elſe to be learned in Numeration? or elſe haue I learned it fully?

Maſter. I might ſhew you here who were the firſt Inuentors of this art, the reaſon of

all

all these things that I haue taught you, but that I will reserue till yée haue learned ouer all the practise of this Art, lest I should trouble you with ouer many things at the first.

Three kinds of numbers.

But yet this must you marke, that there are three kindes of numbers, one called Digits, another Articles, and the third Mixt numbers.

Digits.

A Digit is any number vnder ten, as these: 1.2.3.4.5.6.7.8.9.

Articles.

And 10. with all other that may be diuided into ten parts iust, and nothing remaine, are called Articles, such as are 10.20.30.40.50. &c. 100.200 &c. 1000. &c.

Mixt.

And that number is called Mixt, that containeth Articles, or at the least one Article, and a Digit, as 12.16.19.21.38.107. 1005. and so forth; and for the more ease of vnderstanding and remembrance, marke this; The Digit number is neuer written with more than one figure, but the Article and the Mixt number are euer written with more then one figure. And thus they differ; that the Article hath euermore this Cypher 0 in the first place; and the Mixt number hath euer there some Digit.

Scholar. By these last words I perceiue it much better then I did before, and now (I thinke) I will neuer misse to know those three asunder.

Master. If you remember now all that I haue

haue ſaid, you haue learned ſufficiently this firſt kinde of Arithmeticke called Numeration. Howbeit I will exhort you now to remember both this that I haue ſaid, and all that I ſhall ſay, and to exerciſe your ſelfe in the practiſe of it; for rules without practiſe, are but a light knowledge, and practice it is, that maketh men perfect and prompt in all things.

Vſe maketh maſtery.

And as you haue learned to gather and expreſſe the value of a ſumme propounded, and ſet downe before you; ſo muſt you practiſe to marke, note, or write downe with apt Figures and in due places, any number onely named or recited to you, or of your ſelfe imagined; as for a proofe. How note you or write downe this ſumme, fiue thouſand two hundred, fifty, and ſeauen?

Scholar. This troubleth me now, whether I ſhould begin at the firſt or at the laſt: For reaſon (me thinketh) ſhould cauſe me to begin at the firſt; and yet if I write it as you ſpeake it, I muſt begin at the laſt.

Maſter. When you know your places perfectly, you may begin where you liſt; but the more eaſe for your hand is to begin with the laſt, that is to ſay, as I did ſpeake them; yet for the more ſurety, a while you may begin with the firſt, repeating my words backward thus, Seauen, fifty, two hundred, fiue thouſand: or els ſoũding them al by their digit or value,

valewer, as thus: ſeuen, fiue, two, fiue: for that way is eaſieſt: But then muſt you loke well whether there be any Cypher in your ſumme, that he may be ſet in his place. As if the laſt valewer of your ſumme, (as you ſpeake it) be aboue 9, then is there a Cipher in the firſt place. And if it be an hundred, or aboue, then is there two Cyphers, one in the firſt place. and an other in the ſecond, and ſo forth.

But becauſe this thing is ſuch that cannot be ſet forth without many words, I thinke beſt here now at the end of Numeration to adde a Table eaſie and ready for the firſt exercise of it.

Loe this is the Table.

The left side or hand. *The right side or hand.*

The names of Digits, values certaine, or valuers.

The denominators of the place or value vncertaine.	Vnites.	Tennes.	Hundreds.	Thousands.	X. Thousands.	C. of Thousands.	Millions.	X. of Millions.	C. of Millions.	M. of Millions.	X. M. of Millions.
Nine.	9	9	9	9	9	9	9	9	9	9	9
Eight.	8	8	8	8	8	8	8	8	8	8	8
Seuen.	7	7	7	7	7	7	7	7	7	7	7
Six.	6	6	6	6	6	6	6	6	6	6	6
Fiue.	5	5	5	5	5	5	5	5	5	5	5
Foure.	4	4	4	4	4	4	4	4	4	4	4
Three.	3	3	3	3	3	3	3	3	3	3	3
Two	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
Cipher	o	o	o	o	o	o	o	o	o	o	o
The order of places.	First.	Second.	Third.	Fourth.	Fift.	Sixt.	Seuenth.	Eight.	Ninth.	Tenth.	Eleuenth.

This Table (as yée may sée) hath eleuen places, and in each of them are set all the Digits, whose certaine value is written on the right hand of the Table, and the value vncertaine on the left hand, so that by this Table you may learne both how to expresse any number that you list, if that it excéde not

eleuen places) that is to say, XC. thousand millions, and so may you by helpe of it, value all summes proposed vnder the said number.

For example: take the summe that I proposed before, which was fiue thousand, two hundred fifty and seuen. And if you will expresse it, take the first number (as I speake it) which is [illegible] whose valuer or certaine value is 5. and his vncertaine value or denomination is M. First, you shall seeke at the right hand of the valuer 5. Then seeke along vnder the title of denomination toward the left hand till you finde thousands, and vnder it, right at the foot of the Table is the number of the place that is in the fourth, wherein you must write your digit or valuer 5.

Afterward come to the second part of the number two hundred, whose valewer is 2. and his denomination C. Seeke two at the right hand of the Table, and goe along vnder the denomination toward the left hand, til you come vnder C: then looke to the foote of the Table, and there you shall see the number of the place, that is to say, the third, wherein you must set your digit 2.

Then doe so by your other two numbers that remaine, and you shall finde 5. in the second place for your fifty, and 7. in the first place for your seuen. And thus may you doe with other numbers.

Scholar. Master, I thanke you heartily. I perceiue you seeke to instruct me most plainely

and

and briefly, and not to hide your knowledge with ſubtle words, as many doe. For this rule is ſo plaine that I can deſire it no plainer. And though it ſeeme ſomewhat long, yet I perceiue it to be a ſure way.

Maſter. So it is, and though it be long, yet it is neither to long, neither to plaine for yong learners that lacke practiſe: for the Table is inſtead of a teacher to them that lacke one. But now I truſt I haue ſaid enough of Numeration: which after you haue well practiſed, then may you learne forth.

Scholar. Yet I pray you in one thing to tell me your iudgement. Why doe men reckon the order of the places backward, from the right hand to the left?

Why numbers are written backward.

Maſter. In that thing all men doe agree, that the Chaldees, which firſt inuented this Art, did ſet theſe figures as they ſet all their Letters, for they write backeward, as you learne it, and ſo doe they reade. And that may appeare in all Hebrew, Chaldee, and Arabicke Bookes; for they be not onely written from the right hand to the left, and ſo muſt be read, but alſo the right end of the booke is the beginning of it, whereas the Greekes, Latines, and all nations of Europe, doe write and read from the left hand toward the right: and all their Bookes begin at the left ſide.

Scholar. That reaſon hath ſatisfied me.

Maſter. It neither ſatisfieth me, neither liketh me well, becauſe I ſée that the Chaldees

and Hebrewes doe not so vse their owne numbers, as at another time I will declare. But this plaine reason may best satisfie you presently: that seeing in pronouncing of numbers we keepe the order of our owne reading, from the left hand to the right: and againe, we doe euer name the greater numbers before the smaller: it was reason that the lesser places, containing the lesser numbers, should bee set on the right hand, and the greater places containing the greater numbers, to proceed toward the left hand.

Scholar. This reason is to mee so plaine, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied; hoping that practise shall make me fully ready and expert in it. And in the meane season, I desire to learne the other kindes of Arithmeticke.

Master. That is well said: but what should you next learne? can you tell?

Scholar. I remember you said that Addition was next.

Master. Euen so, and what that is, must you first know.

Addition.

Addition.

Addition is the gathering together and bringing of two numbers or more into one ſumme. As if I haue 160 Bookes in the Latine tongue, and 136 in the Greeke tongue, and would know how many they be in all, I muſt write theſe two numbers one ouer another, writing the greateſt number higheſt, ſo that the firſt figure of the one being vnder the firſt figure of the other: and the ſecond vnder the ſecond, and ſo forth in order.

When you haue ſo done, draw vnder them a right line, then will they ſtand thus:

Now begin at the firſt places toward the right hand alwaies, and put togs-ther the two firſt figures of theſe two numbers, and loke what commeth of them, write vnder them, right vnder the line. As in ſaying 6. and 0 is 6 write 6. vnder 6 as thus:

160	
136	
6	

And then goe to the ſecond figures, and doe likewiſe: as ſaying 3 and 6 is 9 write 9 vnder 6 and 3, as here you ſée.

160	
136	
96	

And likewiſe doe you with the figures that be in the third place, ſaying, 1 and 1 be 2. write 2 vnder them, and then will your whole ſumme appeare thus:

160	
136	
296	

So that now you ſée, that 160. and 136. doe make in all 296.

Scholar. What: this is very eaſie to doe: me thinketh I can doe it euen ſince.

There came through Cheapeſide two droues of cattell: in the firſt was 848 ſheep, and in the ſecond was 186 other beaſts.

Thoſe two ſummes I muſt write as you taught me, thus: Then if I put the two firſt figures together ſaying, 6. & 8. they make 14. That muſt I write vnder 6, and 8. thus:

848	
186	
	14

Maſter. Not ſo, and here are you twice deceiued. Firſt, in going about to adde together two ſummes of ſundry things, which you ought not to doe, except you ſéke onely the number of them, and care not for the things. For the ſumme that ſhould reſult of that addition, ſhould be a ſumme neither of ſheepe, nor of other beaſts, but a confuſed ſumme of both. Howbeit ſometimes yee ſhall haue ſummes of diuers denominations to bée added, of which I will tell you anon: but firſt I will ſhew you where you were deceiued in another point, and that was in writing 14. which came of 6. and 8. vnder 6. 8. which is vnpoſſible; for how can two Figures of two places be written vnder one Figure and one place.

Scholar. Truth it is; but yet I did ſo vnder-

ſtand

ſtand you.

Maſter. I ſaid indéede, that you ſhould wzite that vnder them that did reſult of them both together: which ſaying is alwaies true, if that ſumme doe not excéd a digit. But if it be a mixt number, then muſt you wzite the digit of it vnder your Figures as you haue ſaid before: and if it be an article, then wzite o vnder them, and in both ſozts you ſhall kéepe the article in your mind; and therefoze when you haue added your ſecond Figures, which occupy the place of tens, you ſhall put that one thereto, which you kept in your minde; foz though it were ten indéed, yet in that place it is but as one, becauſe that euery one of that place is ten, foz that it is the place of tens. And in like manner, if you haue in the ſecond place ſo great a number that it amounteth aboue 9. then wzite the digit, and reſerue the article in your minde, euer adding it to the next place following, and ſo of all other places, how many ſoeuer you haue. And if you haue a mixt number when you haue added your laſt figures, then wzite the digit vnder the laſt figures, and the article in the next place beyond them: ſo ſhall your number reſulting of Addition, haue one place moze then the numbers which you ſhall adde together.

A place.

Scholar. Now doe I perceiue you, and the reaſon of this, is, (as I vnderſtand) becauſe that no one place can containe aboue 9. which is the greateſt figure that is, and then all tens

So that now you ſée, that 160. and 136. doe make in all 296.

Scholar. What: this is very eaſie to doe: me thinketh I can doe it euen ſince.

There came through Cheapeſide two droues of cattell: in the firſt was 848 ſheep, and in the ſecond was 186 other beaſts.

Thoſe two ſummes I muſt write as you taught me, thus: Then if I put the two firſt figures together ſaying, 6. & 8. they make 14. That muſt I write vnder 6, and 8. thus:

848	
186	
14	

Maſter. Not ſo, and here are you twice deceiued. Firſt, in going about to adde together two ſummes of ſundry things, which you ought not to doe, except you ſéke onely the number of them, and care not for the things. For the ſumme that ſhould reſult of that addition, ſhould be a ſumme neither of ſheepe, nor of other beaſts, but a confuſed ſumme of both. Howbeit ſometimes yee ſhall haue ſummes of diuers denominations to bée added, of which I will tell you anon: but firſt I will ſhew you where you were deceiued in another point, and that was in writing 14. which came of 6. and 8. vnder 6. 8. which is vnpoſſible; for how can two Figures of two places be written vnder one Figure and one place.

Scholar. Truth it is; but yet I did ſo vnder-

ſtand

ſtand you.

Maſter. I ſaid indéede, that you ſhould wzite that vnder them that did reſult of them both together: which ſaying is alwaies true, if that ſumme doe not excéd a digit. But if it be a mixt number, then muſt you wzite the digit of it vnder your Figures as you haue ſaid before: and if it be an article, then wzite ○ vnder them, and in both ſozts you ſhall kéepe the article in your mind; and therefoze when you haue added your ſecond Figures, which occupy the place of tens, you ſhall put that one thereto, which you kept in your minde; foz though it were ten indéed, yet in that place it is but as one, becauſe that euery one of that place is ten, foz that it is the place of tens. And in like manner, if you haue in the ſecond place ſo great a number that it amounteth aboue 9. then wzite the digit, and reſerue the article in your minde, euer adding it to the next place following, and ſo of all other places, how many ſoeuer you haue. And if you haue a mixt number when you haue added your laſt figures, then wzite the digit vnder the laſt figures, and the article in the next place beyond them: ſo ſhall your number reſulting of Addition, haue one place moze then the numbers which you ſhall adde together.

A place.

Scholar. Now doe I perceiue you, and the reaſon of this, is, (as I vnderſtand) becauſe that no one place can containe aboue 9. which is the greateſt figure that is, and then all tens

or Articles must be put to the next place following: for euery place (as I may sée) exceedeth the other place next before him by 10.

Now (if it please you) I will returne to my example of cattell. But I remember you said I might not adde summes of sundry things together, and that I may sée by reason.

Master. Truth it is, if you séeke the due summe of any thing, but if you onely séeke a bare summe, and haue no respect to the thing, then were it better to name the summe onely without any thing: as in saying, 8 4 8 without naming sheepe or any thing else. And likewise 1 8 6 naming nothing.

Now let mée sée how can you adde those two summes?

Scholar. I must first set them so, that the two first figures stand one ouer another, and the other each one ouer his fellow of the same place: then shall I draw a line vnder them both. And so likewise of other figures, setting alwaies the greatest number highest, thus as followeth.

Then must I adde 6. to 8. which make 14. that is a mixt number, therefore must I take the Digit which is 4. and write it vnder 6. and 8. keeping the Article 1 in my minde thus:

848
186
4

Next that, I doe come to the second figures, adding them together, saying 8. and 4 make 12. to the which I put the one reserued in my minde, and that maketh 13: of

which

which number I write the digit 3 vnder 8 and 4, and keepe the Article in my minde, thus:

848
187

34

Then come I to the third figures, saying, 1 and 8 make 9. and 1 in my minde maketh 10. Sir, shall I write the Cypher vnder 1. and 8?

Master. Yea.

Scholar. Then of 10 I write the Cypher vnder 1 and 8, and keepe the article in my minde.

Master. What needeth that, seeing there follow no more Figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last figures, I must write the Cypher vnder them, and the Article in a further place after the Cypher, thus:

848
186

1034

Master. So now you see, that of 848 and 186 added together, there amounteth 1034.

Scholar. Now I thinke I am perfect in Addition.

Master. That will I proue by this example. There are two armies of Souldiers: in the one are 106800. and in the other, 9400. How many are there in both armies, say you?

Scholar. First, I set them one ouer another, beginning with the first numbers of the right hand thus:

106800
9400

But the nether number will not match the ouer number.

Master:

Maſter. That forceth not.

Scholar. Then doe I adde 0. to 0. and there amounteth 0. that muſt I write vnder the firſt place thus:

106800	
9400	
0	

Maſter. Well ſaid.

Scholar. Then likewiſe in the ſecond place I adde 0. to 0. and there arriſeth 0. which I write vnder the ſecond place thus;

106800	
9400	
00	

Then I come to the third place, ſaying, 4 & 8, make 12. of which I write the digit 2, and kæp the article 1. in my mind, thus;

106800	
9400	
200	

Then I adde 9. to 6. which make 15. to that I adde the article 1. that was in my mind, and it is 16. I write 6. vnder 6. & 9. and kæpe one in minde thus.

106800	
9400	
6200	

Maſter. Why doe you not write both Figures, ſæing you are come to the laſt couple of numbers?

Scholar. Nay, reaſon ſheweth me, that I muſt adde that article that is in my minde vnto the next figure of the ouer ſumme, though there be no more in the nether ſumme.

Maſter. That is well conſidered: then doe ſo.

Scholar. Then ſay I, 0, in the ouer ſumme, and 1. in my minde, maketh 1: that write I vnder

vnder 0. Then followeth there yet one moze in the ouer ſumme, which hath none to be added to it, foz there is none in the nether ſumme, noz yet in my minde: therefoze I thinke I muſt wzite that euen as it is.

Maſter. Yea.

Scholar. Then doth my whole ſumme appeare thus.

Maſter. If you marke this you haue learned perfectly the common Addition of all ſummes which are of one denomination: ſo that ye obſerue this alſo, that in Addition you muſt haue two numbers at the leaſt: oz elſe how can you ſay, that you doe adde? And euer let the greateſt number be wzitten higheſt, foz that is the beſt way, though it be not neceſſary.

106800	
9400	
116200	

And forget not this, that (if you haue many numbers to adde together) you ſhall haue oftentimes an article of a greater value then 10. ſometimes 20. ſometimes 30. ſometimes moze, yea (peaduenture) 100. Therefoze as you did with the article 10. ſo doe with them, reſerving them in your minde, and adding to the number next following ſo many, as their valuer oz value certaine is: that is to ſay 2. foz 20. 3 foz 30. 5 foz 50. 10 foz 100. 12. foz 120. and ſo forth of other like. So that, if the article be 100 then muſt you ſet downe the 0. and keepe 10 in minde, to be carried to the next row of figures oz place, if any ſuch happen

happen to come. For your better vnderſtanding take this example for all:

I would adde theſe thirteene ſumms into one, which I ſet after this manner: then doe I begin and gather the ſumme of the firſt row of Figures, which come to 107 (for I take 9 there tenne times, and that is 90) then 9 and 8 is 17, that is in all 107, of which ſumme I write the 7 vnder the firſt row of figures, and then for that 100 is ten tens, I kéep ten in minde, which ten I muſt adde vnto the next row of figures, which are in the ſecond place:——— which ſecond row of figures (when they are added together with that tenne that I had in my minde) make in all 125. of which ſumme write the digit 5 vnder the ſecond row, and then (for that 120 containeth twelue tens) I kéepe twelue in minde to bée added to the third place or row of figures: which being added together, make in all 60 the cypher 0, I ſet downe vnder the row of figures in the third place.

4889	
4599	
2290	
3699	
2299	
4099	
1099	
3298	
299	
699	
499	
899	
389	

And the figure 6. I kéepe in minde to be added to the row of figures in the fourth place; which (when they are added together) make 29, The figure or digit 9 I ſet downe vnder the fourth place. And, becauſe it is my laſt worke, I ſet downe the 2. alſo that I haue in

my

my mind to the 9 in the fift place; so those summes doe make in all 29057.

4889
4599
2290.
3699
2299
4099
1099
3298
299
699
499
899
389
29057

¶ But (for your more ease in worke) when you haue an addition of so many summes to bee added together, you were best part that sum into two or three parts, and worke them seuerall, and so put their additions together, and this were the best things you could doe when ouer many summes fall to be added.

Scholar. This seemeth somewhat hard by the reason of so many numbers together.

Howbeit, I thinke (if I doe often proue, euen with this same example, either by working of it alone, or else by parting it as you said euen now) that I shall be able to doe so shortly with any other summe.

Master. So shall you. For it is often practise that maketh a man quicke and ripe in all thing; but because, aswell in great summes as in small, there may chance to bee some errour, I will teach you how you shall proue whether you haue done well or no.

Scholar. That were a great helpe and ease.

The proofe of Addition.

Master. Begin first with the highest number, and then to all the other orderly, & adde them together, not hauing regard to their places, but as though they were all vnites: and

and ſtill (as your number increaſeth aboue 9) caſt away 9. Then goe forth, euer caſting away 9 as often as it amounteth thereto: and ſo doe till you haue gone ouer all the numbers that you intended firſt to adde; and whatſoeuer remaineth after ſuch addition and caſtings away of 9. write it in ſome void place by the end of a line, for the better remembrance: and thus is the firſt part of your worke proued. Then ſecondly put together the figures that reſult of the addition vnder the line, ſtill caſting away 9 alſo. And then that that remaineth write at the other end of that line; and if thoſe two figures be like, then haue you well done; but if they be vnlike, then haue you miſſed. As for example, in this preſent ſumme. The firſt figure of the ouer line is 9. let him goe, then 8. and 8. is 16. take away 9. there reſteth 7. and adde that 7. to 4. that followeth, and it maketh 11. from which if you take 9. there reſteth 2. Then come to the next row, whoſe firſt and ſecond numbers are 9. therefore ouerpaſſe them both, and take the 5. to the 2. which did remaine in the firſt row, that maketh 7. put thereto the 4. following, and that maketh 11. thence take 9, and there remaineth 2. Next vnto that, goe to the third line, whoſe two firſt numbers you may let paſſe becauſe they are nines: then take the two figures of 2. which (with the other two that remained, in the ſecond row) make 6. Then goe to the fourth row, whoſe two firſt

numbers

numbers let goe; and take the 6. to the 6, that remained, and that maketh 12. take away 9. and there resteth 3. which with the 3. that is next, maketh 6. And so goe through all the other numbers, and you shall finde that there remaineth 5, after you haue cast away 9, as often as you can finde it: therefore write 5 at the end of the line in a void place, thus:

5———— ——

Then gather all the figures of the totall summe, which is vnder the lowest line, and cast away 9. as often as you can finde it: as thus: 7. and 5. make 12. take away 9. there resteth 3. to that if you adde the 2. that is last, (for you may omit the 9) then doth it make 5. which 5. you must write at the other end of the line that you made in the void place, thus:

5———— ——5

And then you see that those two figures be like, whereby you may know that you haue done well, and so you may proue in any other.

Scholar. (If it please you) I will proue in another summe.

Master. With a good will.

Scholar. Then will I take one of your former examples, which was this.

First in the highest line 8 and 6 make 14 then 9. taken away, there remaine 5. to which I adde the 1 that followeth, & that maketh 6 Then came I to the second line, where I finde first 4.

	106800
	9400
	116200

which

which with 6 maketh 10. from that I take 9 and there resteth 1. the next figure is 9. and therefore I let him alone, so finde I 1 remaining which I set at the end of a line, thus.

1———

Then I come to the totall summe, and there I find that all the figures put together, make ten, from which I take nine, and there resteth 1. also, which I put at the other end of the line thus. 1———1

And because they be like, I know that I haue well added.

Master. So, you know now both how to adde two summes or more together, and also how to proue whether you haue done well or no; and now I will teach you how to adde summes of diuers Denominations together: which thing can neuer bee but when the one Denomination is such that it containeth the other certaine times. And yet you shall adde them to the other, not after this sort (as you did them that were of one denomination) but after such a sort as I will now shew you, that is to say:

Addition of numbers of diuers denominations.

If you haue a summe of diuers denominations, then loke that you set euery denomination by himselfe, with some note or figure of his denomination, as they are wont to be written. Then write your other summes so vnder that first, that euery one bee set vnder the other of the same denominations: As for example, if your denominations bee

pounds,

pounds, shillings, and pence, wyte pounds vnder pounds, shillings vnder shillings, and pence vnder pence: and not shillings vnder pence, nor pence vnder pounds.

Scholar. Now that you haue spoken it, me thinketh it needeth not to warne me of it, for it were against reason so to confound sums: but yet if you had not spoken of it, peraduenture I should haue bin deceiued in it.

Master. If you doe say it is plaine, I will speake no more of it, but with an example make the matter to appeare euidently.

First one man oweth me 22 l. 6. s. 8 d, another oweth me 5 l. 16. s. 6 d. and another oweth me 4 l. 3 s. I would know what this is altogether: Therefore must I first set downe my great summe, and then the other, euery one vnder his Denomination agree ing to the greatest summe, as here you see with a line vnder them.

li	s	d
22	6	8
5	16	6
4	3	0

Then must I begin at the smallest numbers (which must alwaies be set next to the right) and adde them together: and (if the sum will make 1 or 2 or 3 of the next denomination) then must I keepe it in my mind till I come to that place, and vnder that first place must I note the residue (if there remaine any of the same denomination:) but, if there remaine none, then need I to wryte vnder it nothing. And this is all that you must marke in this Addition: for all other things are like to the

manner of Addition before mentioned. *Therefore, the chiefest point of this Addition is, to know the values of Common Coines and rated sums.* As how many shillings be in a pound: how many pence in a shilling: of which (and of other like things) I will instruct you hereafter in teaching of Reduction. But now I may not daine be your wit from the thing that we are about.

Therefore let vs returne to that former example which I propose of three Debtors: which summes when I had set orderly, they stood thus with a line vnder them.

Then to adde them into one summe I must begin at the right hand where the smallest denomination is and add them together, first saying, 6 and 8 make 14 Now, seeing these 14 are pence, which containe one shilling and 2. pence: the 2 pence I set downe vnder the like of pence: and the one shilling I keepe in my minde to carry to the next row beeing the place of shillings.

li	s	d.
22	6	8
5	16	6
4	3	0
		2

Then doe I adde the shillings together, saying, 1. in my minde and 3 make 4 and 6 make 10 and 6 make 16 and 1 in the second place, which standeth for 10 make 26 which is 1 pound 6 s. The 6 s. I set downe vnder the place of shillings, as appeareth in the example. And the 1 pound I keepe to carry to the pounds.

li	s	d.
22	6	8
5	16	6
4	3	0
6		2

Then

Then come I to the pounds, adding them all together saying, 1 that I kæpe and 4 make 5 and 5 make 10 and 2 make 12 The figure or digit 2 I set downe right vnder that place or row of pounds where I gather them, and the article 1 I kæpe to carry to the nert place, saying, 1. in minde and 2 is 3 which 3 I set downe directly vnder the 2. And then appeareth my whole summe thus.

li	s	d
22	6	8
5	16	6
4	3	0
32	6	2

And thus must you doe with any such like sums whatsoeuer, whether they be money, weight or measure, which (if you practise diuers summes) you shall be well acquainted with the feat of Addition.

But now can you tell how to proue this Addition, or such other like of diuers Denominations and to trie whether you haue well done or no?

Scholar. I would I could.

Master. That shall you doe by this meanes: You must make a Crosse which shall haue so many lines as you haue sundry Denominations in your Addition: As if you haue but two Denominations then you may make it thus: that the ouer part and nether part may serue for one Denomination. And if you haue three Denominations (as pounds, shillings, and pence) then must you make

Proofe of Addition of diuers denominations.

three lines, thus: The vpright line may serue for pounds, and the highest thwart line for shillings, & the lowest for pence as for example the summe which we last wrought.

li	s	d
22	6	8
5	16	6
4	3	0

	5	
6		6
2		2
	5	

For the proofe of the which, because it containeth three denominations, I must make a crosse of 3. lines, as in the example before. Then I reckon first at the right hand, the pence: 6 and 8. make 14. from which I take 12. for the next Denomination that is to say, a shilling, and there resteth, 2. which I must write at one end of the nether thwart line.

After that, I gather the summe of the shillings, 3.16.6. which maketh 25. to whom I put 1. that I toke of the pence, & that maketh 26. from those I take 20. the quantity of the next greater Denomination, that is to say, a pound, and there resteth 6. which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds 4.5. and 2. which make 11. to them I adde the one that came of the shillings, and they make 12. from whence I cast 9. and there resteth 3. That 3. I ioyne to the 2. in the next place, and they make 5. which 5. I set at the Crosse

also

also. And thus is my first part of my worke proued.

That done, I come to the totall summe vnder the line, and examine it, beginning at the pence, where I find but 2. and cannot take 9. from him: therefore, I set him at the other end of the nether thwart line: then I come to the shillings, where I find onely 6 which (because it is lesse then nine) I set it at the other end of the line of shillings, that is, the ouermost thwart line.

Last of all, of the 32. li. I take thrée times 9. which is 27. and there remaineth 5. which I write vnder the vpright line: either else I may reckon them simply without any respect of their valuation or place: saying, 2 and 3 make 5 which, (because it is lesse then nine) I set vnder the vpright line as before. Then I consider euery number, comparing it to the number, that is against it: and because I find them to be euery one like his match, I know that I haue well done.

Scholar. This crosse I perceiued doth serue for these 3 denominations, pounds, shillings, pence: but what if I had l.s̃.d.ob.q ?

Master. These lines (as I haue said) doe serue for thrée denominations, such as they be, as here thrée doe serue for pounds, shillings, and pence: but if you haue no pounds in your sum, then may they serue for shillings, pence, and halfe pennies: yea, for d. ob. and q. or in weight for C.q; q l or, in measure, for Elles,

Quarters, and Nailes, if you haue no greater Denomination: so that you remember that the vpright line serueth for the greatest Denomination, and the highest thwart line for the next, and the lowest for the least.

And so, if you haue foure Denominations, you must make your crosse with so many lines. And (if your summe be of more denominations) make so many lines in your Crosse. And thus will I make an end of Addition, saying that here (for the better vnderstãding of this Rule) I haue set you down certain examples both of money, weight, and measures, with their workes and proofes.

Examples of Addition.

li	s	d
23	10	4
45	6	8
37	2	9
15	13	6
121	13	3

li	s	d
130	17	10
28	6	8
13	13	4
120	0	0
292	17	10

The proofes.

C.

C.	q;	li
24	1	3
12	2	2
7	3	4
13	0	13
57	2	22

yards,	q.	nayles.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3

3
2 — 2
4 — 4
3

8
1 — 1
3 — 3
8

Subtraction:

Subtraction.

Scholar.

Hen haue I learned the two firſt kinds of Arithmeticke: now (as I remember) doth follow Subtraction, whoſe name (me thinketh) doth ſound contrary to Addition.

Maſter. So it is indéed: for, as Addition increaſeth one groſſe ſumme, by bringing many into one: ſo contrary wiſe Subtraction diminiſheth a groſſe ſumme by withdrawing of other from it. *Sothat Subtraction or Rebating is nothing elſe but an Art to withdraw and abate one ſumme from another, that the remainer may appeare.*

Subtraction.

Scholar. What doe you call the remainer?

Maſter. That you may perceiue by the name.

Scholar. So me thinketh: but yet it is good to aſke the truth of all ſuch things, leſt in truſting to mine owne coniecture, I be deceiued.

Maſter. So it is the ſureſt way. And (as I ſée cauſe) I will ſtill declare things vnto you ſo plainely, that you ſhall not néed to doubt. Howbeit, if I doe ouer paſſe it ſometimes (as the manner of men is to forget the ſmall knowledge of them to whom they ſpeake) then doe you put mee in remembrance your ſelfe,

selfe, and that way is surest.

And, as for this word that you last asked mée, take you this discription: The remainer is a summe left after one Subtraction made, which declareth the excesse or difference of the other two numbers: as if I would abate or subtract 14. out of 18. there should remaine 4. which is called the Remainer, and is the difference betwéene those two numbers 14. and 18. Remainer.

Scholar. I perceiue then what Subtraction is: Now resteth to know the order to worke it.

Master. That shall you doe by this meanes. First, you must consider, that if you should goe about to rebate, you must haue two sundry sums proposed: the first, which is your grosse summe (or summe totall:) and it must be set highest: and then the rebatement (or summe to be withdrawn) which must be set vnder the first, (whether it be in one parcell or in many) and that in such sort, that the first figures be one iust ouer another, and so the second, and third, and all other following as you did in Addition: then shall you draw vnder them a line, and so are your sums duely set to begin your working.

Then begin you at the right hand (as you did in Addition) and withdraw the neather number out of the higher, and if there remaine any thing, write that right vnder them beneath the line: and if there remaine nothing

(by

(by reason that the two figures were equall) then write vnder them a Cypher of nought. And so doe you with all the other Figures, euermore abating the lower out of the higher, and write vnder them the Remainer still, till you come to the end. And so will there appeare vnder the line what remaineth of your grosse summe, after you haue deducted the other sum from it, as in this example.

I receiued of your Father 48 s. of which I haue laid out for you 36 s. now would I know what doth remaine: And therefore I set my number thus in order. First, I write the greatest summe, and vnder him the lesser, so that the Figures at the right side bee euen one vnder another, and so the other, thus.

48
36

Then do I rebate 6 out of 8. and there resteth two, which I write vnder them right beneath the line thus.

48
36
2

Then I goe to the second figures, & doe rebate 3 out of 4 where there remaineth 1 which I write vnder them right, and then the whole summe & operation appeareth thus.

48
36
12

Whereby it appeareth, that if I withdraw 36 out of 48 there remaineth 12.

Scholar.

Scholar. Now will I proue in a greater summe, and I will subtract 2367924. out of 3468946. those sums I set in order thus

3468946	
2367924	

Then doe I begin at the right side, and deduct 4 out of 6 and there resteth 2 which I write vnder them. Then go I to the second figures, and withdraw 2 out of 4 & there remaineth 2. which I set vnder them also then I take 9 out of 9 and there resteth 0. which I write vnder them (for you say that if the Figures be equall; so that nothing doe remaine, I must write this Cypher 0 vnder them.)

Master. It was well remembred: now goe forth.

Scholar. Then I come to the fourth place, and draw 7 out of 8 and there remaineth 1. which I write vnder them also: Then in the fift place I take 6 out of 6 and the resteth 0. (for it I write vnder them the Cypher 0.) Then in the first place 3 rebated from 4. there remaineth 1. which I write vnder them, and likewise in the seuenth and the last place 2 taken from 3. there is left 1.

Which I write vnder them: so haue I done my whole working, and my summes doe appeare thus. Whereby I see, that (if I doe rebate 2367924. out of 3468946.) there remaineth 1101022.

3468946	
2367924	
1101022	

Master. This is well done. And that you may bee sure to perceiue fully the Art of Subtraction,

traction, let mæ sée how you can subtract 52984732. out of 8250003456.

Scholar. First, I set downe the greatest summe, and after that, I wzite vnder it the lesser number, beginning at the right side, and then my figures will stand thus.

8250003456
52984732

Then take I 2 from 6. and the rest is 4. which I wzite vnder them: then doe I withdzaw 3. from 5. and there remaines 2. which I wzite vnder them. Then take I 7. out of 4. but that I cannot, what shal I now doe?

Note. Master. Marke well what I shall tell you now, how you shall doe in this case, and in all other the like: if any figure of the nether summe bæ greater then the figure of the summe that is ouer him (so that it cannot be taken out of the figure ouer him) then must you put 10. to the ouer figure, and then consider how much it is, and out of that whole summe withdzaw the nether figure, and wzite the rest vnder them. Can you remember this?

Scholar. Yes, that I trust I shall. Now then in mine example where I should haue taken 7. out of 4. and could not, I put 10. to that 4 which maketh 14. from it I take away 7. and there resteth 7. also, which I wzite vnder them.

Master. So haue you done wel, but now must you marke another thing also: that

(whensoeuer

whensoeuer you doe so put ten to any figure of the ouer number) you must adde one still the figure or place that followeth next in the nether line: as in this example there followeth 4. to which you must put 1. and make him 5. and then goe on as I haue taught you.

8	2 5 0 0 0 3 4 5 6
	5 2 9 8 4 7 3 2
	0 1 8 7 3 2

Scholar. Then shall I say, 4. and 1. (which I must put him for the 10. that I added to 4. before) make 5. which I should take out of 3. but that cannot be: therefore I must put to it also 10 and then it will be 13. from which I take 5. and there resteth 8. to be written vnder them: and because of that 10. added to the 3. I must adde one to 8. that followeth in the nether line, & that maketh 9. which I should take out of 0 & cannot: therefore I put thereto 10. & that maketh 10 from 10. I take 9. and there remaines 1. which I write vnder them.

Thus doe I adde 1. likewise to the next figure beneath, which is 9. and that maketh 10. that 10. should I take out of the figure aboue, but I cannot: for it is 0. therefore I put 10 to it, and so take I 10. out of 10. and there resteth 0 to be written vnder them.

Then come I to the next figure, which is 2 and to him I doe adde 1 which maketh 3. that 3 I cannot take out of nought: therefore of that nought I make 10. and thence doe take 3. so there remaineth 7 to be written vnder them: likewise doe I put 1 to 5

that

that 5. & make it 15. from which I rebate 6, and there remaineth 9, which I write vnder them. Now haue I spent all the nether figures, and what shall I doe more.

825000	3456
52	98473 2
819701	8724

Master. You should haue added one to the next figure following (if there had beene any) because you added 10. to the last figure before of the ouer line : but seeing there is no figure following, you must adde that one to the place following. and then deduct that one from the number aboue.

Scholar. Then shall I say, because I borrowed 10 to the ouer 5. I must put 1. in the next place beneath, that is vnder 2. then must I subtract that 1 from 2. and there resteth, 1, to be written vnder that in the ninth place. Now I haue no more to subtract, for there is not any figure remaining beneath, neither yet any Vnite to be added, because I borrowed not 10 to the figure last before: and yet is there 8. remaining in the ouer line, which I thinke (by reason) should be set at the end of the figures in the lowest row, which is vnder the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so indeed.

Scholar. But Sir, I beseech you, shall I alwayes when any number is remaineth alone, as this 8 did, write him vnder the line straight

ſtraight againſt his owne place?

Maſter. Yea, what elſe? Whether they be one or many: and this well remembred, you haue ſufficiently learned Subtraction: Howbeit, becauſe of certaine things that might deceiue you, if you did not take good hēd to your working. I will propoſe to you another example of many numbers to be ſubtracted, as thus. I receiued of a friend of mine to kēpe 2869 crownes, of which at one time I deliuered him againe 500 at another time 368 at another time 440 at another time 80 & another time 64 now would I know how many doe reſt behind? Therefore firſt I ſet downe my groſſe ſumme and vnderneath it I ſet all the parcels thus, and vnder them a double line.

2869 *Crownsreceiued*

500
368
440
80
64
} *Deliuered.*

Then firſt I being at the firſt place I gather together the ſum of all thoſe lines (ſaue the ouermoſt) in their firſt figures: and ſo I doe with all the figures of the ſecond place, and ſo forth, as I did in Addition, ſaue that I leaue out the higheſt row of numbers (as the line warneth mee) and that ſumme ſo gathered betweene the double line, is the ſumme deliuered in all: which ſumme I doe afterwards ſubtract out of the higheſt row of numbers, and the remainer doe I ſet vnder the nethermoſt line:

Note.

as

as for example.

I set the summes as before: then doe I gather the first figures of all the places deliuered, together: where I find but 4. and 8. that maketh 12. (for three Cyphers increase no summe in Addition, as you learned before:) of the 12. therefore, doe I write the Digit 2. betweene the double line, and keepe the Article in my minde, till I come to the second place, where I finde 6. 8. 4. 6. that maketh 24. to them I put the Article in my minde, and it is 25. of which I write 5. vnder the second place, and keepe the Digit 2. in my mind for the third place, where I find 4. 3. 5. that makes 12. to the which I adde the 2. in my minde, and it maketh 14. whereof I write the 4. vnder the third place, & because there remaines no mo figures to be added, I write the Digit in the fourth place, as you see in the example: and so it appeareth, I haue deliuered in all, a thousand foure hundred fifty two crownes.

2869	*Crownes receiued*
500	*Deliuered.*
368	
440	
80	
64	
1452	*Deliuered in all.*
1417	*Rest behind.*

Then come I to the subtracting of this suume betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I proceed to subtract it from the ouermost summe, saying, 2 from 9, remaine 7, to be written vnder them beneath the lowest line.

line. Then in the second place I take 5 from 6, and there resteth 1 to be written vnder them. Then in the third place, 4 from 8, resteth 4. Last of all, in the fourth place, 1 from 2, remaineth 1. And thus I see that after those fiue summes are subtracted from 2869 the Remainer is 1417.

Scholar. This I perceiue: but is there no shorter way and more speedy?

Master. Yea, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest summe. But, if in quantity those numbers added together, exceede the highest summe or vpper number, then shall you (as before hath beene taught you) imagin to borrow 10, 20, or 30 more, as need shall require, and put them to the vpper number, to helpe to further the abatement, referring or restoring the Articles that you borrowed to the next place againe: & so still go forward till you haue ended your worke: as for example. In the last summe proposed, I gather first in the first place 4 and 8 that maketh 12, which 12 I should deduct or take out of 9 in the vpper number aboue the line, but I cannot: therefore I adde vnto 9 an article of 10 and that maketh the vpper number 19 from when I take 12, then there resteth 7; then for the Article 10 I adde to the next place of mony deliuered; saying, 1 that I bring and 6 make 7, & 8 make 15, & 4 make 19 and 6 make 25.

An abridgment of the former manner of Subtraction.

which 25 I should take out of 6 in the vpper number, but I cannot. Therefore I adde 2 tens or 20 vnto 6 in the vpper number, and that maketh 26, then 25 out of 26, resteth 1; then the tens which I borrowed, or haue in mind. I adde to the next row or sum deliuered; saying, 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14; then 14 out of 8 I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to be added, the one that I borrowed, or haue in mind, I rebate from 2 in the vpper line, and there remaineth 1, which I set downe in the remainer line; and so my summe appeareth (as before) to be 1417 Crownes.

Loe thus haue you now a shorter way.

Scholar. I like both waies wel, and I perceiue both well: yet, as in the working séemeth somewhat long, so in the other it leaueth very much (me séemeth) to remembrance, and therefore may cause errour quickly, except a man haue a quicke and an exercised remembrance. But yet for the sharpning of my wit by your patience (if you will giue me leaue) I will try what I can doe in a like summe, to worke it the shortest way: whereupon I would subtract out of 40301964. these three parcels.

Therefore

Therefore I set them first in due order: then I gather the parcels of the first place, which are 8.2 1. that is a 11, which I should take or deduct out of 4 which is ouer him, but I cannot: therefore I adde an article or one ten to 4 which maketh 14, then 11 out of 14, there resteth 3 to be writ-ten vnder the first place betweene the two lines.

40301964	*Charge.*
20003428	*Disch.*
10002432	
10101461	
43	

Then come I to the second place, saying, 1 that I borrowed to haue in my mind, & 6 make 7, & 3 make 10, and 2 make 12. which I cannot take from 6, therefore I adde 10 to 6 which maketh 16, & then 12 from 16 resteth 4 which I write vnder the second place betweene the two lines.

Then come I to the third place, saying, 1 that I borrowed or haue in mind and 4 make 5, & 4 is 9 & 4 make 13, which I should take out of 9 that is ouer them, but I cãnot: therfore I adde 10 to 9 which make 19, then 13 out of 19, rest 6.

Then come I to the fourth place, saying, 1, in minde and 1 is 2 and 2 is 4, and 3 make 7, which because it cannot be taken from 1, I take it from 11, and there resteth 4.

After that, I come to the fift place, where are onely three Ciphers, which make nothing,

vnto which I adde 1 in mind, then should I take that (that is to say) 1 from the figure ouer them, which is also a Cipher: therefore I say thus, I cannot take 1 from 0, but 1 from 10 remaineth 9: so must I write 9 vnder them. Then in the first place I find but 1, and 1 in mind make 2 which I take out of 3 ouer him, and the remainer is 1: that must be written betweene the two lines in the first place. So I goe to the seuenth place, where I find only Ciphers, & in the grosse summe ouer them a Cipher also: therefore must I write the remainer (which is nothing) with a Cipher also. Then in the eight & last place, I gather 1, 1, 2, that maketh 4, which if I take out of that 4 that is ouer them, there will nothing remaine. And that must be noted with a Cipher betweene the two lines (as I haue often said) and so haue I ended my worke, and the figures stand followeth.

But Sir, I remember you taught mee that Ciphers should not come in the last place, for because they serue onely to increase the value of other figures which follow them, and serue not those figures that goe before them: and now in my Example I haue set two Cyphers in the two last places.

Master I commend you for your remembrance. And truth it is, you should not haue set them here, but onely because that I would make you plainly to perceiue the art of Subtraction.

traction. Therefore seeing that you doe now perceiue it, whensoeuer you would write downe a Cypher, looke whether any other figures be yet behind: and if not, then let goe the o also, for it needeth not to write him in the latter places, where no other figure doth follow, except it be (as I did now suffer you) to teach the vse of Subtraction the plainer.

Therefore your figures must stand thus when the worke is ended.

```
40301964 Charge.
--------
20003428 }
10002432 } Disch.
10101461 }
--------
194643 Rest
--------
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Scholar. Sir, I doe thinke with that that you taught me before, and by these two summes that you taught mee last also, that now I could subtract any summe.

Master. So may you, if you haue marked what I haue taught you. But, because this thing (as all other) must be learned surely by often practise, I will propound here two examples to you: wherein if you often exercise your selfe, you shall be ripe and perfect to subtract any other summe lightly, for in them is contained all the obseruances of whole numbers. And because you shall perceiue somewhat both how to doe it, and also whether it be well done when you haue proued to doe it; therefore haue I written vnder them both, the Remainers.

30606.	*Lent.*		308964.	*Debt.*
10354	}		103145	}
10249	}	*Paid.*	102597	} *Paid.*
163	}		101024	}
20766.		*Paid in all.*	02198	*Reſt.*
9840		*Reſt to pay.*		

Scholar. Sir, I thanke you: but I thinke I might the better doe it, if you did ſhew me the working of it.

Maſter. Yea, but you muſt proue your ſelfe to doe ſome things without my aid, or elſe you ſhall not be able to doe any more then you are taught: And that were rather to learne by rote (as they call it) then by reaſon. And againe, there is nothing in theſe examples, or any other of whole numbers, but I haue taught you the rules of them already.

Scholar. Then I truſt by practiſe to attaine the vſe of it. And is this all that I ſhall learne of Subtraction?

Maſter. Yea, ſauing that (as you haue ſéene in Addition) there are numbers of diuers Denominations, in which the working is not much vnlike: yet (without ſome inſtructions be giuen of it) it might ſéeme to a learner more difficult then indéede it is. Therefore I will briefly ſhew you the vſe of it onely by one example or two.

A certaine man owed to me: 14 l, 12 ſ, 8 d;
of which he paid me at one time 4 l, 6 ſ, 8 d,
at another time 3 l, *at another* 2 l, 3 ſ, 4 d,
and laſt of all 6 ſ, 8 d:

Now would I know what remaineth vnpaid yet: therefore I ſet my ſummes thus, euery one in their due place: As pounds vnder pounds, ſhillings vnder ſhillings, pence vnder pence.

li	ſ	d
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholar. Sir, I pray you why doe you write 2 ℔ for the common ſpeech vſeth rather to ſay 40 s.

Maſter. We muſt here vſe the Denomination that is greateſt in any ſumme, ſo that wee may not write according as wee vſe to ſpeake, ſaying, 16 d, 18 d, or likewiſe 7 groats, 8 groats, 24 s, 40 s, 48 s and ſuch other: but we muſt write euery Denomination that is in any ſumme by it ſelfe. Note how the penne differeth from the common order of Counters.

Namely, ſhillings and pounds. So muſt wee write for the laſt ſummes now named, 1 s, 4 d; 1 s; 6 d; 2 s, 4 d; 2 s, 8 d; 1 ℔, 4 s; 2 ℔, 8 s, and ſo forth of other like.

Scholar. So that we may not write in Arithmeticke, pence, when the ſumme amounteth to ſhillings, nor ſhillings, when the ſumme maketh pounds. Now, (if it pleaſe you) end your example.

Maſter. When my ſummes are ſo ſet as I ſhewed, then (according to the rules of Addition) I gather all the particular ſummes which bee paid mee into one totall ſumme, directly to bee ſet vnder them betweene the two lines, not medling with the 14 l, 12 s, 8 d, as the line warneth mee: therefore muſt I begin with the ſmalleſt Denomination, ſaying, 8, 4, 8, is 20, pence, which maketh one ſhilling and 8 pence, the 8 d I ſet downe vnder the place of pence, and the one ſhilling I keepe in mind to carry to the next denomination of ſhillings. Then come I to the ſhillings, and ſay, one that I bring or haue in minde, and 6 is 7, and 3 is 10, and 6 makes 16, which, becauſe it containeth not one pound, I ſet directly vnder the place of ſhillings. Then come I to the pounds, whoſe parcels are 2, 3, 4, that is in all 9, that 9 doe I ſet downe directly vnder the pounds: And ſo the totall or whole Addition of all the particulars paid, amounteth to 9 l. 16 s. 8 d.

li	ſ	d	
14	12	8	
4	6	8	} 2
3	0	0	
2	3	4	
		6	8 }
9	16	8	
4	16	0	*Reſt.*

Now, for the worke of Subtraction, I muſt rebate that totall ſumme of Addition out of the highеſt number, that is to ſay, from

the

the 14. l. 12 s. 8 d.

Therefore to performe the worke, I say, 8 d. out of 8 d. remaineth or resteth nothing, therefore in the place of the rest or remaine, right vnder the denomination, I set downe 0. Then comming to the shillings, where I find 16. which should bee taken out of 12, but I cannot: therefore I imagine to borrow 1 of the next Denomination, that is, of the 14 l, and put that one pound so borrowed vnto 12 s, that maketh 32 s.

Now 16 s, out of 32 s resteth 16 s, which 16 s I set downe directly vnder the place of of the rest.

Lastly, comming to the pounds, saying, one pound in mind that I borrowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest or remaine, appeare to be 4 l. 16 s. 0 d.

This I account the easiest way for a young beginner to practise, though it be something long.

Scholar. Is there any shorter way for this worke also?

Master. Yes, as in this last example I will also shew you, for you may adde together the particular summes as

they

Maſter. When my ſummes are ſo ſet as I ſhewed, then (according to the rules of Addition) I gather all the particular ſummes which bee paid mee into one totall ſumme, directly to bee ſet vnder them betweene the two lines, not medling with the 14 l, 12 s, 8 d, as the line warneth mee: therefore muſt I begin with the ſmalleſt Denomination, ſaying, 8, 4, 8, is 20, pence, which maketh one ſhilling and 8 pence, the 8 d I ſet downe vnder the place of pence, and the one ſhilling I keepe in mind to carry to the next denomination of ſhillings. Then come I to the ſhillings, and ſay, one that I bring or haue in minde, and 6 is 7, and 3 is 10, and 6 makes 16, which, becauſe it containeth not one pound, I ſet directly vnder the place of ſhillings. Then come I to the pounds, whoſe parcels are 2, 3, 4, that is in all 9, that 9 doe I ſet downe directly vnder the pounds: And ſo the totall or whole Addition of all the particulars paid, amounteth to 9 l. 16 s. 8 d.

li	ſ	d	
14	12	8	
4	6	8	1
3	0	0	1
2	3	4	1
	6	8	1
9	16	8	
4	16	0	*Reſt.*

Now, for the worke of Subtraction, I muſt rebate that totall ſumme of Addition out of the higheſt number, that is to ſay, from

the 14. l. 12 s. 8 d.

Therefore to performe the worke, I say, 8 d. out of 8 d. remaineth or resteth nothing, therefore in the place of the rest or remaine, right vnder the denomination, I set downe 0. Then comming to the shillings, where I find 16. which should bee taken out of 12, but I cannot: therefore I imagine to borrow 1 of the next Denomination, that is, of the 14 l, and put that one pound so borrowed vnto 12 s, that maketh 32 s.

Now 16 s, out of 32 s resteth 16 s, which 16 s I set downe directly vnder the place of of the rest.

Lastly, comming to the pounds, saying, one pound in mind that I borrowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest or remaine, appeare to be 4 l. 16 s. 0 d.

This I account the easiest way for a young beginner to practise, though it be something long.

Scholar. Is there any shorter way for this worke also?

Master. Yes, as in this last example I will also shew you, for you may adde together the particular summes as

li	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

they are set in order, beginning with the pence, saying, 8,4 8, make 20 d. which 20 d. you should take out of the 8 d. aboue the line, but you cannot, therefore shall you borrow 1 of the next denomination, that is to say 1 of the shillings, & put it to the 8 d. that maketh 20 d, now 20, out o 20 d resteth o which Cypher I set downe directly vnder them.

Then one shilling that I borrowed or had in mind, and 6 make 7. and 3 make 10, and 6 make 16 the 16 out of 12 I cannot take, therfore of the next Denomination I doe borrow one l, and put it to 12 s. which maketh 32s, then 16 s, out of 32 s resteth 16 s.

Lastly, I came to the pounds, saying, 1 l in minde, or that I borrowed, and 2 make 3. and 3 is 6 and 4 is 10, then 10 out of 14 there resteth 4.

So doth my remainer or rest appeare as before to be 4 l, 16s, 0d.

Scholar. This doe I perceiue very well, and if there bée none other things to bee learned in Subtraction, then may I come to Multiplication, for that you reckoned to be next in order.

Master. We haue done indéed with the Art of Subtraction, as touching the working But yet before we goe to Multiplication, I will

will instruct you how to examine your work, whether it be well done or not. For the performance whereof, if you marke what I said in Addition, you may easily perceiue what is to be done for the proofe of Subtraction, which is best made by the aid of Addition thus. Proofe of Subtraction.

Draw vnder the lowest number (which is your Remainer) a line, and then adde this Remainer and all the other that you did substract before, together, and write that that amounteth vnder the lower line: and if the summe that commeth thereof, be equall to the highest of the Substraction, then is the Substraction well wrought, or else not. As you may see for example in the summes set downe before, and first in summes of one Denomination, whereof one was this.

Where the number	8 2 5 0 0 0 3 4 5 6
52984732. is substracted	5 2 9 8 4 7 2 4
from 8250003456. the	
Remainer is 8197018724.	8 1 9 7 0 1 8 7 2 4

Now to proue whether it be truely wrought or not. I adde the Remainer and the number Substracted, together, beginning at the right hand; and first I say 4 and 2 is 6 which is set vnder the line. Example in a summe of one denomination.

The number giuen	8 2 5 0 0 0 3 4 5 6.
The number to Substract	5 2 9 8 4 7 3 2
The Remainer	8 1 9 7 0 1 8 7 2 4
The Proofe.	8 2 5 0 0 0 3 4 5 6

Then againe in the second place I say 2 and 3 is 5, which I write vnder, next that in the third

third place, 7 and 7 are 14 of which I write the Digit 4, and keepe the Article 1. in my minde. Then in the fourth place 8 and 4 is 12 and 1. in my minde maketh 13, whereof I write downe the Digit 3 and keepe the Article 1. in my minde. Againe in the fift place, 1 and 8 is 9, and 1 in my minde is 10. Whereof I set downe 0 and keepe the 1. in my minde. And so going on to the rest (as it is taught in Addition) when I haue made an end, I see that the lowest line of numbers & the highest be alike: wherefore I know that I haue well done.

So likewise the proofe is to be made in numbers of diuerse Denominations: as for example in our summe of that kinde which in the first forme of working, stoode thus; (all the particulars number to bee subtracted, being drawne into one.)

Example in a summe of diuerse denominations.

Where, in the title of pence, I find 8 and 0; the 8 I set downe directly vnder in that place of pence.

Then in the place of shillings I find 16 and 16 which make 32 shillings, wherein is contained 1 l. and 12 s, the 12s, I set downe directly vnder them in the due place

li	s	d
14	12	8
4	6	8
3	0	4
2	3	0
	6	8
Paid in all. 9	16	8
Rest.	4	16 0
Proofe	14	12 8

of

of ſhillings and one pound I kéepe.

Then comming to the pounds, I ſay, I that I kéepe, and 4 is 5, and 9 is 14, which 14 in oue order I ſet downe directly vnder them as this figure ſheweth. And the whole ſumme is 14 l. 12s, 8d, agréeing with the vpper number aboue. So I find the worke is good, and the Subtraction well wrought.

The ſame thing is to be done for the latter forme of Subtraction, (where the particular ſummes are not gathered together into one groſſe.) For the Remainer and all the particular ſummes ſubtracted, being added together if the ſumme that commeth thereof bée equall to the higheſt number aboue, then is the Subtraction well wrought, or elſe not.

As for example alſo in the laſt ſumms which ſtood thus.

First, in the title of pence, I adde 8 4. 8, that maketh 20 d, which containeth one ſhilling and 8 pence.

The 8 I ſet downe vnder the loweſt line in the row of title of pence, and that one ſhilling I kéepe to carry the next Denomination or place of ſhillings.

li	ſ	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

Example of a proofe in the latter forme of Subtraction.

Then returning to the ſhillings, ſaying: one

one in minde, or that I kéepe, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 ſhillings, which amounteth to one pound, 12 s, the 12 s I ſet downe vnder the title of ſhillings, and 1 pound I kéepe or haue in minde to carry to the next Denomination or place of pounds. Then come I to the pounds, ſaying, 1 that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14, then doe I write 14 vnder the pounds, and ſo haue I ended the Addition: and I ſée that the loweſt line is like vnto the vppermoſt line in number, wherefore I know that I haue well done.

And thus haue I taught you the Art of Subtraction, and the meanes to proue whether it bee well wrought or not. Therefore now will I make an end thereof, and will inſtruct you in Multiplication.

Multiplication.

Multiplication.

Multiplication is an operation whereby two ſummes produce the third: which third ſumme ſo many times ſhall containe the firſt, as there are Vnites in the ſecond. And it ſerueth in ſteade of many Additions. As for example. When I would know how many are 30 times 48. if I ſhould adde 48 thirty times, it would bee a long worke. Therefore was this worke of Multiplication deuiſed, which ſhall doe that at once that Addition ſhould doe at many times.

Multiplication what it is.

Scholar. I perceiue the commodity of it partly, but I ſhall not ſee the full profit of it till I know the whole vſe of it. Therefore Sir, I beſeech you, teach mee the working of it.

Maſter. So I iudge it beſt, but becauſe that great ſummes cannot be multiplied, but by the Multiplication of Digits, therefore I thinke it beſt to ſhew you the way of multiplying them As when I ſay, 9 times 8 or 8 times 9 &c. And as for the ſmall Digits, vnder 5 it were but folly to teach any rule, ſeeing they are ſo eaſie that euery childe can doe it: but for the Multiplication of the greater Digits, thus ſhall you doe.

Multiplication of Digits.

First, ſet your Digits one right, ouer the other

other then from the vppermoſt downeward, and from the nethermoſt vpward, drawe ſtraight lines, ſo that they make a croſſe, commonly called Saint Andrewes croſſe, as you ſée here. Then loke how many each of them lacketh of 10, and write that againſt each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I muſt write thoſe Digits thus.

The difference.

Digit difference

8

×

7

Then doe I loke how much 8 doth differ, from 10 and I find it to be 2: that 2 doe I write at the right hand of 8, at the end of the line thus.

Digit difference

8 2

×

7

After that I take the difference of 7 likewiſe from 10, that is 3, and I write that at the right ſide of 7, as you ſée in this example.

Digit difference.

8 2

×

7 3

Then do I drawe a line vnder them, as in Addition thus

Laſt of all, I multiply the two differences, ſaying 2 times 3 make 6, that muſt I euer ſet vnder the differences, beneath the line: then muſt I take one of the differẽces (which I will, for all is like) from the other digit (not from his owne) as the lines of the Croſſe warne

warne me, and that that is left must I write vnder the digits. As in this example, if I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write vnder the digits, and then there appeareth the multiplication of 7 times 8 to be 56. And so likewise of any other digits, if they be aboue 5, for if they be vnder 5, then will their differences be greater then themselues, so that they cannot be taken out of them. And againe, such little summes euery child can multiply, as to say: 2 times 3, or 4 times 5 and such like.

Digit *difference.*

8 2

7 3

5 6

Scholar. Truth it is. And seeing mee seemeth that I vnderstand the multiplying of the greater digits, I will proue by an example how I can doe it. I would know how many are 9 times 6.

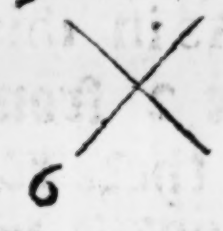
Master. It is all one in value to say 9 times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying, 6 times 9, and so of all other summes.

Scholar. Then would I know how many are 6 times 9: therefore I set the digits thus, and make the crosse, thus.

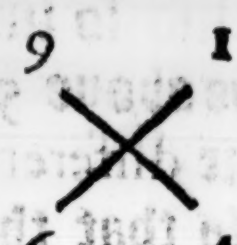
9

6

Then doe I set their differences from 10 at the right side, the difference of 9 which is 1, against it, and the difference of 6 which is 4, against it also, as in this example.



And vnder them draw a line, Then doe I multiply the differences together, saying: 1 time 4 maketh 4, that 4 doe I write vnder them thus.



Then take I one of the differences from the other digit, as, 1 from 6, or else 4 from 9, and each waies there resteth 5, which I doe write vnder the digits thus. And so appeareth the multiplicatiõ of 6 times 9 to be 54. Thus I see the feat of this manner of multiplication of digits.

Master. Now might you goe straight to the multiplication of great numbers, saue that both for your ease and surety in working I will draw you here a Table, whereby shall appeare the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seeke your first or last Digit in the greater figures, and from it goe right forth toward the right hand, till you come vnder the number of your second Digit, which is in the highest rowe, and then the number that is in the meeting of the rowes of little squares (which come directly from both your propounded Digits) is the Multiplication that amounteth of them. As if I would know by this table the multiplication of 7 times 9, seeke first 7 in the greater figures, and then go right forth toward the right hand, till you come vnder 9 of the highest row, in which place where you so come vnder the other digit (as here for example you come vnder 9,) is alwaies contained the of come or product, which you seeke,

and that place wee terme to bee in the common angle, in respect of the two numbers so taken on the outsides: as here in that common angle, where the rowes of little squares directly proceeding from 7 and 9 doe meet, you haue 63, which 63 is the summe of the multiplication of 9 by 7.

To multiply greater sums.

Scholar. This is very good and ready. And so may I find the multiplication of any digits: but now how shall I do in greater summes?

Master. *When you would multiply any summe by another, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: and likewise the lesser number, vnder it, for that is the place of the Multiplier or Multiplicator, that is to say, the number by which the Multiplication is made, and is in English alwaies put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word Times, is that which must bee multiplied.*

Multiplier.

Times.

Therefore when I would multiply one number by another, I must write the greatest highest, and the lesser vnder it, as in Addition. And vnder them must I drawe a line. As for example: If I would multiply 264 by 29, I must set them thus.

264	
29	

¶. Of which numbers thus set downe to be multiplied, may be formed a question, as thus. There are 29 men, and each man hath

264

264 Lambes. The queſtion is, how many Lambs they haue in all.

To the perfoꝛmance whereof, I muſt multiply euery figure of the higher roꝛw, by euery figure of the nether roꝛw: and that that amounteth, I muſt ſet vnder the line, as thus.

Firſt I doe multiply 4 by 9, ſaying: 9 times 4, (oꝛ 4 times 9 which is all one) and that maketh 36, as the Table beſoꝛe of digits doth declare, of that 36 I muſt wꝛite the 6, that is the Digit, vnder the 9, and the Article 3 I kéepe in mind to carry to the next place.

	264
	29
	———
	6

Then come I to the ſecond figure of the higher roꝛw, which is 6, and ſay: 9 times 6 make 54, and with the 3 in my minde make 57. the 7 I ſet doꝛwne vnder the 2, and 5 I kæpe in minde.

	264
	29
	———
	76

After that I come to the next figure, which is 2, and multiply it by 9, and that maketh 18 and with 5 that I haue in mind, maketh 23: wheꝛefoꝛe because it is the laſt woꝛke of the Multiplier, I ſet it doꝛwne in oꝛder as you ſée:

	264
	29
	———
	2376

And ſo haue I ended the firſt figure of the Multiplier. Wheꝛefoꝛe I giue it now a fine daſh with my pen.

Then begin I with the next figure, and multiply it into all the higher figures, as thus:

```
264
 29
----
2376
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First, 2 times 4 make 8, that 8 doe I write vnder the second place: for euermore the Digit or first figure of the Multiplication that amounteth of the figure of the higher number, must be set vnder the Multiplier of it, the other in their order toward the left hand,

Scholar. I vnderstand you thus, that the Digit of the summe amounting of the Multiplication of the first figure of the higher row, by the first figure of the lower row, or Multiplier, must bee set vnder the first place: and that that amounteth of the same first figure by the second Multiplier; must be set vnder the second place, and so of the other, if there be more Multipliers.

Master. So meane I indeed: and if there amount but a Digit, then must it be set vnder the Multiplier.

And now to goe forth: I multiply by the same 2, the second figure of the higher row, which is 6 saying: two times 6, make 12, whereof I write the digit 2 vnder the third place, and the Article 1 I keepe in minde.

```
264
 29
----
2376
  28
```

Then doe I multiply the last figure of the higher summe by that same 2, saying: two times 2 is 4, and with the 1 that I haue in minde maketh 5, which 5 I write vnder the fourth place. And so

so haue I ended the whole Multiplication: wherefore I also giue the 2 a dash with my pen, thus: and so I doe euer as soone as I haue dispatched any Digit by which I multiply: and the summes stand thus.

Then must I drawe a line vnder all those summes that mount of the multiplication, and must adde all them into one summe, as in the example you may see.

264
25
2376
528
264
25
2376
528
7656

Where in the first place I find but 6 and therefore write I it vnder the line. Then in the second place 8 and 7 make 15. where of I write 5 and keepe one in my mind, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656 which amounteth of the Multiplication of 264. by 29, and that is the iust number of Lambes the 29 men had.

Scholar. If there be no more to bee obserued in it, then can I doe it, I suppose, as by this example I shall proue.

¶ There is a peece of ground which containeth 1365 yards in length, and 236 yards in breadth: I would know how many yards square there is in all this peece of ground: which numbers I set downe with the greater aboue, and the lesser vnder, as you see

1365
236

Then doe I multiply 5 by 6, saying: 6 times 5 make 30, of which I write the Cipher in the first place and the Article 3 I doe kæpe in minde to carry to the next place.

1365
236
0

Then doe I by the same 6 multiply the second figure of the higher summe, which is 6, saying, 6 times 6 make 36, and 3 in my minde make 39, of which I write the 9 vnder the second place and the Article 3 I kæpe in minde.

1365
236
90

Then doe I multiply the third figure, which is 3 by the same 6, and that maketh 18, and 3 in my minde make 21. The 1 I set downe, and kæpe 2 in mind.

1365
236
190

Then come I to the last figure of the higher summe, and multiply it by 6, saying, 6 times 1, make 6, and 2 in my mind make 8, that 8 doe I write vnder the fourth place. And so haue I ended the first Multiplier, and dash him sleightly with my Pen.

1365
236
8190

Then beginne I with the second Multiplier, and say, first 3 times 5 that maketh 15, of which I set the 5 vnder the second place, because that the Multiplier, is there, and the Article 1 I kæpe in minde.

Then

Then come I to the ſecond Figure, that is 6, and multiply it by 3, which maketh 18, and with one in minde maketh 19, the 9 I ſet downe vnder the third place, and 1 I kéepe in minde.

Then come I to the third Figure, which is 3, and multiply it by 3, ſaying: 3 times 3 make 9 and with 1 in minde make 10, the Cypher I ſet vnder the fourth place, and the Article 1 I kéepe in minde.

And then comming to the laſt figure 1, I multiply it by 3: and it maketh 3, and with the one in minde, it maketh 4, which 4 I ſet in the firſt place, and then haue I ended two of the Multipliers, and the ſummes ſtand as you may ſée in the latter end of the page going beforе, and then I giue 3 his dash.

```
1365
 236
8190
4095
```

Then come I to the third Multiplier, and multiply it into euery figure of the higher ſum, and firſt I ſay, 2 times 5 make 10, of which I ſet the Cypher vnder the Multiplier in the third place, and the Article 1 I kéepe in minde.

```
1365
 236
8190
4095
   0
```

And ſo multiplying the ſecond figure 6 by

that same 2, there amounteth 12, and 1 in my mind maketh 13, whereof I write the Digit 3 vnder the fourth place, and the Article 1 I kæpe in minde.

Then doe I multiply the said 2 by the third figure of the higher somme, which is 3, and that maketh 6, and the one in minde make 7, which 7 I set downe vnder the fift place, as appeareth by the example.

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the first place, and then doth the somme stand thus.

And so haue I ended the whole Multiplication.

But now (as you taught me) to know what this whole sum is, I must adde all those parcels together, and then vnder the line will appeare, as you may see, the grosse or total summe is, 322140. Wherby I know there is so many yards square in that peece of ground.

Master. This is well done.

Scholar. Then me thinketh I could call it well done, when I know, whether I had well done or no.

Master

Maſter. It is to be proued by 9 as Addition was, but the ſureſt proofe is by Diuiſion, and therefore I will reſerue that proofe by Diuiſion, till you haue learned the Art of Diuiſion. And anone I will ſhew you how it is commonly proued.

¶ *But firſt, for your further inſtruction in this exerciſe of Multiplication, I will with one example more try your cunning, and ſo make an end: And the queſtion is this. I would know how many dayes it is ſince the Nativity of our Lord and Saviour Ieſus Chriſt, vnto this yeere* 1630. *Which to performe; you muſt multiply this preſent yeare* 1630. *by the dayes in one whole yeere, which are* 365.

Scholar. Now, for that you haue giuen me ſo much light into the queſtion, you ſhall ſée I will handſomely finiſh the worke, for according to your former inſtructions, I ſet them downe with a line vnder them, thus.

1630	
365	

Then ſay I, 5 times 0 is 0, which I ſet downe vnder the firſt place, as here appeareth. Then ſay I, 5 times 3 make 15, the digit 5 I ſet downe in the ſecond place vnder 3, and the Article 1 I kéepe in minde to bee added to the next Multiplication. Then ſaying fiue times 6 make 30, and 1 in minde 31, the 1 I ſet downe in the third place, and 3 I kéepe in minde. Then comming to the laſt figure, I ſay once 5 is 5, and 3 in mind make 8, that 8 doe I ſet downe vnder the

the fourth place: and thus haue I ended my first Multiplier, and therefore I giue it a dash with my Pen.

Then come I to the second Multiplier, which is 6, & dos likewise multiply it into the vpper number, saying, 6 times 0 is 0, which I set downe in the second place right vnder his Multiplier: then say I, 6 times 3 make 18, the 8 I set down vnder the third place, and 1 I kape in minde. Then say I 6 times 6 make 36, and 1 I kœpe in minde make 37. the Digit 7 I set downe in the fourth place, & 3 I kœpe in minde. Then say I, 6 times 1 is 6, & once 6 is 6, and 3 in mind make 9, which I set down nert, & so haue I ended two Multipliers: wherefore I dash the 6 with my Penne.

1590	
365	
7950	
9540	

Then I begin to multiply the third Multiplier into the ouer number, saying, 3 times 0 is 0; the 0 I set down in the third place right vnder his Multiplier. Then say I 3 times 3 make 9, which I set down in order nert, then say I, 3 times 6 is 18, the 8 I set downe, and 1 I kape. Lastly, I say, once 3, is 3 and 1 I kœpe is 4, which I set downe orderly nert: And so haue I ended the Multiplication, and my figures stand thus.

	or thus
1630	1630
365	365
8150	815
9780	978
4890	489
594950	594950

Master.

Maſter. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one ſumme it wil be 594950; which is the graſſe or totall ſumme of that Multiplication, and declareth the number of daies ſince our Lord and Sauiour his incarnation, vnto the end of 1630. *yeeres, beſides* 407 *dayes, and twelue houres for leape yeeres.*

Scholar. This is marueilous me thinke, that ſuch great matters may ſo eaſily bee atchieued by this Art, which heretofore I euer thought had beene impoſſible, as infinite ſorts of people are of that minde.

Maſter. Truth it is, that knowledge hath no greater enemy then ignorance, for this is one of the leaſt of ten thouſand things that may be done by this Art, as hereafter you ſhall be able to iuſtifie.

Scholar. The manner of Multiplication I perceiue, if there be no more in it.

Maſter. *Yes, there are other formes and helpes for eaſe and ſhorter labour of the worke of Multiplication: but I will remit them till you haue a little taſted Diuiſion, where alſo the like helpe into Diuiſion may be vſed: and ſo therefore vnder one example for both, will I ſhew you both eaſe in Multiplication, and alſo in Diuiſion.*

But ſith the other formes and workings doe nothing differ from theſe works in effect, but onely in ſetting of the numbers, I will ouer-paſſe them till a more meete place and

time.

time. And now wil I instruct you in Diuision, so that you thinke your selfe sufficiently to perceiue what I haue taught you.

Scholar. Yes Sir, I thanke you, but I doe not perceiue how to examine my worke, to try whether I haue well done, or no: therefore as you promised me ere-while, I pray you first shew me how I shall prone it.

Master. *That is commonly vsed by the proofe of 9, as you learned before in Addition, sauing that it differeth from that forme in diuers respects: As for example.*

First, you must make a cresse after this manner:

Proofe of Multiplication.

Then must you examine your summe that should be multiplied, & loke what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoeuer remaineth in it after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, and loke what amounteth therof, if it be vnder 9 write at the higher part of the crosse: but if it be aboue 9, then take thence 9 as oftenas ye can, & write the rest at the head of the crosse: As for example, we will proue the example you put forth of the peece of ground that contained 1365 yards in length, and 236 yards in bredth.

Therefore first I cast away all the nines from the summe to be multiplyed, saying, 5

and

and 6 make 11, caſt away 9 reſt 2: then 3 and 2 makes 5, and 1 is 6, that 6 I write at one ſide of the croſſe thus.

Then doe I examine the Multiplier, which is 2 3 6, wherein when the 9 is caſt out, there remaineth 2, that 2 therefore I ſet at the other ſide of the Croſſe.

Then doe I multiply 6 by 2, and it maketh 12, from which 12, I withdraw 9, then reſteth 3; that 3 doe I ſet at the head of the croſſe. Then doe I examine the groſſe ſumme amounting of the Multiplication, which is 322140, where I find 9 once, and 3 remaining; that 3 I ſet at the foote of the croſſe, and then I ſee it to agree with the other 3 at the top of the croſſe, and ſo know I that I haue done well: for if they two did differ, then were my worke vaine, and the Multiplication falſe.

This is the common proofe but the moſt certaine proofe is by Diuiſion, of which I will anone inſtruct you.

A ſure proofe of Multiplication.

Scholar. Sir, what is the chiefe vſe of Multiplication?

Maſter. The vſe of it is greater then you can yet vnderſtand: howbeit, theſe plaine commodities it hath, that if you would reſolue any great and whole value into many ſmall and leſſe portions, as if you would change

pounds

pounds in shillings and pence, or any other greater or smaller parcels by Multiplication, ye shall doe it spedily and easily. Also if you should neede to adde one summe to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more spedily, readily, easily, and surely, then by often and sundry Additions. Take you these commodities grossely shewed for an answer at this time, and hereafter I will more abundantly make you to perceiue the vse of it.

Diuision.

Diuiſion.

Scholar.

WEll Sir, then in Diuiſion I pray you to inſtruct mee. But me thinketh by the name of it, that it ſhould bee all one with Multiplication: for I call that Diuiſion, when any thing is parted into diuers and many parts.

Maſter. You take it as it is taken commonly: howbeit, if you marke well, you ſhall perceiue that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but contrary wayes, it bringeth many parcels into few, but yet ſo, that theſe few taken together, are equall in value to the other many: for by Diuiſion, pence are turned into ſhillings, and ſhillings into pounds: As for example, of 120 ſhillings it maketh 6 pounds, ſo are 120 turned into 6, which is a ſmaller number: but then if you conſider the Denominators, you ſhall ſée that they are ſuch that one of the latter is equall to 20 of the firſt, and ſo in value the ſummes are one, though in number they doe differ, and the latter ſumme is the leſſer, and ſo it is alwaies in Diuiſion: howbeit, yet

in the worke, the ſumme is parted by another, and thereof doth it take the name.

Scholar. I thinke I ſhall better vnderſtand the reaſon of the name when I know the vſe of the worke, therefore now would I gladly learne that.

Diuiſion what it is.

Maſter. *Diuiſion is a diſtributing of a greater ſumme by the vnites of a leſſer: Or, Diuiſion is an Arithmeticall producing of a third number, in reſpect of two propounded numbers; which third number ſhall ſo often containe an vnite, as the greater of the two pounded numbers can containe the leſſer.* So that as Multiplication did ſeeme to ſerue inſtead of many Additions, ſo Diuiſion may ſeeme to be in place of many Subtractions: Becauſe that third number briefly expreſſeth how many times the leſſer of your two propounded numbers may be ſubtracted from the greater: as in practiſe will more plainely appeare. Therefore (as you may perceiue) vnto Diuiſion are required three numbers: the firſt, which ſhould be diuided, and that muſt (generally) bee the greater: and the ſecond, by which the other muſt be diuided, and that is (generally) the leſſer, and is called, the Diuiſor: And the third, which anſwereth to the queſtion (How many times?) and therefore is called the Quotient.

A generall rule for placing the figure.

The firſt muſt be firſt written, and the ſecond ſo ſet vnder it, that the laſt figure of the lower number be right vnder the laſt of

the higher, contrariwiſe to the worke of o-ther kinds of Arithmetike: for in them the two firſt figures were ſet euer méet, one vn-der the other: but in Diuiſion, the laſt figures muſt be ſet méet, except it chance ſo that the laſt figure of the Diuiſor be greater then the laſt of the higher number, for then you ſhall ſet the laſt of the Diuiſor vnder the laſt ſaue one of the higher number, as for example. An excep-tion.

If you ſhould diuide 365 (which are the ſumme of the daies of a yeere) by 28 which are the daies of a com-mon moneth, then ſhould you ſet them thus.

365	
28	

But if you ſhould diuide theſe 365 daies by 52, which is the number of weekes in one yéere, then ſhould you ſet them thus.

365	
52	

Likewiſe, if I would diuide the ſame 365 by 4, which is the ſumme of the quarters of yéeres, then muſt I ſet them thus.

365	
4	

Scholar. Sir, this doe I vnderſtand, but now how ſhould I doe to diuide the one by the other?

Maſter. You muſt begin with the laſt fi-gure next the left hand, and ſée how many times the laſt figure of the Diuiſor may bée taken out of the laſt figure of the other number, and that ſhall you note within a crooked line toward your right hand. As for example, I would diuide 365 by 28,

28, then set I those two sums thus.

```
365(
 28
```

And I loke how many times I may find 2 (which is the last figure of the Diuisor) in 3, (which is the last of the number to be diuided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1 and that is called the Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 ouer 3, and deface or cancell the 3 and the 2, then will the figures stand thus:

Quotinet number.

```
 I
365(I
 28
```

Then come I to the next figure of the Diuisor, and take it likewise so many times out of the figures that be ouer it, and loke what doth remaine, that I must write ouer them, and cancell them, as in this example.

Therefore now doe I take once 8, out of 16, and there remaineth 8, which I must set ouer the 6, and cancell or crosse out the 16 and the 8 of the Diuisor: and then will the figures stand thus. And so haue I once wrought.

```
 18
365(I
 28
```

Scholar. So I perceiue that you take the nether figure, not onely out of the other that is right ouer him, but out of that with the other, also

ſo that remaineth before, and are written ſtward the left hand.

Maſter. So muſt you doe: for you muſt ſo take the Diuiſor out of the ouer number, that there remaine not ouer it ſo great a ſumme as it ſelfe is, for then were your worke in vaine.

But yet againe here muſt you marke, that when you ſéeke how many times the laſt figure of the Diuiſor may be found in the number ouer him, that you loke alſo whether you may as often find all the figures following in thoſe that are aboue them (conſidering all the remainers, if there be any) if not, take your Quotient leſſe by one, and then proue againe, and ſo ſtill till you find a mete Quotient: and by that méete Quotient muſt you alwaies multiply your Diuiſor, and ſet the product vnder your Diuiſor, ſo that the firſt figure ſtand vnder the firſt figure of your Diuiſor, and the ſecond vnder the ſecond, and ſo forth: and then ſubtract that product from the number to be diuided that ſtandeth directly ouer it, as you haue ſéene mée doe.

When you haue thus wrought once, then muſt you begin againe, and write your Diuiſor anew, néerer toward the right hand by one place, as in this example, you ſhall ſet 2 vnder 8, and 8 vnder 5, thus.

Then (as before) ſéeke how many times you may take

18	
365	(1
288	
2	

your diuisor out of the number ouer him now.

Scholar. That may I doe here 4 times.

Master. Truth it is, that you may finde 2 foure times in 8: but then marke whether you can find the figure following so many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholar. No, neither yet once.

Master. Therefore take 2 out of 8 once lesse.

Scholar. That is three times.

Mark how to consider this kinde of Remainer.

Master. Well then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following, make 25, in which summe I find 8 three times al-

so: and therefore I take 3 as a true Quotient, and write it within the croked line of the quotient before the 1, thus.

Then say I 3 times 2 make 6, then 6 out of 8 re-steth 2, therefore I cancell the 8. and write ouer it the 2 that doth remaine, thus:

	2
	x 8
	365 (13
	288
	2

Then doe I take 8 as many times out of 25, saying: 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1, so then I cancell 25 and 8, and ouer the 5 set 1: thus.

Or, you might (after you found 3 to bee a fit Quotient) ſtraight way haue multiplied the whole Diuiſor 28, by that at once: which giueth 84, which beeing ſet vnder 28, and duly ſubtracted from 85, of the number diuided, giueth 1, the remainer of the whole Diuiſion, as before you had. Worke which way you liſt, there you ſee alſo the forme.

And now haue I done with the diuiding, for I cannot find my Diuiſor 28 no more in the ouer ſumme.

Scholar. No, except you would part the 1, that remaineth into 28 parts.

Maſter. That is well ſaid, and ſo muſt wee doe in ſuch caſes, when there remaineth any thing: but I will let that paſſe now, and will make you perfect in Diuiſion of whole numbers, and will hereafter teach you particularly of broken numbers, called Fraction. Now if you doe perceiue the order of Diuiſion then doe you diuide this ſumme 1 3 6 2 8 0 by 145.

Scholar. Firſt, I ſet downe the number that ſhould bee diuided, then doe I ſet the Diuiſor vnder the laſt figure of the ouer number. Then will it be thus.

	2
	1 8 1
	3 8 5 (1 3
	2 8 8
	2

1 3 6 2 8 0	
4 5 2	

Maſter. Can you take the laſt of your Diuiſor (which is 4) out of 1, which is the laſt of the ouer number ?

Scholar. I had forgotten, becauſe the laſt of the Diuiſor cannot be taken out of the laſt of the ouer number, in ſo much as it is the greater, therefore muſt I ſet the Diuiſor one place more forward toward the right hand thus.

	136280
	452

And then muſt I loke how often I may find the laſt figure of the Diuiſor (that is 4) in 13, which I may doe 3 times, therefore do I ſay, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doe I make at the right hand of my ſummes a crooked line, and write before it my Quotient 3, and I cancell 13 and 4, and ouer the 3 I ſet the 1 that remaineth, and then the figures ſtand thus:

	1
	136280 (3
	452

Then I multiply the ſame Quotient into euery figure of the Diuiſor, & withdraw the ſumme that amounteth out of the numbers ouer them, as firſt I ſay, 3 times 5 make 15, which I take from 16, and there reſteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remaineth thus,

	11
	136280 (3
	452

Then doe I ſay likewiſe, 3 times 2 make 6, which I take out of 12 and there reſteth 6, therefore

therefore I cancel the 12 and the 2, and ouer the 2 I write the 6 that remaineth thus.

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XX6
23628O ( 3
452
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Then ſhould I ſet forward the Diuiſor into the next place toward the right hand thus.

```
XX6
X3628O ( 3
4522
45
```

Maſter. But you may ſée that ouer the 4 is no figure, therefore I muſt ſet the Diuiſor yet forwarder by another place.

And marke, whenſoeuer it chanceth ſo, that you ſhould ſet forward the diuiſor, and that it cannot ſtand there, becauſe there is no number ouer the laſt place, or if there be any, it is leſſer then the laſt figure of the diuiſor, then muſt you remoue the diuiſor, yet once againe: and becauſe that his firſt place of remouing ſerued not to ſubtract him ſo much as once, therefore you ſhall write in the Quotient a cipher, and if you ſhould by chance nád to do ſo oft times, for euery time write a Ciphe in the Quotient. The reaſon of this will I ſhew you hereafter.

Scholar. Then muſt I ſet my ſummes thus.

```
XX6
23628O ( 30
4522
45
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And becauſe I remoued the diuiſor, ſo that I ouerſkipped one place, I muſt write a Cypher in the Quotient: and then muſt I ſéeke a new Quotient, as

in this example I muſt ſay; How many times 4 is there in 6? (and ſith it can bée but once) therefore doe I write 1 in the Quotient: and then ſay I, 1 times 4 taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 ouer them thus.

2
4 4 6 3
4 3 6 2 8 0 (301
4 5 2 2 2
4 5 5
4

Then ſay I againe, once 5 out of 28, remaineth 23: I let the 2 ſtand as it did, and ouer that 8 I ſet 3, cancelling the 8 and the 5 vnder it, thus.

2
4 4 6 3
4 3 6 2 8 0 (301
4 5 2 2 2
4 5 5
4

Maſter. You might as well haue ſaid, once 5 out of 8, and ſo remaineth 3, but now goe forward:

Scholar. Then once 2 out of 0 cannot be: What ſhall I now doe?

Maſter. Borrow of the next number that is behind (for there is 230) and doe as you learned in Subtraction in a like caſe.

Scholar. Then muſt I borrow 1 of the 3 comming behind next, and make that 0

to

to be 10, and then take 12 out of 10, and there resteth 8. And because I borrowed one of the 3, I must cancell the 3, and write 2 over it: then doth the Figure stand thus.

```
22
4 4 6 3 8
4 3 6 2 8 0 ( 301
4 5 2 2 2
4 5 5
4
```

Master. Now haue you done, and yet remaineth 228, and your Quotient sheweth you, that if you diuide 136280 by 452, you shall find your Diuisor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example (where I diuided 365 by 28, the Quotient was 13, and 1 remained, whereby I knew that in a yeare (which containeth 365 dayes) there are 13 moneths, reckoning 28 dayes (or 4 weeks) iust to a moneth, and 1 day more.

Scholar. Why then doe wee call a yeare but twelue moneths?

Master. Of that at a more conuenient time will I fully instruct you: but now it is not conuenient to intangle your minde with other things then doe directly pertaine to your matter. Therefore if you remember what you haue heard, you haue learned a short manner of diuision, which I would haue you often to practise, so that you may be perfect in it, and hereafter I will shew you certaine other proper points touching it.

Scholar. Then I pray you tell mee how

I ſhall examine and try my worke, whether I haue done well or no, that though no man be by me to tell me, yet I may perceiue it my ſelfe.

Proofe of Diuiſion.

Maſter. Some men (yea and commonly moſt) doe try it by the rule of 9, as in all the other kinds, ſaue that their order is: Firſt, they caſt away 9 as often as they can out of the diuiſor, and that remaineth they ſet at one ſide of a croſſe, as in our firſt example the diuiſor was 28 from which you may take 9 three times, and 1 remaineth: which they ſet by a Croſſe thus.

Then they likewiſe examine the Quotient (which in our example is 13) and from thence they caſt away 9 as often as they can, and the remainder they ſet at the other ſide of the Croſſe, and then they multiply together thoſe two remainers: and to it that amounteth they adde the remainer of the Diuiſion, if there were any from that whole ſumme they withdraw 9 as often as they can and the reſt they ſet at the head of the croſſe, as in our example the Quotient is 13, from which take 9 and there remaineth onely 4, and therefore muſt you ſet 4 at the other ſide of the Croſſe thus

Then multiply 4 by 1, and it yeldeth but 4, thereto adde the remainer of the Diuiſion (which was 1) and it will be 5, which ſumme doth

doth not amount to 9, and therefore muſt be ſet wholly at the head of the croſſe as you ſée héére.

And this number on the head of the Croſſe is the firſt proofe, to which if you find another like in the number that was diuided, then haue you done well.

Therefore now ſhall you likewiſe examine the whole ſumme that was diuided, and take away 9 as often as you can, and that that remaineth, ſet at the foot of the Croſſe: and if it be equall to that in the head of the Croſſe then haue you done well, elſe not.

As in our example the whole ſumme was 365, which maketh 14: from that take 9 and there reſteth 5, which ſet at the foote of the Croſſe, thus:

And you ſhall ſée that they agrée: therefore haue you well done.

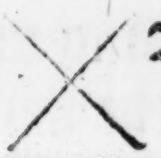
Now will I likewiſe examine our ſecond example, where the diuiſor was 452. which maketh 11; from thence I take 9, and the 2 that remaineth I ſet at the right ſide of the croſſe thus:

Then examine I the Quotient, which was 301, where I find but onely 4: that I ſet at the other ſide of the croſſe, thus:

Then I multiply 4 by

2,

2, and it maketh 8: to that I adde the remainer of the diuision (which was 228, and it maketh 12) and they two make 20, wherein I find twice 9, and 2 remaining: that 2 must I set at the head of the Crosse thus:



Then I examine the whole number to be diuided, which was 136280, where I find twice 9 and 2 remaining which I set at the foote of the Crosse thus:

And because it doth agree with the figure at the head of the Crosse, I know that the diuision was well wrought.

The proof of Diuision more certaine by Multiplication.

Master. This is the common proofe. Howbeit, the more certaine working is by the contrary kind: as, to proue diuision by Multiplication thus:

Multiply the Quotient by the diuisor, and if the summe that amounteth, bee equall to the summe that should be diuided, then haue you well diuided: else not.

Howbeit, this must you marke, that if there remained any thing after the diuision, that must you adde to the summe that amounteth of the Multiplication. As in our first example our Quotient was 13, and the diuisor was 28: Now multiply the one by the other, & the summe wil be 364: to that if you adde the 1 that remained after the diuision, then

then will it be 365, which was the ſumme that ſhould be diuided: and therefore I know that I haue well done.

Scholar. Now will I proue the ſame in the ſecond example, whoſe diuiſor was 452 and the Quotient 301: theſe doe I multiply together, and there amounteth 136052: to which if I adde the 228 that remained, then will it be 136280, which was the whole ſumme to be diuided: and therefore I perceiue that I haue well done.

Maſter. This is the ſureſt way, to examine diuiſion by Multiplication: and contrariwiſe the ſureſt proofe of Multiplication is by diuiſion.

And therefore (according to my promiſe) now will I ſhew you how you may proue Multiplication by diuiſion.

Proofe of Multiplication by Diuiſion.

When you haue ended Multiplication, and would know whether you haue well done or not, ſet the groſſe ſumme that amounteth of the Multiplication ouermoſt, and diuide it by the Multiplier: and if the Quotient be the ſame number that ſhould bee multiplied, then haue you well wrought, elſe not, as in that example where wee multiplied 264 by 29, the groſſe ſumme was 7656.

Now if you will know whether that Multiplication be true, you ſhall diuided that 7656 by the multiplier, 29, and you ſhall perceiue that the Quotient will be 264, and that is a token that you haue well wrought.

Scholar.

Scholar. By your patience I will proue that, and firſt ſet downe the groſſe ſumme and the multiplier, not after the rule of Multi. cation, but after the rule of Diuiſion, for now that number is become the diuiſor, that was before the Multiplier, I ſhall ſet them therefore thus:

```
7656
29
```

Then ſhall I ſeeke how many times, 2 in 7, that may be 3 times, and 1 remaineth: but then may not 9 be found ſo often in 16, therefore muſt I take a leſſer Quotient, that is to ſay 2: then ſay I, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then doe I cancell 7 and 2, and ouer 7 I write 3, and in the Quotient I ſet 2: ſo the figures ſtand thus:

```
   3
7656(2
  29
```

Then ſay I forth, two times 9 make 18, which I abate out of 36, and there reſteth 18: then cancell I 3, and ouer him ſet 1, and likewiſe I cancell 6 and 9, and ouer them I ſet 8: ſo that thus ſtand the Figures:

```
   1
   38
7656(2
  29
```

Then I ſet forward the diuiſor by one place, and ſeeke a new Quotient, that is to ſay, how many times 2 are in 18, which I find 9 times: but then can I not find 9 [illegible]y times in 5, therefore I take a leſſer Quotient, as to ſay 8: but yet that is too great: for if I take 8 times 2 out of 18, there remaineth

remaineth but 2, and I cannot find 8 times 9 in 25: therefore yet I take a lesser quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seeke a lesser quotient, as to say 6, then say I, 6 times 2 make 12, that I take out of 18, & there remaineth 6, so I cancel 18, and the 2, & write 6 ouer 8 thus:

Then say I forth, 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the Figures stand thus:

Then must I set forth the Diuisor againe, and take a new quotient, which will be 4: for though I may find 2 in 11, 5 times, & 1 remaine, yet I cannot find 9 so often in 6, therefore I set the figures thus:

And the 4 in the quotient I multiply into the figures of the diuisor saying, foure times 2 makes 8 which I take out of 11, and there rests 3: therefore I cancell the 11, & the 2, and set 3 ouer the first place of 11, thus:

And then doe I say forth, 4 times 9 maketh 36, which I take from 36, & there remaineth nothing:

	16
	38
	7656 (26
	299
	2
	1
	16
	381
	7656 (26
	299
	2
	1
	163
	381
	7656 (264
	2999
	22

ſo that the Quotient of this Diuiſion alſo (where 7656 is diuided by 29) is 264: Which doth declare, that if 264 be multiplied by 29, the ſumme will be 7656. And thus I perceiue now how both Multiplication is proued by Diuiſion, and Diuiſion alſo by Multiplication.

Maſter. Now haue I ended the fiue common kinds of Arithmeticke. For (as touching Mediation, Duplation, Triplation, and ſuch other) they are no ſeuerall kinds of Arithmeticke, but are contained vnder the other. For Mediation is contained vnder Diuiſion, and is nothing elſe but diuiding by 2: and ſo are Duplation & Triplation contained vnder Multiplication: for Duplation is nothing elſe but multiplying by 2, and Triplation is multiplying by 3, of which I will onely propoſe an example for the rules you haue heard already.

An example of Mediation.

If you would mediate or diuide into 2, this ſumme 4531010, you ſhall ſet 2 for the diuiſor, and worke as you learned before, as thus:

```
4531010(
2
```

Then I find 2 in 4 two times, therefore my Quotient muſt be 2: ſo I cancel 4 and 2, and remoue the Diuiſor forward thus, as the worke requireth, and as before Diuiſion hath bene declared.

```
4531010(2265505
 22222222
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Which mediation or diuiſion by 2 being finiſhed

finished, you shall haue for your Quotient 2265505, which is the halfe of 4531010, as you may trie by Duplation; for double that Quotient, or multiply it by 2, and the same number will amount. Duplation

I will no longer tarry about these, seeing they are but members of the other kinds. But here now (according to my promise) I will teach you certaine easie formes both of Multiplication and of Diuision. And first of Multiplication,

If you would therfore multiply any summe by 10, you shall need to doe no more but adde a cipher before his first place: as for example, 36 multiplied by 10, make 360. Easie formes of Multiplication.

Likewise, if you would multiply any summe by 100, put two cyphers at his beginning. So if you would multiply any summe by 1000, adde three Cyphers to the beginning of it.

Scholar. This doe I well perceiue, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmetick also: and as to our matter now, loke, as I haue told you, that you both remember it, and also often practise it.

And now you haue learned how to multiply easily by 10, 100, 1000: and of like manner may you doe with any other of that sort.

But now if you will multiply by 20, 30, 40, and so forth, or by 200, 300, and such like, where there is one Cypher in the first place,

or many orderly in the first places, you shall take away those Ciphers and multiply the summe onely by the other figure, or figures, (if they be many) and then at the beginning the summe that amounteth, you shall set so many Ciphers as you toke away.

Example of 2873, which I would multiply by 300. First I omit the 2 Ciphers from the Multiplier, and I multiply the summe by three onely that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omited or toke away, and then is it 861900. And that is the summe that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures beside the Cyphers, I must only take away the Cyphers, and multiply by the other figures, as I learned before: as if I would multiply 93648 by 25000, I should take away the three Cyphers, and multiply the same by 25, and then at the beginning of that totall summe should I adde the 3 Cyphers againe.

Maister. Euen so: but if it chance the number that should bee multiplied, or both the summes, as well the number that should be multiplied, as the Multiplier, to haue Cyphers in their first places, euermore omit the ciphers & worke by the rest. But remember to restore as many Cyphers to the amounting summe as you bated before, as in this example: 30200 shall bee multiplied by 206 I shall onely take away two Cyphers from the grea-

ter

ter number, and then multiply 302 by 206, and afterward adde the two Cyphers againe. But if I would multiply the ſame 30200 by 2060, I ſhall not onely take away the two Cyphers from the number that ſhould be multiplied, but alſo I may take away the one Cypher from the Multiplier, and then muſt I adde 3 Cyphers to the ſumme that amounteth: but take hæd that you take away no Cypher that commeth after any ſignifying figure, as in the laſt example, you may not take away that in the fourth place of the higher number, neither that in the third place of the Multiplier: howbeit, yet thus you may doe: If one Cypher or more come in the midſt of your ſummes you may multiply by the other Figures, and ouerſkip them: but ſo, that you giue euery figure his due place: as thus, I will multiply 3026 by 2004, therefore I ſet them thus:

```
3026
2004
-----
12104
  52
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And thus I doe multiply them. Firſt 4 times 6 make 24, I ſet the 4 vnder the firſt place, and kæpe the 2 ſtill in my minde. Then ſay I againe 4 times 2 maketh 8, and the 2 that is in my minde maketh 10, I ſet downe the Cypher 0, and kæpe the article 1 in my minde: Then 4 times 0 is 0 and the 1 in my minde maketh 1, I ſet downe the figure 1, and ſay againe, 4 times 3 is 12, I ſet downe 2, and kæping the 1 ſtill in my minde (hauing no more places of the vpper number to multi-

tiply it withall) I put it downe next 2 in the fifth place.

But now when I come to the next place (being a Cypher o) I let it go, because it multiplieth nothing, & likewise the second cipher.

But then, when I come to the 2, and multiply it into the 6 of the ouer number, you must take héed (according as I taught you in Multiplication) that the first number amounting of the Multiplication be set right vnder the Multiplier, and the other orderly toward the left hand, according as you may sée in this example. which being finished, with the addition thereof gathered together, will stand as this example, sheweth.

	3026
	2004
	12104
	6052
	6064104

Which is indéed wrought so much the soner and shorter by ouerskipping of the 2 Cyphers: which otherwise (if the same example were wrought at length) it would haue had 2 workings more, as by the same example here also set downe doth appeare.

	3026
	2004
	12104
	0000
	0000
	6052
	6064104

Scholar. Sir I thanke you, for I sée great ease in this way of Multiplication: (and if you can shew me such like in diuision) you shall greatly further me.

Master.

Maſter. Yes, I will teach you ſome eaſie waies in Diuiſion alſo, and firſt this: If you would diuide any ſumme by 10, you ſhall onely with your pen make a ſquare line betwæne the firſt figure of your ſumme and the ſecond, and then haue you done; for the whole number that followeth the line, ſtandeth for the Quotient, and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10, 3 6 4 |8. Where 3 6 4 is the Quotient, and betokeneth that ſo many times are 10, in 3648, and the 8 after the line is the remainer, which cannot be diuided into 10, but by breaking it into fractions, wherewith I will not meddle yet. Easie formes of Diuiſion.

And ſo likewiſe if you would diuide any ſumme by 100 with your pen, you ſhall cut away the two firſt figures, and if you would diuide by 1000, you muſt cut away the three firſt figures, and ſo of any other diuiſor, whoſe laſt figure is 1, and the other ciphers, loke how many ciphers the diuiſor hath, and ſo many figures at the beginning ſhall you cut away with the ſquare line, and they ſtand alwaies for the remainer, becauſe they are leſſe then the diuiſor, and cannot be diuided by it, and the other Figures that are behind the line ſtand for the Quotient.

But now if your Diuiſor haue any other figure in his laſt place then 1, and in all his other places haue cyphers, loke how many

Cyphers they be, cut away ſo many of the firſt figures of the number that ſhould be diuided, and diuide the reſt that followeth the line by that Figure that is in the laſt place, as if it were the whole diuiſor.

Example of 64284, which I would diuide by 300, here muſt I cut away the two firſt figures (for ſo many Cyphers my diuiſor hath) and muſt diuide the reſt by 3, which is the figure in the laſt place of the diuiſor: Firſt therefore I part away the two firſt figures, and the ſumme ſtandeth thus:

642 | 84 (2
3

Then doe I diuide 642 by 3, and the quotient is 214: for in 6 I find twice 3 and in 4 once, and 1 remaining, which 1 with the 2 next before doth make 12, wherein I find 3 foure times: and this is a ready way to turne ſhillings into pounds: for ſith one pound doth containe 20 ſhillings, I muſt diuide the whole number of ſhillings by 20. Therefore eaſily to doe it, I ſee that my diuiſor hath one Cypher, and therfore I cut away one figure from the beginning of the whole ſumme of ſhillings and then I doe mediate or diuide by 2 the other figures or ſumme that followeth.

Scholar. I will put an example.

If you would diuide 64287 ſhillings by 20: that is to ſay, if I would turne ſo many ſhillings into pounds, I muſt cut away the firſt figure, that is 7, and diuide the reſt, that is 6428 by 2, ſo ſhall the quotient be 3214, whereby

whereby I know that 64287 ſhillings make 3214 pounds, and 7 ſhillings remaining.

Maſter. Now proue by Multiplication whether you haue well done or no.

Scholar. The quotient is 3214, which I doe multiply by the diuiſor 2, and it doth amount to 6428.

Maſter. Hereby you may perceiue not onely that you haue well done, but alſo how by diuiſion you may turne ſhillings eaſily into pounds: and contrariwiſe by Multiplication you may turne pounds into ſhillings.

But here ſhall you ſee amongſt diuers men diuers formes of ſuch diuiſion: but if you marke what I haue told you, you ſhall perceiue eaſily all the wayes. For ſome men doe not cut away ſo many of the firſt figures of the ſumme that they would diuide, as there are Cyphers in the firſt places of the diuiſor: but they ſet all their Cyphers orderly vnder the firſt places of the number that they would diuide; and then with the other figure or figures (if there be many) they diuide the reſt of their ſumme. Another manner of the Argument.

Example. If they would diuide 725931 by 3400, they doe ſet their ſummes thus:

725931		
3400		

And then doe they diuide orderly till they come to the Cyphers: for there they ſtay and end their worke, as in this example.

They ſeeke how often 3 may be found in 7. which is two times, and 1 remaining: therefore

therefore they set 2 in the Quotient, and cancel, 3 and 7, and ouer 7 they set the 1 that remaineth, thus:

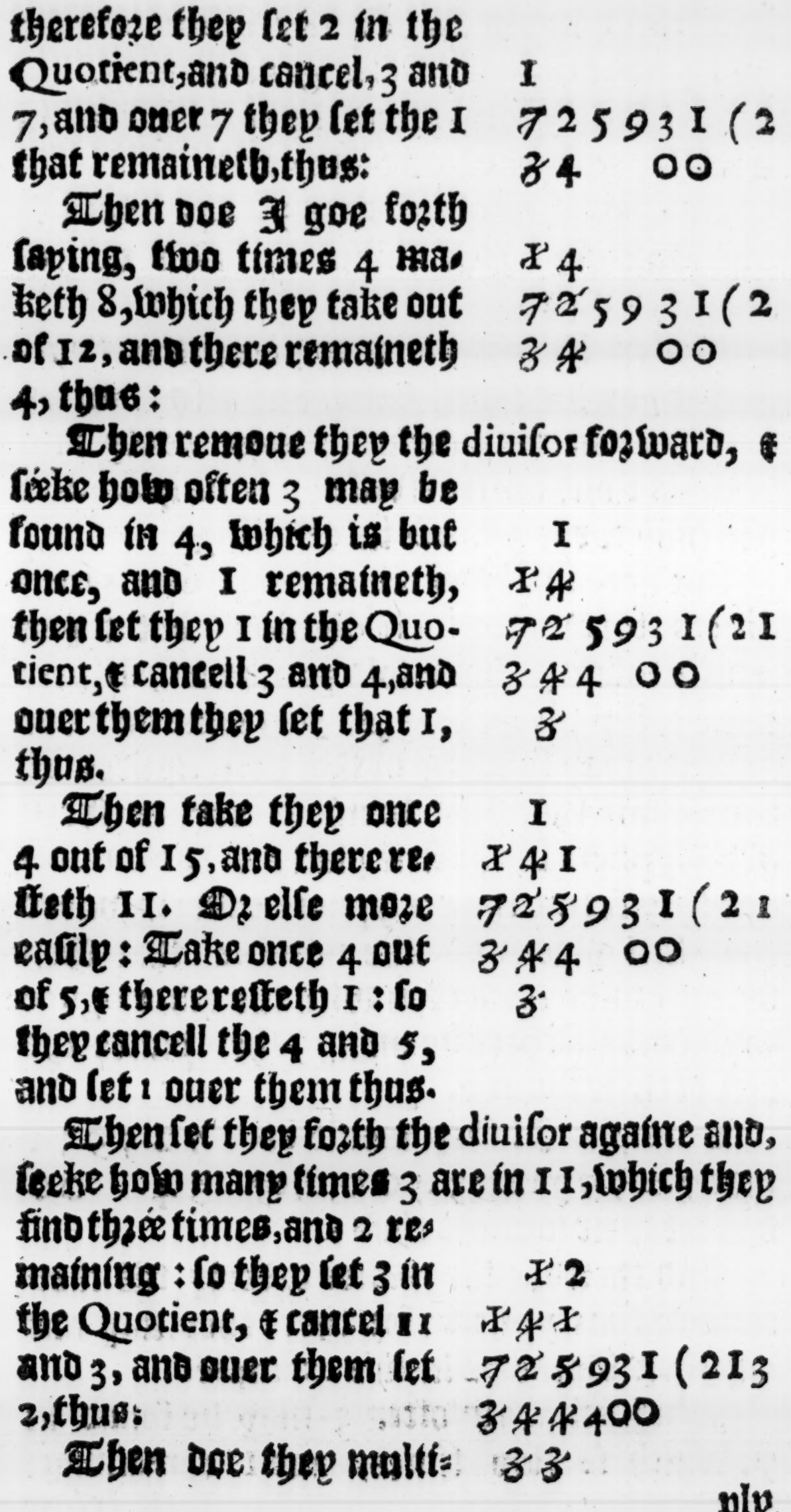
Then doe I goe forth saying, two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus:

Then remoue they the diuisor forward, & seeke how often 3 may be found in 4, which is but once, and 1 remaineth, then set they 1 in the Quotient, & cancell 3 and 4, and ouer them they set that 1, thus.

Then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5, & there resteth 1: so they cancell the 4 and 5, and set 1 ouer them thus.

Then set they forth the diuisor againe and, seeke how many times 3 are in 11, which they find three times, and 2 remaining: so they set 3 in the Quotient, & cancel 11 and 3, and ouer them set 2, thus:

Then doe they multi-



ply

ply 4 by 3, which ma-
keth 12; that with-drawe
they out of 29, and there
resteth 17, of which the
7 must bee set ouer the
9, and the 1 ouer the 2,
thus.

1
12
141
7
72593 1 (213
3433400
33

And now are the two Cyphers next ensuing, so that the Diuisor can no more be set forward, and therefore is the Diuisor ended, and the Remainer is 1731.

Now the Quotient which is 213, doth declare, that if you diuide 725931 by 3400, you shall finde it therein 213 times, and there remaineth 1731: so shall you finde it, if you worke as I taught you, by cutting away the 2 first figures, because of the 2 Cyphers.

But this must you marke (as you may perceiue by this last example) that if there be left any other remainer in the summe that was behind the squire line, that the remainer, must be set to the latter end of the first Remainer, which was cut away with the squire line: as if you would diuide 725931 by 3400, after the forme that I taught you, then would your summes appeare thus. Note.

1
12|
141|7
7259 |31(213
3444
33

So that 17 which remaineth after the line, must be set to the 31 (that was cut a-

way

way with the line) in higher places, as you se here: where that 17 with the 31, doe make 1731.

Scholar. Sir, is there no other forme of Diuision in practise but this?

Master. Yes verily, there are other formes in practice, but because I loue brevity, I will declare onely one, which I first learned of, and is practised by that worthy Mathematician, my ancient and especiall louing friend Master Henry Brigges, wherein not any one figure is defaced or cancelled. As if I should diuide 72 by 6, first place them thus:

Then if you please, you may write the diuisor in a loose paper that it may more easily without cancelling or defacing of the worke be applyed to, and removed from the diuidend at pleasure; then apply your diuisor 6 to 7, the first figure of the diuidend, and enquire how oft it may be had in 7, and seeing 6 is but once in 7, set 1 in the quotient line, thus:

Write the Diuisor in a loose paper, to remoue at pleasure.

Then multiply the diuisor 6, by the quotient 1, and set the product 6 vnder 7 thus:

Then draw a line vnder 6, and subduct

6)72.

6)72(1

6)72(1
6

6 out of 7, setting the remainer 1 vnder 6 thus:

Then bring downe the next figure of the diuidend, and set it with the Remainer 1 vnder the line thus:

And bring the moueable diuisor 6 vnder the 2, and as before enquire how oft 6 is in 12, and finding it to be twice in 12 set 2 in the quotient thus:

```
6) 72 (1
    6
   --
    1

6) 72 (1
    6
   --
    12

6) 72 (12
    6
   --
    12
    12
   --
    0
```

And multiply 6 by that new quotient 2, setting the product 12 vnder the other 12, and subducting it out of the vpper number, there resteth nothing. And since the vnities of this product doe stand vnder the vnities of the diuidend, the diuision is ended: otherwise you should proceed as before, bringing downe the next figure; removing the diuisor, diuiding, multiplying, subducting, &c.

Scholar. This is very easie, but if there be greater numbers propounded, is the operation the same?

Maister. If the numbers be neuer so great, the worke is the same without any difference, as shall appeare by this example.

Diuide

Diuide 7890 by 33.

First let them thus, then bring the Diuiſor vnder 78, and ſee how oft it is there found, which it twice, and therefore ſet 2 in the Quotient, by which multiply the Diuiſor 33, and ſet the product 66 vnder 78, & ſubduct it out of it thus.

Then bring the next figure 9 downe, and ſet it with the Remainer 12, is maketh 129, and remouing the diuiſor 33 thereto, enquire how often 33 is contained in 129, and I finde it but thrice, (though at the firſt it made a ſhew of more) therefore ſet 3 in the Quotient, and multiplying 33 by 3, ſet the product vnder 129, ſubducting that product out of the number aboue, and proceed as before:

```
33 ) 7890 ( 2
33)7890(239 3/33
 66
----
 129
  99
----
  300
  297
----
    3
```

Then ſhall you finde the diuiſor 9 times in the Remainer, therefore ſetting 9 in the Quotient, multiply, and ſubduct as before, and at the laſt you ſhall finde onely 3 remaining, which muſt be ſet aboue a line after the Quotient, and the Diuiſor vnder, as aboue appeareth.

Scholar. Is there no more difficulty in the

the whole Rule?

Maſter. Not any, although your number be neuer ſo great, as before I haue ſaid.

¶ And here will I make an end of Diuiſion, (ſauing that I doe requeſt you to exerciſe your ſelfe well herein by many ſummes, till you haue attained ſome expertneſſe therein.)

For the reaſons and concluſions thereof are ſo many, and ſo auaileable for all ſorts of men whatſoeuer, that if I ſhould ſpeake of the infinite vſes thereof, I ſhould rather lacke words then matter. And therefore recommending it to your iudgement hereafter, vpon your further trauell into the Art, I will here end this Treatiſe, repreſenting vnto you one example or ſimple queſtion of Diuiſion and Multiplication, in ſtead of many, which is this.

A queſtion offshooting in Ordinance.

There are foure braſſe Peeces. The firſt of them at a ſhot ſpendeth 9 pounds of powder, the ſecond ſpendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed againſt the battery of a hold, & there is allowed by the Maſter Gunner 700 pounds of powder to be ſpent by theſe foure Peeces in this aſſault. The queſtion is twofold: The firſt how many ſhot each Peece ſhall iuſtly make about with this 700 pounds of powder? And laſtly, how many pounds of powder ought iuſtly to be allowed to each Peece for his true proportion?

Scholar.

Scholar. Why Sir, you make me ſmile, to beare me in hand, that theſe two demands may be ſimply reſolued by Multiplication and diuiſion.

Maſter. Truly that they may, and that you may by and by worke your ſelfe with a little labour: firſt adde together their quantities of powder, that is, 9 pounds, 5 pounds 4 pounds and 2 pounds, al which make 20: Diuide the 700 pounds of powder by that 20, and your quotient glueth 35. as here appeareth, which ſheweth for moſt certainty that they ſhall make iuſt 35 ſhootes about.

X	
700	
200	(35
2	

Scholar. Sir, all this haue I done, and I ſée it is ſo, but whether it be true or not, I cannot tell.

Maſter. To trie the truth of the ſame, multiply the firſt peece that ſpends 9 pounds by 35, & you ſhall ſée his allowance, which is 315 pounds of powder. Multiply alſo the ſecond peece that ſpends 5 pounds by 35, & you ſhall finde 175 pounds his allowance: then 4 by 35 and you ſhall finde 140 pounds his allowance. Laſtly, multiply 2 by 35 and you ſhall finde 70 pounds his allowance. All which foure particular ſumms you ſhal adde together by Addition, as here appeareth, and it maketh iuſt 700 pounds, & ſo is the queſtion truly abſolued.

315
175
140
70
700

Scholar.

Scholar. Truly ſir, theſe excellent concluſions doe wonderfully moze and moze make mée in loue with the Art.

Maſter. It is an Art, that the further you trauell, the moze you thirſt to goe on fozward. Such a Fountaine, that the moze you dzaw, the moze it ſpzings. And to ſpeake abſolutely in a wozd (excepting the ſtudy of Diuinity, which is the ſaluation of our Soules) there is no ſtudy in the World comparable to this, foz delight in wonderfull and godly exercíſe: Foz the ſkill hereof is well knowne immediately to haue flowed from the wiſedome of God into the heart of man, whom hée hath created the chiefe Image & Inſtrament of his pzaiſe and glozy.

Scholar. The deſire of knowledge doth greatly encourage me to be ſtudious herein: and therefoze I pzay you ceaſe not to inſtruct me further in the vſe héereof.

Maſter. With a good will. And now therefoze foz the further vſe of theſe two latter, that is, Multiplication and diuiſion; I will bziefly ſhew you the feate of Reduction.

Reduction.

Reduction.

Reduction what it is.

REduction is, by which all ſummes of groſſe denomination may bee turned into ſummes of more ſubtile denomination. And contrawiſe, al ſummes of ſubtile denomination, may be brought to ſummes of groſſer denomination.

Scholar. What call you groſſe denomination, and ſubtile denomination?

Groſſe denomination.

Maſter. That I call a groſſe denomination, which doth containe vnder it many other ſubtiller or ſmaller: as a pound, (in reſpect to ſhillings) is a groſſe Denomination: for it is greater then ſhillings, & containeth many of them.

Subtile denomination.

And ſhillings (in comparison to pounds) are a ſubtile denomination, for becauſe they are leſſer then pounds, and many of them are contained in one of the other: and ſo likewiſe of other things, whatſoeuer thing is compared to other, if it be greater, and containeth many of them, it is a groſſer denomination: but if it be leſſer (ſo that many of them are in the other) then are they called the ſubtile denominations: whereby you may perceiue that one denomination may bee called a groſſe denomination, and alſo a ſubtile, (that is to ſay, a great & a ſmall) in diuers compariſons. For ſhillings compared to pounds, are a ſubtile or ſmall denomination: but compared to pence,

they

they are a groſſe or great denomination.

Scholar. Now I vnderſtand the name, I pray you teach me the vſe.

Maſter. The vſe is eaſily learned, if you remember what you haue learned before. For if you will reduce any ſumme of a groſſe denomination into a ſumme of a ſmaller or ſubtiler Denomination, you muſt conſider how many of that ſubtiler Denomination do make one of the groſſer Denomination, and by that number or Numerator doe you multiply the ſumme: as if you would reduce 20 pounds into ſhillings, you muſt conſider that in a pound are included 20 ſhillings, therefore multiply the one 20 by the other 20, and there will amount 400, whereby you may know that in 20 pounds, are contained 400 ſhillings. Likewiſe, you would reduce 30 ſhillings into pence, conſidering that in a ſhilling are 12 pence, you muſt multiply 30 by 12, and it will be 360, whereby you may finde that in 30 ſhillings are contained 360 pence. And thus may you reduce any groſſe denomination into a more ſubtiller, by multiplication, if you know how many of the leſſer doe make the greater: of which thing I will anon giue you a briefe Table for the moſt accuſtomed kinds of money, weight, meaſures, and time, and ſuch like: whereby you may know how often each ſubtile denomination is contai-

To Reduce groſſe denomination to ſubtile.

To reduce subtile denomination to groſſe.

ned in the groſſer, when you ſhall need it for the foreſaid kind of Reduction. And alſo the ſame ſhall ſerue you, if you would reduce any ſumme of a ſubtiler denomination, into a ſumme of a groſſer denomination. For in ſuch Reduction you muſt conſider (as in the other forme) how many of the ſmaller doe make the greater: and by that number you muſt diuide the other ſumme, and the Quotient will declare how many of the greater denomination are comprehended in that ſumme; as for example. If you would know how many ſhillings are contained in 3240 d, conſider that 12 pence doe make 1 s: you muſt diuide that 3240 by 12, and your Quotient will be 270, whereby you know that ſo many ſhillings are in 3240 d. But if you would know further how many pounds are in theſe 270 ſhillings, ſeeing that euery pound containeth 20 ſhillings: diuide that 270 by 20, & it will be 13, and 10 remaining, whereby you may know that in 3240 d, (or 270 ſhillings) are 13 pounds and 10 ſhillings. For euermore the remainer muſt be named by the name or denomination of that ſumme that was diuided, which in this place were ſhillings. And thus may you doe with any other kinds of Denominations.

☞ ¶ Wherefore to the intent you may haue certaine light or knowledge in moſt common coynes, weights, and meaſures (which is the chiefe and principalleſt thing in traffick to be knowne),

knowne) I haue in each reduction as they come in order, set down certain instructiõs incident thereunto. And first I haue hereunto added this Table, wherein is comprehended, not onely our currant and common coynes, but also the most part of the vsuall coynes of Christendome, with their iust weights and value currant in this Realme of England, intending at the latter end of my Addition to this Booke, to write of the ordinary money vsed in diuers places, and their common values currant for trafficke, with the manner of their exchanges from place to place, &c.

A Table of the names and now valuation of the moſt vſuall Gold-coynes throughout *Chriſtendom, with their ſeuerall weight of Pence and Graines: and what they are worth* of currant Engliſh money this preſent yeere 1630.

The names and titles of the Gold	The waight in Pence.	Grains.	The value in Shill.	Pence.
Great Soueraigne.	10	0	33	0
Double Souer. K.H	8	1	22	0
Double Souer of Q E	7	7	22	0
Royall.	4	23	16	6
Halfe Royall.	2	11.d	8	3
Olde Noble.	4	6	14	8
Halfe Noble.	2	3	7	4
Angell.	3	8	11	0
Halfe Angell.	1	16	5	6
Salute.	2	5	6	11 ob
2. parts of Salute.	1	11	4	7
George Noble.	3	0	9	9 ob
Halfe George Noble.	1	12	4	11 q
Firſt crowne K.H.	2	9	6	11 ob
Baſe crowne K H.	2	0	5	6
Souer. K H. beſt.	3	14	11	8 ob q
Souereigne K.H	4	0	11	0
Edward Souer.	3	15 d	11	0
Elizabeth Souer.	3	15 d	11	0
Elizabeth Crowne.	1	19	5	6
Halfe Crowne.	0	19 d	2	9
Vnite.	6	12	21	0

Double

Double Crowne.	3	6	11	0
Britane Crowne.	1	15	5	6
Thistle Crowne.	1	7	4	4ob q
Halfe Crowne.	0	19d	2	9
Crosse dagger	3	6d.	11	0
Halfe crosse dagger.	1	15	5	6
Rose Royall.	8	12	33	0
Spurre Royall.	4	10d	16	6
The Angell.	2	23d	11	0
Halfe Angell.	1	11d	5	6

¶ *All the seuerall peeces of Gold heretofore mentioned are set downe according to their valuation by the Kings Maiesties Proclamation for Gold, Dated the 23. of Nouember,* 1611.

A Table of forraigne gold coyne, according to their ancient valuation and seuerall weight in pence and graines.

The names and titles of the Gold.	The weight in Pence, Graines.		The value in Shil. Pence.	
Vnicorne of Scot.	2	10	6	0
Scottish Crowne.	2	5	6	0
French Noble.	4	16	13	4
All sorts of French Crownes.	2	5	6	0
Flanders Riders	2	6	6	6
Gelders Riders	1	2	3	6
Philips Riders	3	10	10	0
Philips Crowne.	2	5	5	0
Collen Gilden.	2	2	4	8
New And Gild.	2	2	5	0

The names and titles of the Gold.	The weight in Pence, Graines.		The value in Shil. Pence.	
Flaunders Noble.	4	10	12	0
Halfe Flaun. Noble.	2	6	5	0
Fiem : Angell heft	3	6	9	0
Flaund. Royall orkey.	3	10	10	0
Carolus Gilden.	1	21	3	6
Flaunders Royall.	2	6	5	0
Saxon Gilden.	1	2	4	8
Flaunder Crowne.	2	5	6	0
Philips Gilden.	2	3	4	2
Halfe Phil. Gilden.	1	1	2	1
Golden Lyon.	2	16	7	8
3 parts of gold Lyon.	0	21	2	5
$\frac{1}{3}$ parts of gold Lyon.	1	19	4	11
Dauds Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old Andre Gilden.	2	3	4	10
Crusa long crosse:	2	6	6	0
Crusa short crosse.	2	6	6	2
Milreyes.	4	20	13	4
Halfe Milreyes.	2	10	6	8
Portigue 1 ounce.	2	16	68	0
Golden Castilo.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungary Ducket.	2	7	6	4
Double Pistolet.	4	9	11	8
Single Pistolet.	2	4 d.	5	10
Ducket of Floren.	2	5	6	4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
Double duc of Rome	4	13	2	8

It is to be vnderſtood (gentle Reader) that whereas in theſe Tables, the weight is called by the name of a penny, it is not ment a penny of ſilver money, but a penny of Gold-ſmiths weight, which containeth 24 Barlycornes. Concerning which, ſee Troy weight in folio 133.

So if a man haue not the weight wherewith to weigh any peece of gold, he may doe it with Barleycornes, being dry, and as it is ſaid, fol. 133.

The prices of Gold which the bringers in of *forraine Gold ſhall receiue at the Mint*, according to the Kings Maieſties *Proclamation. Dated the* 14 of May, Anno 1612.

For an ounce of *French* crowns, being 22 Karects fine.	3 li, 6 ſ.
For euery ounce of *Spaniſh* Piſtolets, being 21 Karects, 3 graines and a halfe fine.	3 li, 6 ſ.
For Duckets of *Spaine*, being 23 Karects, 1 graine fine at leaſt the ounce.	3 li, 8 ſ, 8 d
For Milreas Cruſado long croſſe. Cruſado ſhort croſſe, the ounce.	3 li, 6 ſ, 2 d
For *Hungary* Duckets being 23 Karects, 1 graine fiue at leaſt the ounce.	3 li, 9 ſ,
For the Checkeen of *Venice*, being 23 Karects, 1 graine fine at leaſt the ounce.	3 l, 10 ſ.

For

For *Barbary* Gold, being 23 Karects and digraine fine, at the leaſt the Ounce.	3 li, 9 ſ,

¶ *And if the ſaid Barbary Gold bee of leſſe fineneſſe, abatement to be made according to the rate.*

For Sultaines being 23 Karects, 1 grain fine at leaſt the ounce.	3 li, 8 ſ, 8 d.
For all other Gold, being 12 Karects fine the ounce.	3 li, 6 ſ.

¶ *And being finer, a greater price according to that rate, and being courſer, a leſſe, ſo that the bringer in ſupply the leſſe fine, with the more fine, in ſuch ſort, that in the totall it make good the ſaid rate of 22 Karects fine.*

The price of Siluer, which the bringers in of forraine Siluer ſhall receiue at the Mint, according to the *Kings Maieſties aforeſaid* Proclamation.

FOr the Ounce of *Spaniſh* ſiluer money of *Ciuill*.	5 ſ.
For the Ounce of *Mexico* money.	4 ſ, 10 d.

For

For Ingots of Siluer, being 11 Ounces, 2 d, weight fine, according to the Standard of *England*, the Ounce. —— 5 £

¶ *And for other Siluer of more fineneſſe, a better price according to that rate, and for courſer a leſſe: ſo that the bringer in ſupply the leſſe fine with the more fine, in ſuch ſort, that in the totall it make good the ſaid rate of* 11 *Ounces,* 2 *pence weight fine, according to the Standard of England.*

Of Siluer Coynes currant in this Realme.

The *Edward* Crowne of 5 ſ.
The *Edward* halfe Crowne of 2 ſ, 6 d.
The *Edward* Shillings, halfe Shilling, and the three pence.
Phillip and *Maries* Shilling, and halfe Shil.
The *Mary* Groat, and *Mary* two pence.
Queene *Elizabeth* Shilling. 9 d, 6 d, 4 d, 3 d, 2 d, 1 d, three farthings, and halfe penny.

HEre would I now expreſſe the values of ſundry other Coynes of diuers Countries, but for three cauſes I now refraine. The firſt and cheefeſt is, becauſe they are not currant by the Statutes of this Realme. Another cauſe is, by reaſon they are ſo vncertaine, that they be neuer long at one rate.

rate. And againe, they are so different in so many places, that it were matter enough for a great Booke to speake sufficiently of them all. Howbeit, because you shall not be altogether ignorant of them, I will shew you the values of some that are most in vse, and first of France.

French Coynes

The most common money are Deniers Soulx, and Frankes: 12 Deniers makes 1 shilling, 20 Soulx make 1 Franke: so that as you see these 3 kindes are like in the rate to pence, shillings, and pounds with vs; but that this is the difference, that their Denyer is but the ninth part of our penny, and so their Soulx, (commonly called Souses) go 9 to our shilling, and 9 of their Frankes to an English pound of money. So that 3 of their Frankes make a Noble. And by those three may you practise how to reduce French money into English money, according as I haue set forth here following-

2160 } Deniers make { 240 d, or 20 s.
3240 } { 360 d, or 30 s.
8352 } { 929 d, or 2 li, 17 s, 4 d

2160 Soulx make 240 shillings. And so of other in like rate. As for the rest of their Coynes, I omit them till hereafter, that you haue some vnderstanding in broken numbers.

Flanders Coynes.

But now as for the Coynes of Flanders they be so changeable, that you must know them from time to time: else you cannot reduce them into our money certainely: but yet that

that you haue an example of their money to exercise you withall, you shall take those that be most common: as Stiuers both single and double, Groats Flemish, Carolus, and Gyldens. A Flemish Groat is a little aboue 3 Farthings English. A single Stiuer is 1 d, ob. q; halfe farthing. The double Stiuer Carolus is 4 4 d, ob. halfe farthing. Then there is also the Carolus Gylden, which is worth 20 stiuers. And the Flemish Noble is worth 3 Carolus Gyldens, and 12 Stiuers.

So that if you would conuert Flemish money, or any other kind of money whatsoeuer it be, iustly into sterling, you must reduce it first into the smallest part of English money that is in that Coyne, as for example. If I would reduce 398 double stiuers into English money (considering that a double stiuer containeth 3 d, farthing) you shall first loke how many farthings be in the double stiuer, & you shall find them 13: therefore multiply the summe of the Stiuers by 13, and then haue you their value in farthings, which is 4784. Now, if you diuide that by 4, then there will appeare the number of pence: but better it were to diuide it by 48 (for so many farthings are in 1 shillings) and then will the Quotient declare the summe of the shillings.

Likewise if you would reduce any summe of single Stiuers into English money, you must multiply the summe first by 13, and then haue you reduced them into a certaine summe, that is

is to wit, halfe farthings, which ſumme if you diuide by 8, then will amount the ſumme of pence: or if you diuide it by 96, the ſumme of ſhillings will appeare.

But marke this in all Diuiſion: when yee doe reduce to bring one Denomination into another, if there be any Remainer after the Diuiſion, that muſt be named by the Denomination of the groſſe ſumme that was diuided. As for example. I would bring 254 farthings into pence, therefore I diuide that 254 by 4 (for ſo many farthings make a peny) and the Quotient is 63, which is the ſumme of the pence, and then remaineth yet 2, which are farthings ſtill, as one may proue by diuiding. And this muſt be marked in all Diuiſion, namely, when it is done for Reduction.

Note wel:

¶ Touching Danske Money, they haue their Soulx, whereof 20 is a Liuer: which is 2 ſhillings ſterling. They haue alſo their Graſh, whereof 80 make a Gilden, which is 4 ſhillings ſterling. They haue alſo Dollors, & their common or old Dollor is 35 Graſh. New Dollors they haue, which be diuers, ſome valued at 24 Graſh, ſome at 26, and ſome at 30. And thus much I thought good to adde to the Authour, touching Danske Money.

Danske Money:

Concerning Spaniſh Money, whereof the moſt common are Cornados, Marueides, Ryals, and Duckets: ſixe Cornados make a Marueide, 4 Marueides make a Ryall, and

Spaniſh Money:

11 Ryals

11 Ryals make one Ducket, so the Ducket containeth 374 Marueides, which is about 5 shillings, 10 pence, sterling. Therefore if you would conuert 124 pounds, 5 shillings, sterling into Duckets, consider that pence is the least value or Denomination named in this question: therefore reduce 124 pounds, 5 shillings into pence, and it maketh 29820 pence: which if you diuide by pence that a Ducket is worth, (which is 70,) you shall haue for your Quotient 426 Duckets, your desire.

In Venice they haue bettes, Souldyes, Lieures. 5 bettes make an English penny, 60 bettes a shilling, which is 2 Souldyes, and 20 Souldyes a Lieure of Venice which is a pound sterling. Venice money.

Thus much haue I said of Money. Now will I shew you in like sort, the distinction of weights. Weights.

After a Statute made Anno 11. H. 7. there ought to be but one sort of weights. As 24 Barley-cornes dry and taken out of the middest of the Eare, doe make a penny weight, 20 of those penny weights make an ounce, and 12 ounces a pound of Troy weight, by which is weighed Bread, Gold, Siluer, Pearle, Silke, and such like. But commonly there is vsed another weight called Haberdupoise; in which 16 ounces make a pound. Therefore when you would reduce ounces into pounds, you must consider whether your weight bee Troy weight or Haberdupoise: and if it be Troy weight, you must diuide

Troy weight. A Penny weight. An ounce A Pound Troy. Haberdupois weight.

diuide your ounces by 12, to bring them to pounds, but if it be Haberdupoise, you muſt diuide them by 16. Now againe, there be greater weights, which are called a hundred, halfe a hundred, and a quarterne, and alſo a halfe quarterne, &c.

A hundred weight.

Scholar. Why: ſo there may be reckoned 20 pound. 40 pound, 200 pound, and ſuch innumerable.

Maſter. All theſe are numbers of waight, but they haue not common waights made to their rate, as the other haue. And againe theſe that I did name, are not iuſt in number as they ſeeme by their name: for an hundred is not iuſt 100, but is 112 pound. And ſo the halfe hundred is 56: the quarter 28, and the halfe quarter 14. And theſe be the common weights vſed in moſt things that are ſold by weight.

Wooll. Weights, Todde. Stone.

Holwbeit there are in ſome things other names, as in Wooll, 28 pound is not called a quarterne, but a Todde: and 14 pound is not named halfe a quarterne, but a Stone, and the 7 pound halfe a ſtone. Other names becauſe they differ in many places, and agree in few, I let them paſſe.

Sacke of wooll.

But a ſacke of Wooll by the Statutes, is limited to be 26 ſtone.

Cheeſe weights.

¶ Now in Cheeſe, though it be ſold by the hundred, and by the ſtone in ſome places, yet the very weights of it are Cloues, and weyes. So that a cloue containeth 8. pound, and

a

a wey 32 Cloues, which is 256 pound, that is 12 score and 16 pound, and so much weigheth the wey of Suffolke Cheese, and the weight is 02 should be the Barrell of Suffolke butter.

The Wey of Essex Cheese containeth six score and sixteene pound: and so much is also the Barrell of Essex butter.

Moreouer the weight is vsed by the Apothecaries in their Physicall composition, and mixture of medicine, wherein the last is a graine.

And	20 Graines	make	A Scruple	thus charactered	℈
	3 Scruples.		A Drachme or Dragme		ʒ
	8 Dragmes.		An Ounces		℥
	16 Ounce.		A Pound.		lb.

The Apothecaries weights

Now of weights are made other measures both for graine & liquor. For a pound in Troy weight, maketh a pint in measure, so that 8 pound or 8 pints do make a gallon: halfe a gallon is named a pottle, and halfe a pottle is called a quart, which containeth two pints. Now aboue a Gallon the next measure is a Firken: then the Tercian a Kilderkin, or halfe Barrell, & a Barrell And by these measures are sold commonly Ale, Beere, Wine & Oyle, Butter and Soape, Salmon, Herrings, and Eeles.

Measures for liquor.

A Pint.
Gallon.
Pottle.
Quart.
Ferkin.
Tertian.
Kilderkin
Barrell.

But as these be vnlike things, so the measures of their vessels doe differ, for the measures of them all are as followeth.

Of Ale	the Firkin	containeth	8	Gallons.
	the kilderk.		16	
	the barrel.		32	

Ale measure.

Beer measure.

Of Beer	*the Firkin.*	*containeth.*	9	*Gallons.*
	the kilderk.		18	
	the barrell.		36	

Sope measures.

Sope measures, both Firkin, Kilderkin, and Barrell should be equall to Ale measure.

Moreouer the Statutes doe limite the weight of euery of those three vessels being empty.

A Barrell	*to weigh empty*	26	*pounds.*
Halfe a Barrell		13	
A Firkin		$6\frac{1}{2}$	

Herrings.

Herrings also sold by the same measures that Ale and Soape be sold by.

Herrings are sold by the tale, 120 to the hundred, ten thousand to the last.

Salmon & Eeles.

Salmon and Eeles haue a greater measure,

Salmon & Eeles	*the Butte*	*holdeth*	84	*gallons:*
	the Barrell		42	
	halfe Bar.		21	
	the Firkin		$10\frac{1}{2}$	

Howbeit, some Statutes did limite Eele vessels equall with Herring vessels.

Wine measures.

Now as for wine vessels they are seldome smaller then Hogsheads, which are of 63 Gallons: Euery Hogshed, is two Barrels: yet there are many other wine vessels, but of them all see this Table; and marke the measures one by another.

Of

Of wine and oyle	the Runlet	holdeth	18½ Gallons.
	the Barrell		31½
	the Hogsh,		63
	the Tercian		84
	the Pipe		126
	the Tunne		252

But you shall marke that there bée other kinds of Tertians: for there be Tertians, (that is to say) Thirds of Pypes, of Hogsheds, and of Barrels, as well of other things as of wine. Tertians.

Also Malmseyes, and Sacke, &c. the halfe Tun is not called a Pipe, but rather a Butte. A Butte.

And thus much haue I thought méet to tel you at this time.

Scholar. And is that alwaies true?

Maister. I haue told you how it should be, but how it is, I may not say: how they doe differ daily from their iust measure, that Gaugiers can tell you better then I. But I will let this passe now, and speake briefely of the other measure.

And as of weights there did spring the liquid measures, (whereof I spake last) so of the same springeth dry measures, as Pecks, Bushels, Quarters, and such like, whereby are measured Corne and like graines, also Salt, Lime, Coales, and other like. And this is the order and quantitie of them. Drie measures.

A Pecke is the measure of two Gallons.

A Bushell
A Quarter.
A Wey.

A Bushell } *A Quarter* } *A Wey* } containeth { *foure Peckes.* *eight Bushels.* *six quarters.*

These are the common names and measures but in diuers places there be diuers sorts.

Strike: Cornock

The Bushell in many places is two Bushels, but then is that Bushell there called a Strike: and in some places halfe a quarter is called a Cornocke. But these diuersities are too many to tell you briefly them all: and againe, sith they are against the law and statutes, I count them vnmete to be vsed.

Measure to mete length, breadth, & thicknesse.

But now remaineth yet another kind of measures, whereby men mete length, breadth and thicknesse, and these are, an Inch, a Foote and such other: whose names and qualities this Table sheweth.

An Inch.
Foote.
Yard.
Elle.
Perch.

3 *Graines of Barley in length make an inch.*
12 *Inches.* } *make* { *a Foote.*
3 *Foote* } *make* { *a Yard.*
3 *Foote and 9 Inches* } *make* { *an Elle.*
5 *Yards and a halfe* } *make* { *a Perch.*
1 *Perch in breadth, and 40 in length, doe make a rodde of Land, which some call a rood, some a yard land, some a Farthendele.*

Acre.

2 *Farthendels* } *make* { *halfe an acre of ground.*
4 *Farthendels* } *make* { *an Acre.*
More, 40 rods in length doe make a furlong, 8 furlongs make an English mile, which containeth 320 Perches.

So that an English mile, grounded vpon the Statute, is in length 1760 yards, 5280 foote,

foote, and 6 3 3 6 0 Inches.

Some what greater then the Italian mile of 1 0 0 0 paces, and 5 foote to a pace.

Here might I tell you many things else touching measures, and also how to reduce strange measures to our measures, but because it cannot be wel done without the knowledge of Fractions, which as yet you haue not learned, I will let them passe till an other time, that I haue taught you the knowledge of broken numbers.

Scholar. But yet sir of the parts of time, I pray you tell me somewhat. The parts of time.

Master. You know that a naturall day hath 24 houres, and euery houre hath 60 minutes. It needeth not to tell you, that 7 dayes make a weeke, & 4 weekes make a common moneth, and 12 moneths make a yeare, lacking one day, & certaine houres & minutes: but of that I shall instruct you hereafter. A day. An houre, Weeke Moneth. Yeare.

Here will I make an end of Reduction for this time, which though it be counted no kinde seuerall of Arithmeticke, you see it is no lesse needfull to be knowne, or easier to be done, then any of the other.

Scholar. Marry sir, it seemeth vnto me much harder then any other sort, for it requireth the knowledge of so many things: but now sir when you see time, I am ready to learne forth: as much of Reduction as you haue taught mee, I remember; but and if I doe at any time forget, I shall haue re-

course to the Tables which you set forth for me.

Master. So doe you, for it will not be remembred without exercise. But in as much as you understand so much as wee have intreated of, I will now instruct you in Progression.

Progression.

Progreſsion.

ALthough vntill this day the moſt part of Writers haue defined Progreſſion as a compendious kinde of Addition, yet truly it is not ſo: for Progreſſion (as the very nature of the word doth informe any man) is a going forward and proceeding in numbers, and that regularly and orderly, whoſe place is aptly choſen to be very neere, or rather next after the expoſition of the foure principall parts of Arithmeticke, for in it after a moſt eaſie manner, are all the foure former parts exerciſed and practiſed: and not onely Addition, as cuſtomably is done. Which cuſtome hath beene the cauſe, why it hath ſo ſpecially beene named a kinde of Addition, and defined to be a quicke and briefe Addition of diuers ſummes, proceeding by ſome certaine and reaſonable order. What progreſſionis,

You ſhall alſo vnderſtand there are infinite kinds of Progreſſions, **but for you (as yet) two are ſufficient to be exerciſed in, of which the one I call** Arithmeticall, **and the other** Geometricall.

Arithmeticall Progreſſion is a rehearſing or placing downe of many numbers, number after number, in ſuch ſort, that betweene euery two next numbers rehearſed or placed downe, the difference, diuerſity, or exceſſe, be equall and alike. Arithmeticall Progreſſion.

Scholar. Sir, I thanke you for that you haue both opened vnto mee what Progreſſion is truly, and alſo why it is here placed.

But I pray you with an example make plaine your definition.

Maſter. examples cannot want, ſeeing all reaſonable creatures naturally vſe the order of one kind of Arithmeticall Progreſſion (which therefore is alſo named naturall) whenſoeuer they diſtinctly do count or number any multitude by one, ſaying, 1.2.3.4.5.6, whereby the proceeding from number to number, & euery one ſurmounting and exceeding his fellow next before by a like quantity (which here is 1) declareth the ſame to be Arithmeticall Progreſſion. And for the more plainneſſe, I ſet it downe in this manner.

The common Exceſſe. 1 1 1 1 1

The Progreſſion: 1 2 3 4 5 6

Scholar. This is moſt euident. And I thinke that I am able to tell you now of any Progreſſion Arithmeticall propounded, what is that common exceſſe or difference whereby it proceedeth, if this order be kept in it.

Maſter. What ſay you of 3,6,9.12,15?

Scholar. They exceede eache other by 3: And that may I ſet downe in ſuch euident order, as you did your example of naturall Progreſſion, in this wiſe.

The common exceſſe

The Porgreſſion. 3 6 9 12 15

Maſter. And doe you not alſo now perceiue, that the whole Table of Multiplication may be made by the order of Progreſſion Arithmeticall? either if you will begin at the firſt number of any of them on the left hand, and ſo proceed right ouerthwart: or at any of the firſt numbers of the vpper row, and goe directly downe-ward.

Scholar. I pray you let mee conſider the thing a little, and I will anſwere you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now very wel, for the common exceſſe or difference between any two next, is continually as much as the firſt number of euery row, either from the left hand ouerthwart taken, or from any

of

of the vppermoſt ouerthwart rowes downeward.

To know the totall ſumme of an Arithmeticall Progreſſion.

Maſter. Now then, if of any ſuch Progreſſion, you would ſpædily know the totall ſumme much quicklier then by common Rules of Addition: firſt tell how many numbers there are (which numbers here we call places, or parcels) and if they be odde, write their ſumme downe by it ſelfe; as in this example, 2,4,6,8,10,12,14, where the numbers are 7, as you may ſee: therefore ſet downe 7 in a place alone, then adde together the firſt number and the laſt, as in this example; Adde 2 to 14, and that maketh 16, take halfe of it, and multiply by the 7 which you noted for the number of the ſame places, and the ſumme that amounteth, is the ſumme of all thoſe thoſe figures added together: As in this example, 8 multiplyed by 7, maketh 56, and that is the ſumme of all thoſe figures.

Scholar. That will I worke by another example. I would know how much this ſumme is, 5. 8, 11, 14, 17, 20, 23. 26, 29 I tel the places and there are 9, that I note. Then I put the firſt number 5, and the laſt 29 together, and they make 34, I take the halfe of it, that is 17, and multiply by 9, and it maketh 153. That you ſay is the ſumme of all the numbers,

Maſter. So ſhall you find it if you try it.

Scholar. How ſhall I try it?

Maſter. By your common Addition, for

if

if you adde all the parcels together, you ſhall ſe the ſame ſumme amount, if you did worke well. And that manner of Addition tryeth all kinds of ſumming any Progreſſion.

Scholar. Then can I ſumme any Progreſſion, if the nembers of the parcels be odde, But what if they be euen: as in this example, 1,2,3,4,5,6,7 8?

Maiter. When the number of the parcels is euen, then note that alſo as you did before, and likewiſe adde the firſt ſumme to the laſt, and by the halfe of the number of the places doe you multiply it: as in our example, the parcels are 8, that I note: then adding the firſt ſumme to the laſt, there amounteth 9, that doe I multiply by halfe of the parcels, that is, by 4, and it maketh 36, which is the ſumme of the parcels.

But if you will take one Rule for theſe both, doe thus: Multiply the halfe of the one by the other whole, and the ſumme will amount all one. For ſometime it chanceth that the number of the parcels be odde, ſo their halfe cannot be taken: and that ſometime it chanceth the Addition of the firſt number and the laſt, doe bring forth an odde number, ſo that halfe of it cannot be taken: but they will neuer be both odde: A generall rule.

Scholar. Then I perceiue this, if there be no more belonging to it.

Maſter. As accuſtomably it hath béene taught, this hath béen the chiefe and oneiy exerciſe

exercise in Progreſſion vſed. But that you may perceiue how diuers wayes, and to how great profit ſo ſimple a thing (as this Arithmeticall Progreſſion is) may bée conſidered and vſed, I will héere propound you ſix propoſitions: of which foure of them were inuented by a friend of mine, and neuer before this publiſhed: and the two firſt, were neuer to my knowledge written of but by thrée Men.

Scholar. This doth greatly encourage me to be attentiue vnto your words, ſéeing I ſhal not onely be inſtructed at your hands in the common knowne Rules of this excellent Art, but beſides that, ſo abundantly in other new rules enformed, as my very entrance ſhall ſéeme to paſſe a great many mens further ſtudy and longer continuance. Therefore Sir, I beſéech you, let me know your ſix Propoſitions.

Maſter. Theſe they are.

1 *To know the laſt number without proceeding by continuall Addition, till you come vnto it, ſo that the common exceſſe, the firſt number, and the number of the places be knowne.*

2 *The firſt number of the Progreſſion and the laſt being knowne, with the common exceſſe to finde the number of the places.*

3 *The exceſſe being giuen, and the firſt or laſt, to know the quantity of any middle number, whoſe place is giuen from the firſt or laſt.*

4 *The totall ſumme being giuen, and the firſt and laſt, to finde out the number of the places.*

The

The totall ſumme of any Arithmeticall Progreſſion being giuen, and the firſt and laſt, to find out the common exceſſe. 5

The totall ſumme being giuen, and the mutuall exceſſe, with the number of the places, to giue the firſt or laſt number of the ſame Progreſſion. 6

Many moe conſiderations could I propound you in theſe *Arithmeticall Progreſſions*, but theſe are ſufficient, to giue you occaſion to thinke, that Rules of knowledge and *Arts* are infinitely capable of enlargement.

Scholar. Happy were I, if I did but well vnderſtand that which is already inuented and written: And yet in my ſimple fantaſie, theſe things offer themſelues (in manner) to be ſtudied for about Progreſſion, therefore I pray you to proceede to the Rules anſwering to theſe propoſitions.

Maſter. I will orderly for euery of theſe ſixe propoſitions giue you Rules, and with euery one an example, vnleſſe the plainneſſe and eaſineſſe néede no farther exemplifying.

For the Solution of the firſt Multiply the exceſſe by a number leſſe by 1 *then the number of the places, and the off-come adde to the firſt number, ſo you ſhall haue the laſt number, which is ſought for.* 1 Propoſition.

As for example. If there were ſeuen places in a Progreſſion Arithmeticall, whoſe continuall increaſe or mutuall exceſſe were 4, and the firſt number were 5, and I would know what the laſt and ſeuenth number is: I multiply 6, which is one leſſe then 7, (the

number

number of the places) by 4, thereof commeth 24, which I adde to 5, that maketh 29: and that is the laſt number which I deſire to knowe. And this you may ſtraight way proue by continuall proceeding from 5 till the ſeuenth place, encreaſing euery one by 4, as thus.

5 9 13 17 21 25 29.

Loe here, the laſt, being alſo the ſeuenth, is 29.

Scholar. I perceiue already one good property in this Rule, which in all workes is to be deſired: that is, it will eaſe one from great labor, if a Progreſſion were propounded of a hundred or two hundred places, or moe: And alſo it is very eaſie to worke, and moſt neceſſary for the total ſumme finding, in a very long Progreſſion.

Maſter. It is true, and therefore now let me ſee if you can anſwer me this queſtion by this propoſition.

A Marchant buyeth 50. pounds of Spices, & agreeth to pay for the firſt pound 4 pence, for the ſecond 7 pence, for the third 10 pence, for the fourth 13 pence, &c.

The queſtion is how much hee muſt pay for the laſt pound, and then how much the 50 pound commeth to?

Scholar. According to the propoſition, I multiply 49, (which is leſſe by one then the number of the places) by the exceſſe, which is

3, to the product 147, I adde the firſt number which is 4, it maketh 151 pence, the price of the laſt pound Now I adde 4, the price of the firſt pound, to 151 the price of the laſt pound it maketh 155, which I multiply by halfe the number of the places, which is 25 the product 3875 pence is the totall ſumme or price of the 50 pounds of Spices. as appeareth

49	Places 1 leſſe.	151	laſt
3	exceſſe	4	firſt
147		155	
4	firſt	25	halfe places
151	the laſt	775	
		310	
		3875	totall ſumme

which amounteth to

li	ſ	d
16	2	11

Maſter. It is truely wrought,

Scholar. Then I intreate you proceede to your ſecond propoſition.

Maſter. *The ſecond Rule is this. From the laſt ſubtract the firſt, the remainder diuide by the common exceſſe, to the* Quotient *adde* 1, *and you haue the number of the places, which you would know: As in this Progreſſion.*

6 11 16 21 26 31

If I know onely 6 and 31, and that they encreaſe

encreaſe by 5, then according to the rule, from 31 I ſubtract 6, there remaineth 25: which 25 I diuide by 5, (the common exceſſe) the Quocient commeth forth 5, to which I adde 1, that maketh 6: and ſo many are the places, as you ſee.

Scholar. This Rule is ſo eaſie, that I were much to blame, if I could not remember it.

3 Propoſition.

Maſter. *The third Propoſition may alwayes thus be ſolued. Multiply the* exceſſe *by a number leſſe by one, then the diſtance of the* place *is from the* firſt, *or the* laſt number giuen: *the of-come adde to the* firſt, *if the* diſtance *be reckoned from the* firſt, *and the* firſt *alſo knowne, or ſubtract from the* laſt, *if the diſtance be from the* laſt counted, *and the* laſt giuen *alſo, and that which commeth forth, either in that* Addition to the firſt, *or* ſubtraction from the laſt *is the number* ſought. As for example, *I propound you this* Progreſſion.

8 15 22 29 36 43 50 57

And for the apt conſidering the manner of this queſtion, I will note ouer euery place his diſtance from the firſt, and vnder euery place his diſtance inclusiuely from the laſt, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now if the exceſſe whereby this Progreſ-

ſion

sion standeth, be knowne to be 7, and the first number giuen, being 8, I would know what number standeth vnder 4, that is to say, in the fourth place. I multiply 7 by 3, (which is lesse by 1 then the number of the place propound) that yeldeth 21, to which I adde 8, the first number so commeth 29: which I say to belong to the fourth place, as ye see in the example it also doth: or if in the third place from the last, you would know what number in this example should stand, the last number being knowne to bee 57, and the common excesse 7, then by 2 which is lesse by 1 than the place propounded, I multiply 7, that giueth 14, which appertaineth to the third place inclusiuely reckoned from the last, and so my example giueth you.

Scholar. I perceiue right good vse of this rule: for if I had forgotten what the first number were, and remember still but the last, the common excesse, and the number of the places, then might I come by the knowledge of my first number againe.

And me thinketh, that it differeth not much from the first proposition, sauing that which you make here a middle number, there was made the last: and also in this point it differeth, that in it the last was onely sought, and no consideration had in numbring the places from the last, as here I marke in your numbers noted vnder your Progression.

Master. And thinke you not, the middle

numbers of Progreſſion ſtanding of a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continuall encreaſing from the firſt by the common exceſſe, or abating from the laſt continually the common exceſſe as the very ſmall numbers in a ſhorter progreſſion would doe?

Scholar. Yes ſir, that I thinke right well, & therfore I am glad of this new framed propoſition and the manner of the working of it.

4 Propoſition.

Maſter. *The rule of the fourth is this. Adde the firſt and the laſt together, and by the of-come diuide the totall ſumme. Double the quotient, and that will be the number of the places.*

Scholar. Then if in a Progreſſion, whoſe ſumme were 207, and the firſt number 12, & the laſt 57, if I adde 57 and 12 together, that maketh 69, and by it I diuide 207, the Quotient will be 3, which I double: and ſo I haue 6 and ſo many muſt be the number of the places that this Progreſſion ſtandeth on.

Maſter. Whether it bée ſo or no, how will you trie?

Scholar. Halfe 6, which is 3, being multiplied by 69, muſt make 207, the total ſumme; if 6 be the number of the places. For ſo the whole worke of your rule in ſumming any Arithmeticall Progreſſion did enforme me. I will then multiply 69 by 3, thus,

It commeth forth iuſtly.

69
3
207

Maſter:

Maſter. I muſt much héreín commend your promptneſſe, both in memorie and in wel applying your rule: although in manifeſt words it did containe no ſuch matter.

Scholar. Sir I pray you beare me frame one example more.

Maſter. I am well pleaſed, ſo that ye bée ſhort, for you make me more longer héere than willingly I would haue béen: but I cannot perceiue how I could haue omitted any thing as yet, without your great lacke thereof.

A queſtion of money

Scholar. If I had receiue 85 pounds of certaine men, but of how many I haue forgotten, yet I remember that the firſt gaue me 7 pound, and the laſt 27 pound, and euery payment after other did riſe by a like ſum. And the man for whom I receiued this mony, conditioned with me, that of euery payment I ſhould haue twelue pence for my labour: now vnleſſe I can by Art finde the truth of this caſe, I am like to loſe the moſt part of my reward.

Maſter. I perceiue you can handſomely frame an example, which ſhould concerne your owne gaine: I pray you let me ſée how you would do iuſtice in this point.

Scholar, I adde the firſt and the laſt together, that maketh 34: by which I diuide 85, thus:

1		
27	17	
85	(2—	
34	34	

Why how now: Sir, here is a remnant of 17. in which 34. cannot bée had:

had: ſo that now I am in the byars for doubling of my Quotient, and farewell then both my Iuſtice, and a good lumpe of my gaines.

Maſter. Yée are neuer the farther from the matter, though it fall into a Fraction. For you ſhall vnderſtand, that the Fraction which of any ſuch worke proceedeth, is euer halfe of one ſuch, as the vnites of the Quotient before are. And that you may trie, if you double that which ſo remaineth, for then it will be equall to your Diuiſor, as if yée double 17 (the remnant) it maketh 34, & your Diuiſor alſo was 34, this noteth the remainder to be halfe of one.

Scholar. Now I am glad of this hard example. For with it I haue a generall rule for the Fraction that may hap in this worke. So that the Quotient being two and a halfe, I & double that, it maketh 5, therefore ſhould my gaine be 5 ſhillings. And to bée ſure, (by your leaue) I will trie it, for I will multiply halfe of 34: (which is the firſt and laſt number ioyned together) by 5, thus.

	17
	5
	85

It is moſt true (I ſée) that I ſhould léeſe nothing by the former working.

Maſter. *The fift porpoſition hath this rule appertaining vnto it: By the fourth rule finde the number of the places, that being done, from the laſt ſubtract the firſt, & the reſidue diuide by a number leſſe by 1, then the number of the places, and the quotient will ſhew the exceſſe which is ſought for.*

5 Propoſition.

An

An example hereof ſhall be this: If ye had diſburſed 685 pounds to a certaine number of men, you neither can tell how many they were, or how much the ones money exceeded his next before, but you are ſure that the exceſſe was equall betweene euery two next: and alſo you remember that the firſt had 19, and the laſt 118 pounds, how would you finde the number of the mane & the exceſſe, continually obſerued in the ſucceſſion of their payments. Example.

Scholar. Your rule doth plainlie bid, firſt to finde the number of the places, which I will doe according to the fourth rule: I adde 19, and 118 together thus.

	118
	19
	137

By this 137, I diuide 685 thus.

Séeing there is no fraction but a whole number, being 5, I double that, and then muſt the number of the places bée 10. Now from the laſt I ſubtract the firſt, as 19 from 118, thus: And ſo remaineth 99.

	x3
	685(5
	x37
	118
	19
	99

This 99 I diuide by a number leſſe by one then the number of the places, and ſéeing the places were 10, I diuide 99 by 9, thus:

the Quotient is 11, and ſo was the exceſſe, if I haue followed

	99
	99(11

your rule right.

Maſter. You haue wrought euery part of this queſtion both well in order, and truely in the practiſe of your rules.

Scholar. I will then ſet it downe alſo formably, ſo that the number of the places, the exceſſe and the totall ſumme may ſtraight appeare, as your firſt example ſtood.

The common exceſſe. 11 11 11 11 11 11 11 11 11 11

The progreſſion. 19 30 41 52 64 73 85 96 107 118

That the places be 10, and that from the firſt to the laſt, the common exceſſe is 11, I perceiue moſt euidently: but whether the totall ſumme be 685. I haue not yet proued, which I will now doe: I adde 19 and 118 together, that maketh 137: I multiply that by halfe the number of the places, thus.

137
5
685

All things agree moſt exactlie, ſo that I am perfect enough in theſe rules, if I forget them not againe.

Maſter. Vſe maketh all things perfect.

6 Propoſition. *Your ſixth rule is this. By the number of the places diuide the totall ſumme, double the quotient, and that will be the firſt and laſt ioyned in one ſumme. Then by a number leſſe by* 1 *then the number of the places, multiply the exceſſe, that of-come ſubtract from the firſt doubled*

doubled quotient, and the halfe of the reſidue is the firſt number. The laſt number you may diuerſly finde out, as by the firſt of our ſixe rules, or by ſubtracting this firſt number from the ſumme which here contained both the firſt and the laſt ioyntly, (or thirdly) by continuall adding the exceſſe.

Scholar. I pray you make this ſomewhat more plainer with an example.

Maſter. If euery moneth in the yeere (counting them now as 13) you gained cleerely 40 ſhillings more then you did the moneth next going before, and at the yeeres end you finde the whole gaine 5720 ſhillings, but yee remember not how much either the gaine of the firſt moneth or the laſt was, by this rule it may be tried out. Example of gaine

Scholar. So that here yée ſéeme to apply the 13 moneths to thirteene places, the 40 ſhillings enery one more then the other next before it, to be the common exceſſe, and 5720 ſhillings to the total ſumme.

Maſter. It is true: by 13 then I diuide 5720 in this manner.

I double this Quotient, ſo haue I 880 for the firſt and the laſt ſumme ioined together; by 12 which is leſſe by one then the num-

x	
x5	
x720	(440
x3 3 3	
x x	

ber

ber of the places; I multiply 40 (the common exceſſe) ſo commeth 480

This 480 I ſubtract from 880, ſo remaineth 400: halfe whereof is the firſt number which we deſired to know: that is 200.

40
12
80
4
480

And as for the laſt number, I can giue you it three waies: As by the firſt of my ſix rules I multiply, the exceſſe by a number leſſe by 1, then the number of the places, as 40 by 12 that giueth 480, which I adde to the firſt, being 200 ſo ſhall the laſt be 680.

The ſame ſumme commeth forth, if yée ſubtract 200 from 880:

And thirdly, If I beginne at 200, and ſo proceed, encreaſing by 40, I ſhall at the thirteenth place haue 680, as thus:

200 240 280 320 360 400 440
480 520 560 600 640 680

Scholar. I thanke you moſt heartily for theſe ſixe rules. Now if it be your pleaſure I would heare and learne ſomewhat of Progreſſion Geometricall.

Maſter. There are yet very many rules and propoſitions, which fall into this Arithmeticall Progreſſion.

And for the vſe and practice of them, I will propone vnto you certaine pleaſant and neceſſary queſtions of Arithmeticall Progreſſion and to the performance of their workings,

such

ſuch neceſſary rules and documents, as are requiſite for the better vnderſtanding of them, or any ſuch like.

A certaine Mercer ſold 20 *yards of Veluet to be paid in* 12 *weekes, by Arithmeticall proportion: that is to wit, to receiue the firſt weeke* 6 *ſhillings, the ſecond weeke* 12 *ſhillings, the third weeke* 18 *and ſo forth, encreaſing the number of weekes by* 6 *ſhillings, till the twelfth and laſt weeke were expired.* The queſtion is how many pounds hæ had for 20 yards of Veluet. A queſtion of Veluet.

To the performance of this queſtion and ſuch other the like, I ſet forth the 12 payments in ſuch ſort, as for example, hære appeareth.

Then touching the adding together of theſe ſummes, without the aid of Addition, according to the rules I taught you in Progreſſion Arithmeticall, I note the number of the places, which are 12, then adding the laſt number of the Progreſſion, which is 72, and the firſt number together, make 78; and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 23 pounds 8 ſhillings. And ſo much hath the Mercer for his 20 yards of Veluet, which is nigh about 23 ſhillings, 5 pence, a yard.

	6
	12
	18
	24
	30
	36
	42
	48
	54
	60
	66
	72

Scholar. I vnderſtand this worke very well: but is there any proofe for the iuſtifying hæreof,

hereof, as you haue of other worke:

Maſter. The worke of it ſelfe (being ſo perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantitty, is all that is demanded for iuſtifying of the ſame: yet notwithſtanding, becauſe your requeſt is reaſonable, I will propone an example for the proofe hereof.

The proof of the laſt queſtion.

A certaine man is bound to pay for 30 *yards of veluet, the ſumme of* 23 *pound,* 8 *ſhillings and it is to be paid weekely, in* 12 *weekes or termes by Arithmeticall Progreſſion.* The queſtion is therfore to know with what number the ſame Progreſſion is to be begun and continued in ſuch equall proportion Arithmeticall, that in 12 weekes the ſame may iuſtly bee accompliſhed.

For the reſolution whereof, and of all ſuch other like, reduce 23 pound 8 ſhillings, all into ſhillings, which maketh 468 ſhillings.

A generall rule.

Then adde 1 vnto 12, the number of the termes, it maketh 13, which 13 you ſhall multiply by halfe the number of the termes, which is 6, it maketh 78; then diuide 468 by 78, & you ſhall find 6 in the Quotient, which is the true number that ſhall begin and continue the ſaid Progreſſion. That is to ſay, the firſt week 6 ſhillings, the ſecond 12 ſhillings, & the third weeke 6 ſhillings more, which is 18 ſhillings, and ſo euery weeke as they riſe, 6

ſhillings

ſhillings more then the weeke before, as is manifeſt in the queſtion aforeſaid.

A Farme is to be ſold to be paid by the weekes in a yeere, the firſt weeke to pay 4 ſhilling, the ſecond weeke 8 ſhillings, the third weeke 12 ſhillings & ſoforth, increaſing each number by 4, till the number of 52, (which are the number of weekes in a yeere be expired.) The queſtion is, what the price of the Farme commeth to? A queſtion of a Farme

Scholar. I doubt not, but by that you haue already taught me, to end this queſtion very well; wherefore I ſet forth the Progreſſion with his exceſſe 52 times.

Maſter. May ſtay a while: And héere for your further eaſe, (to abridge you of great labour that appeareth to fall out in this queſtion, and ſo may do in any other the like) if a queſtion, were propoſed of 100 or 200 places or moe, and that this queſtion, nor any other the like can be ended, vnleſſe you know abſolutely what the laſt number of the Progreſſion at the 52 place is, or ought to bee) I will giue you a generall rule how to know the laſt number of any Progreſſion Arithmeticall, as well as if you had ordinarily procéeded by continuall Addition, till you had come to the laſt worke, which is this.

Multiply the exceſſe by a number leſſe by one then the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall haue your deſire. A generall rule.

Scholar.

Scholar. This rule is well worth the noting: for if I vnderſtand you aright, I conſider that my excelle is 4, which I multiply by 51, which is one leſſe then the number of the places, and it maketh 204: whereunto I adde the firſt number of the Progreſſion, which is 4, and then it is 208. which you ſay is, or ſhould be the laſt number of the Progreſſion.

Maſter. This is a moſt approued truth, if there were neuer ſo many places.

Scholar. This rule is ſo eaſie, that I were much to blame, if I doe not remember it. For by the benefit hereof, I haue ſuch an eaſe and light into this excellent Art, that my firſt entrance doth ſeeme to paſſe a great many mens further ſtudy, and longer continuance.

Maſter. Many moe conſiderations could I propound you in theſe *Arithmeticall Progreſſions*; but theſe are ſufficient for a taſte, to giue you occaſion to thinke that *Rules* of knowledge and *Arts*, are infinite capable of enlargement.

Scholar. Happy were I, if I did but well vnderſtand that which is already inuented and written. But theſe things, in my ſimple fantaſie, offer themſelues to bee greatly beneficiall vnto the aid of Progreſſion. Therefore now I will goe forward with your queſtion.

Now conſidering that the 52 and laſt place is 208, I adde thereunto the firſt number

number of the Progreſſion, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, and it amounteth to 5512 ſhillings. And ſo much is the totall ſumme or addition of this Progreſſion: which maketh 275 pounds, 12 ſhilling, as appeareth here by my Tables.

Maſter. I like well your labour, and commend you for your diligence; I will here propone one example more, and therewithall for this time will end Progreſſion Arithmeticall.

A queſtion of Holland

A certaine man bought 20 elles of Holland, to be paid in 17 weekes or termes by Progreſſion Arithmeticall. And the firſt weeke to pay 1 ſhilling 8 pence, the ſecond weeke 3 ſhillings, 4 pence, the third weeke 5 ſhillings the fourth weeke 6 ſhillings, 8 pence, and ſo forth, each weeke ſucceeding 20 pence more then the weeke before. The queſtion is, what the ſumme of his 20 Elles commeth to?

Scholar. Becauſe here is mention made both of ſhillings and pence, I feare there is ſome harder matter contained herein, then in the other before: therefore I pray you worke it your ſelfe, and I will diligently marke your labour.

Maſter. There is no more to be done in this, then in the other before; but becauſe your requeſt is ſo reaſonable, bee attentiue vnto mee.

Firſt by the generall Rules I ſeeke to find out the laſt number of the 17 place, what this

this Progreſſion ought to bée. Therefore héere in my Tables multiplying the exceile 20 by 16, which is one leſſe then the number of the termes or places, and it commeth to bée 320; and thereunto adding the firſt number of the termes or places, and it commeth to bée 320; and thereunto adding the firſt number of the Progreſſion, which is 20 pence, all is 340 pence, or 28 ſhillings 4 pence: for ſo much ought the laſt number of the payments to bée.

Then finally to know what the whole 17 places amount vnto, I adde the firſt number of the Progreſſion and the laſt together, which make 360. Now becauſe 17 is an odde number, whoſe halfe cannot bée taken, I take the halfe of 360, which is 180, and multiplying 180 by 17, commeth to 3060 pence, which maketh as you ſée by Diuiſion 12 pound, 15 ſhillings. And ſo much is the buyer to pay for his 20 elles of Holland. Which 3060 pence if you diuide by 20, the number of elles that was bought, you ſhall finde 12 ſhillings, 9 pence, and ſo much paid he for an elle one with another.

The Proofe.

A queſtion of debt. *A Certaine man doth owe* 12 *pound*, 15 *ſhil. to be paid in* 27 *weekes or termes by Arithmeticall Progreſſion. The queſtion is, to know with what number he ſhall begin and continue the Progreſſion in ſuch equall proportion, as the ſame may be truly*

paid

paid and ſatisfied in 17 *weekes.*

The Anſwer.

Firſt I reduce 12 pounds 15 ſhillings, all into pence, which as you ſée héere in my Tables, make 3060 pence, that I let ſtand by a while.

Then I adde 1 to 17, the number of the places or terms, which maketh 18, which I ſhould multiply by halfe the number of the weekes or termes, which is $8\frac{1}{2}$ which $8\frac{1}{2}$ multiplied by 18 cannot well be done, vnleſſe you were acquainted with Fractions or broken numbers, therefore you ſhall let that paſſe and multiply 17. by the halfe of 18: which is 9, (for that is all one with the Multiplication of $8\frac{1}{2}$: and the Multiplication of 9 into 17 maketh as you ſée 153, with which number you ſhall diuide the 3060 pence beforeſaid, and the Quotient bringeth forth 20 pence, which is the firſt number or payment to begin the Progreſſion with all: and ſo each weeke ſucceéding to riſe 20 pence more then the weeke before, and thereby in 17 weekes ſhall 12 pound 15 ſhillings be paid: as before was ſufficiently declared. Thus much for Progreſsion Arithmeticall.

Scholar. Certainly Sir, I know not how to render you condigne thankes for theſe benefits ſhewed mée, which me thinketh are ſo eaſie, delightfull, and pleaſant, that I count my ſelfe happy to be in your company.

Maſter.

Maſter. I am glad you delight ſo well herein, which is an Art of wonderfull dexterity to all ſorts of men of what degrée or profeſſion ſoeuer they be. And now will I proceede to Progreſſion Geometricall wherein I will be more briefe, both becauſe I haue béene ſo lóng in this part of Arithmeticall Progreſſion, and alſo for that it would require the knowledge of Rootes and ſurd numbers, (whereof ye haue learned nothing) if I ſhould frame the like propoſitions in them as I haue done in theſe. Therefore I will onely teach you two practiſes about it, and ſo end the conſiderations and workes of theſe Progreſſions.

Progreſſion Geometricall.

PRogreſſion Geometricall is when the numbers increaſe by a like proportion, that is, if the ſecond number containe the firſt, 2, 3, or 4 times, and ſo forth: then third containeth the ſecond ſo many times alſo: and ſo

the fourth the third,	3	6	12	24	48
and the fift the	1	3	9	27	81
fourth, wherefore I	2	10	50	250
ſet theſe 3 examples					

Here in the firſt example you ſee, that euery number containeth the other (that goeth next before him) two times: and in the ſecond example three times, and in the third example fiue times. Now if you will know how to finde eaſily the ſumme of any ſuch number, do thus: Conſider by what numbers, they be multiplied,

multiplied, whether by 2, 3, 4, 5 or any other, and by the ſame number multiply the laſt ſumme in the Progreſſion.

To finde the totall ſumme in any Geometricall Progreſſion.

Scholar. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, which I haue framed by proceeding from 2, and continuall multiply by 4.

Maſter. Then muſt I multiply the laſt ſumme (which is 2048) by 4 alſo, and it will be 8192. Now muſt I abate from this ſumme the firſt number of the Progreſſion, which here is 2, then reſteth 8190: which ſumme I muſt diuide by 1 leſſe then was the number that I multiplied by. Seeing then I multiplied by 4, I muſt diuide by 3, ſo diuiding 8190 by 3, the Quotient will be 2730; which is the ſumme of all the Progreſſion. And now to proue whether you can doe the ſame, I giue you theſe numbers to adde by this rule 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tell by what number this Progreſſion doth increaſe.

Maſter. In any ſuch doubt doe thus: Diuide the ſecond number by the firſt, and the quotient will ſhew you the number that engendreth the Progreſſion.

Scholar. Then is that number in this example 5, for ſo many times is 3 in 15.

Maſter. So is it. Now worke as I taught.

Scholar. The laſt number is 46875, which I multiply by 5, and it yeeldeth 234375, from

which I abate the laſt number of the Progreſſion, that is 3, and there reſteth 2 3 4 3 7 2, which I diuide by 4, for that is one leſſe then 5, and the Quotient is 5 8 5 9 3, which is the whole ſumme of the Progreſſion.

Maſter. If you remember well this, you haue learned the Art of Progreſsion both Arithmeticall, and alſo Geometricall, which you may proue either by ſubtracting of each number alone from the ſumme and ſo will there nothing remaine: or elſe by adding together of all the parcels, for ſo will the ſame ſumme amount.

Aqueſtion of Satten. *A Mercer hath* 12 *yards of Satten, which he valueth at* 16 ſhillings *the yard, and ſelleth the ſame* 12 *yards to another man to be paid as followeth: That is to wit, for the firſt yard to haue one ſhilling, for the ſecond yard two ſhillings, for the third yard foure ſhillings, for the fourth yard* 8 *ſhillings, &c. doubling each number following, till the twelfth and laſt yard.* The queſtion is, who hath made the better bargaine of the buyer or the ſeller.

Firſt you may ſet downe 12, the number of the yards as you ſée héere in this Example. And againſt each number the number of ſhillings due to be paid, as the order of Duplation or Multiplication by two teacheth.

Then reſorting to the adding vp or ſumming of this Progreſſiõ, where I conſider that the increaſe of this ſum proceeded by the Multiplication of 2, & therefore after I haue drawne

8	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4095	

a line vnder the 12, I worke and multiply the laſt ſumme by 2 alſo, & it yéeldeth 4096: from whence I abate the firſt number of the Progreſsion, which is 1, and then reſteth 4095: which I ſhould diuide by one leſſe then I did multiply by, but ſéeing it is 1, I néede not to diuide it: for 1 (as I haue ſaid before) doth neither multiply nor diuide; therefore I take that ſumme 4095 for the whole ſumme of the ſhillings, which by Reduction amounteth to 204 pounds, 15 ſhillings, and ſo much hath the Mercer for his twelue yards of Satten: which is 17 pound, 1 ſhilling, 3 pence a yard. But I thinke you will buy none ſo deare.

Scholar. No ſir by the grace of God this yeare.

A queſtion of a Horſe.

Maſter. *Then what ſay you to this queſtion? If I ſold vnto you a horſe hauing 4 ſhoes, and in euery ſhoe 6 nailes, with this condition, that you ſhall pay for the firſt nayle one* ob: *for the ſecond nayle two* ob: *for the third nayle foure* ob: *and ſo forth, doubling vntill the end of all the nayles.* Now I aſke you how much would the price of the horſe come vnto?

Scholar. Firſt to know the number of the nayles, I muſt multiply 6 by 4, and it maketh 24 Then will I doe thus: I will write the number of the nayles euery one in order from 1 to 24, and againſt each number of the nayles the ſumme of halfe pence duely, as the order of Duplation or Multiplication by 2 teacheth, and as in this next figure following appeareth.

Then doe I reſort to the Rule of ſumming vp the Progreſsion, where I conſider that the increaſe of this ſumme proceedeth by the Multiplication of 2, as the laſt example did. And therefore multiplying the laſt ſumme by 2 alſo, and it yeeldeth 16777216, from which I abate the firſt number which is 1, and then reſteth 16777215, which I ſhould diuide by one leſſe than I

did

1	1	did multiply: but seeing
2	2	that it is 1, I need not to
4	3	diuide it, for 1 (as you
8	4	haue before said) doth nei-
16	5	ther multiply nor diuide,
32	6	therefore I doe take the
64	7	number 16777215 for
128	8	the whole summe of the
256	9	half pence, which by Re-
512	10	duction I find to be 699050
1024	11	shillings, and 7 pence, halfe
2048	12	penie: that is 34952
4096	13	pounds, 10 shillings, 7
8192	14	pence, ob.
16384	15	Master. That is wel done,
32768	16	but I think you wil buy no
65536	17	horse of the price.
131072	18	Schol. No sir, if I be wise.
262144	19	Master. Well then an-
524288	20	swer me to this questi-
1048576	21	on:
2097152	22	*A Lord deliuered to a*
4194304	23	*bricklayer a certaine number*
8388860	24	*of loades of brick, whereof he*

16777216

A question of bricks.

willed him to make twelue walles, of such sort, that the first wall should receiue two thirdels of the whole number, and the second two thirdels of that which was left; and so euery other, two thirdels of that that remained: and so did the Brick-layer: and when the 12 walles were made, there remaineth one load of Brick.

Now I aſke you, how many load went to each wall, and how many load was in the whole?

Scholar. Why Sir, it is impoſſible for mée to tell.

Maſter. Nay, it is very eaſie, if you marke it well. Marke well that I ſaid, that euery wall ſhould receiue two thirdels of the ſumme that was left. Now take away two thirdels from any ſumme, and you muſt néeds grant that that which remaineth, is one thirdell of the ſumme laſt before: Example of 9, from which if you take two thirdels, there will remaine three, which is one thirdle of 9. Likewiſe from three bate two thirdels, and there will remaine 1.

Scholar. This is true, and now I perceiue the leaſt wall had but two load of brick.

Maſter. And by the ſame reaſon may you know how many loade euery wall had, according as this figure following doth ſhew, and likewiſe what the whole ſumme of brickes was: for if you make 12 ſummes, multiplying by 3, ſtill from the laſt remainer, as you may ſée héere on the left ſide of the Table, there will appeare all the remainers of the whole wall; and if you multiply the laſt of 12 ſummes by 2 alſo, then will that bée the ſumme of the loads which was deliuered to the Brickelayer.

	1	1 2 2		
The remainer after euery wall.	3	1 1 6	*Loads due to*	
	9	1 0 1 8	*each wall.*	
	27	9 5 4		
	81	8 1 6 2		
	243	7 4 8 6		
	729	6 1 4 5 8		
	2187	5 4 3 7 4		
	6561	4 1 3 1 2 2		
	19683	3 3 9 3 6 6		
	59049	2 1 1 8 0 9 8		
	177147	1 3 5 4 2 9 4		

Summe of the loades 5 3 1 4 4 0 *deliuered.*

Againe, if you double euery Remainer, as you may ſée at the right ſide of this Table, thoſe numbers will ſhew the ſumme of loades that went to each wall, whereby you may perceiue that each wall was three times ſo great as the next leſſer.

Scholar. Loe now it appeareth eaſie enough. Now ſurely I ſée that Arithmeticke is a right excellent Art.

Maſter. You will ſay ſo when you know more of the vſe of it: For this is nothing in compariſon to other points that may bée wrought by it.

Scholar. Then I beſéech you ceaſe not to inſtruct mée further in this wonderfull cunning.

The Golden Rule or, Rule of Proportion direct, called the Rule of Three.

Maſter.

The Rule of proportion.

BY order of the Science (as Men haue taught it,) there ſhould follow next the extraction of Rootes of number, which becauſe it is ſomewhat hard for you yet, I will let it paſſe for a while, and will teach you the feate of the Rule of Proportion, which for his excellency, is called the Golden Rule.

The Golden Rule.

Whoſe vſe is, by three numbers knowne to finde out any other vnknowne, which you deſire to know, as thus.

Queſtion of boarding.

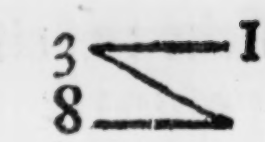
If you pay for your board for three moneths ſixteene ſhillings, how much ſhall you pay for eight moneths?

To know this and all ſuch like queſtions, **you ſhall conſider which** two of your numbers **be of one** denomination, **and ſet thoſe two the one ouer the other, ſo that the vndermoſt be it that the** queſtion **is of: as in my** queſtion 3 **and** 8, **be both of one** denomination, **for they both be** moneths; **and becauſe** 8 **is the** number **that the** queſtion **is aſked of, I ſet the one ouer the other, and** 8 **vndermoſt thus, with ſuch a** crooked draught **of lines. Then doe I ſet**

3
8

the

the other number which is 16, against 3 the at right side of the line, thus.



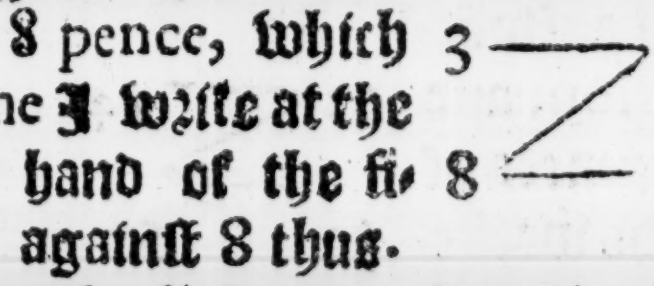
And now to know my question, this must I do: I must multiply the lowermost on the left side; by that on the right side, & the summe that amounteth, I must diuide by the highest on the left side: or in plainer words, thus, I shall multiply the number of which the question is asked (which is called the third number) by the number of another denomination (which is called the second, & the summe that amounteth, must I diuide by the summe of like Denomination, (which is called the first) Then for the knowledge of this question, I multiply 8 into 16, and there amounteth 128, which I diuide by 3 & it yeldeth 42 shillings, and 2 shillings remaineth, which I turne into pence, and they bee 24 pence, of which third part is 8 pence, so the third part of 128 shillings, is 42 shillings, 8 pence, which summe I write at the right hand of the figure against 8 thus.

Note.

The third number

The second number.

The first number.



Hereby I know that if three moneths boarding, doe come to 16 shillings, that 8 moneths boarding will come to 42 shillings, 8 pence, & likewise of any other like question.

But here must you marke, that the first number and the third be of one denomination, & also the second and the fourth, for which

you

you ſéeke: or elſe be of ſuch denominations, that youin worʒking may bʒing them into one; As if a man ſhould aske me this queſtion.

Queſtion of expences.

Twelue weekes iourneying coſt mee 14 French crownes at 6 ſhillings, the peece, how many pounds is that in one yeare? Here you ſee no two numbers of one denomination, but yet in worʒking you may turne them into like denomination: as thus, turne the one yeare into 52 weekes, and the fourth ſumme will bee French Crowns, by the order of the working. Then to knowe this queſtion, multiply the third ſumme 52, by the ſecond 14 and the ſumme will be 728: that diuide by your firſt number, 12, and the Quotient will bee 60 Crownes, and 8 Crownes remaining: which if you turne into ſhillings, they wil be 48. ſhillings, which if you diuide by your firſt number 12 the quotient will be 4, which ſignifieth 4 ſhillings: put thoſe 60 French Crownes, which make 18 pounds with the 4 ſhillings, for the ſumme that anſwereth to the queſtion, and it is the iuſt expences of a yeere: And the worʒke will bée thus.

12	14	
52	60	ſhillings

A general Rule.

And take this euermore for a generall rule touching this whole Art, that the doubtfull or vnknowne number that you would be reſolued of, ſhall alwaies be ſet in the third place. Note alſo the firſt number and the third, muſt euer be of one nature & denomination, or elſe muſt in working be brought

to like denomination, and then of neceſſity muſt the other number be in the ſecond place.

Remember alſo that the place of the firſt number is higheſt on the left ſide, and the place of the ſecond, right againſt it on the right ſide; the place of the third number is vnder the firſt, as by thoſe examples you haue ſeene.

Scholar. This I truſt I can doe.

Maſter. But and if the queſtion be asked thus: *In 8 weekes I ſpend 40 ſhillings, how long will 105 ſhillings ſerue me?* Heere you ſée that 8 weekes anſwers himſelfe, & ſaith 40 ſhillings. But how long time 105 ſhillings will ſerue you know not. Therefore you ſhall ſet 105 in the third place, according as I told you euen now. And the firſt place muſt alwaies be of the ſame nature or Denomination that the third is of, which here is 40 Then muſt 8 néeds be that other. Now multiply 105 by 8, and it will be 840, which if you diuide by 40, it will yéeld 21, which is the fourth number, and ſheweth how many weekes 105 ſhillings will ſerue, if you ſpend 40 ſhillings in 8 weekes.

The figure of this queſtion is this: as if you ſhould ſay: if 40 ſhillings ſerue for 8 weekes: 105 will ſerue for 21 weekes

Shillings.	*Weekes.*
40	8
105	21

Other diuerſities there be of working by this rule, but I had rather that you would learne this one well, then at the beginning

to trouble your minde with many formes of working, sith this way can doe as much as all the other, and hereafter you shall learne the other, more conuenently.

Note. ¶ And for your further aid and instruction, to make you better acquainted with this Golden Rule, I haue here proponed six questions, and their answers, which I thinke most conuenient and meete to preferre the desirous to perfect vnderstanding. The first foure are all branches of one question sprung out of the best tree (for a yong learner to taste of) that groweth in this *Ground* of *Arts* : for that no manner of question in the Rule of Three whatsoeuer it be, can be proponed, but it must be comprehended vnder the reason or stile of one of these foure.

The Questions.

If 15 *elles of cloth cost* 7 pound 10. shillings *what comes* 27 *elles to at that rate* ? Answer 13 pound 10 shillings.

If 27 *elles cost* 13 pound 10 shillings *what are* 15 *elles worth*? Answer: 7 pound 10 shilling

If 27 *elles cost* 13 pound 10 shillings: *how many elles shall I haue for* 7 pound 10 shillings Answer: 15 *Elles.*

If I sell 15 *elles for* 7 pound 10 shillings: *how many elles are to be deliuered for* 13 pound 10 shillings? Answer; 27 *Ells.*

If 8 *pound of any thing cost* 16 shillings 6 pence,

pence: *what money is to be receiued for* 49 pound? Anſwer: 5 pound, 1 ſhilling od $\frac{3}{4}$.

If 4 pound *of any thing coſt* 17 pence: *what money will* 8765 *pound of that commodity coſt?* Anſwer: 155 pound, 4 ſhillings, 3 pence, q;

Of all which queſtions, I omit the worke of purpoſe, that you ſhall whet your wit thereby at conuenient leaſure, to clime each branch, and gather the fruit of them, and doe minde now, before we make an end of this Rule, to giue you ſome inſtructions of the backer rule of three, whoſe order is quite contrary to this that you haue learned.

Scholar. I thanke you heartily for the ſix Queſtions, which I will God-willing practiſe at conuenient times: I pray you proceed therefore to the Backer or Reuerſe Rule,

The

The Golden Rule, or Rule of Proportion Backward, or reuerſe.

Maſter.

IN the former Rule euermore looke how much the third number is greater then the firſt, ſo much the fourth number is greater then the ſecond. And contrariwiſe: looke how much the firſt ſumme is greater then the third, (if it doe chance ſo) ſo much is the ſecond ſumme greater then the fourth.

Note this well.

But in this rule, there is a contrary order, as this: That the greater the third ſumme is aboue the firſt, the leſſer the fourth ſumme is beneath the ſecond; and this rule therefore you may call the Backer or Reuerſe Rule, as in example.

If I haue bought 30 yards of cloth of two yards breadth, and would haue Canuas of three yards broad to line it withall, how many yards ſhould I neede?

Scholar. Why, there is none ſo broad.

Maſter. I doe not care for that, I doe put this example onely for your eaſie vnderſtanding: For if I ſhould put the example in other meaſures, it would be harder to vnderſtand. But now to the matter: If you would know this queſtion, ſet your numbers as you

did

did before: but you ſhall multiply now the first number by the ſecond, and that ariſeth thereof, you ſhall diuide by the third: which thing if you doe here, I meane if you multiply 30 by 2, it will be 60: which ſumme if you diuide by 3, there will appeare 20: wherby I know; that if 30 yards of cloath of two yards broad, ſhould be lined with canuas of three yards broad, 20 yards of canuas would ſuffice as this figure ſhelweth. Note.

Breadth. Length.

2 — 30

3 — 20

And now becauſe ye found fault with my Example, how ſay you, perceiue you this?

Scholar. Yes Sir, I ſuppoſe.

Maſter. *Then anſwer me to this queſtion: how many Elles of canuas of elle breadth, will ſerue to line 20 yards of Say, of three quarters broad?*

Scholar. In good faith Sir I cannot tell, for I know not how to bring the ſummes to like Denominations:

Maſter. Than will I tell you, ſith there is mention hereof quarters, and againe euery one of the meaſures both elles and yards may bee parted into quarters, part them ſo both in the breadth and length, and then put forth the queſtion by quarters,

Scholar. Then I ſhall ſay thus. How many quarters of canuas of 5 quarters broad, will line 80 quarters of 3 quarters broad?

Maſter.

Maſter. Now anſwere to the queſtion.

Scholar. Firſt I will ſet them downe in their foꝛme thus: foꝛ 5 is ioyned with the queſtion, and is therefoꝛe the third number: then is 3 the number of the ſame Denomination, I meane becauſe they be both referred to breadth. Now I multiply 80 by 3, and it is 240, which I diuide by 5, and it yéeldeth 48. Then ſay I that 48 quarters of 5 quarters bꝛoad, will ſuffice to line 80 quarters of three quarters bꝛoad.

Breadth.	*Length.*
3	80
5	

Maſter. Turne the quarters againe into elles and yards.

Scholar. Then I ſay, that 9 elles and thꝛée quarters of a yard of elle bꝛoad, will ſerue to line 20 yards of thꝛée quarters bꝛoad, as this figure ſhewеth.

Bredth.	*Length.*
3	80
5	48

¶ Maſter: *Now what ſay you to this queſtion: I lent my friend* 400 pound *for* 7 *moneths, how much money ought he to lend mee againe for* 12 *moneths to recompence my curteſie ſhewed him: can you anſwer to this?*

Scholar. Yes Sir, I ſuppoſe, foꝛ I will ſet downe my Numbers thus: where I multiply 7 into 400 and it maketh 2800, which I diuide by 12, and it yéeldeth

Moneths.	*Pounds*
7	400
12	

yéldeth 233 pound, and there is 4 pound remaining of my Diuision, what shal I do therewith?

Master. Turne the same 4 pound into shillings, & then diuide it by 12 as you did before.

Scholar. Well Sir, it shall be done: so haue I 6 shillings, for my Quotient, and yet remaineth 8 shillings vpon my Diuision.

Master. You must also reduce that 8 shillings into pence, which maketh 96, and diuide that also by your Diuisor.

Scholar. So haue I done, and I finde 8 pence for my quotient, and nothing is left.

Master. This must you alwaies doe when any thing remaineth vpon your Diuision, whether it be money, weight, measure, or any kind of thing whatsoeuer. This Rule is so profitable for all estates of men, that for this rule only (if there were no more but it) all men were bound highly to estéeme Arithmeticke.

By this rule may a Captaine in warre, worke many things, as Master Digges in his Stratioticos doth declare: Only now in this my simple addition, for a taste & incouragement, I wil inlarge the Author with a questiõ or two more, wishing you and euery my Countrimen or Gentlemen whatsoeuer, that by nature be any thing giuen to Military affaires, to be familiar and acquainted with this Excellent Art, the which he shall find not onely at the Sea, but also in the Campe and Field-seruice, abundantly to aid him, either in fortification, pay-

ing of Souldiers wages, charges of Ordinance, Powder, Shot, Munitions, and Inſtrumens, whatſoeuer, as for example.

If it ſhould chance a Captaine which hath 40000 *Souldiers to be incloſed with his Enemy, that he could haue no freſh purueyance of victuals, and that the victuals which hee had would ſerue that Armie but onely three moneths, how many men ſhould he diſmiſſe to make the victuall to ſuffice the reſidue eight moneths?*

Scholar, As you taught me, I ſet the numbers thus, ſaying: if three moneths ſuffice 40000, to how many will eight ſuffice.

Moneths	*Men.*
3	40000
8	

To know this; I multiply the firſt number 3 into the ſecond 40000, and it yeeldeth 120000, which ſumme I diuide by 8, and there will be in the quotient 15000, which if I doe ſubtract from 40000, the remainer will declare that he muſt diſmiſſe 25000 as this figure ſheweth.

Moneths,	*Men.*
3	40000
8	15000

A queſtion of a Fort.

I Maſter. *Now anſwere mee to this queſtion: If* 136 *Maſons in a moneth bee able to build a Fort to preſerue the Souldiers from the Enemy, and ſuch expedition requireth that I would make the ſame finiſhed in eight dayes: how*

how many workemen ſay you is there to be appointed.

Scholar. As you taught me; I ſet the numbers thus, ſaying: If 28 dayes require 136 Maſons, what number of men by the like propoztion will 8 dayes require?

To know this, I multiply the firſt number 28 into 136; and it yéeldeth me 3808: which I diuide by 8, & my Quotient is 476: which is the iuſt number of Maſons that ſhall ſupply this woꝛke. And now me thinke theſe queſtions are very eaſie.

Maſter. Truely if you take delectation herein, you ſhall finde this Art not onely eaſie, but wonderfull pleaſant and pꝛofitable. Now therefoꝛe one queſtion moꝛe I will pꝛopoſe, and ſo leaue off this Rule in whole numbers, vntill we come to the vſe of it in broken numbers: foꝛ had you the vnderſtanding of broken numbers perfectly, not onely in this Rule, but in all other, the queſtion that in the ſight oꝛ appearance ſéemeth to be 100 times moꝛe harder to abſolue, may thereby be wꝛought as ſoone, oꝛ ſooner then this.

Scholar. Your woꝛds doe greatly encouꝛage mé to bée ſtudious to attaine whole numbers: but might I once attaine to bée a practitioner in broken numbers, I ſhould thinke my ſelfe happy.

Maſter. *What ſay you then to this queſtion? If* 48 *Ioyners in two daies make* 200 *light horſemens ſtaues, (eſteeming they worke but* 12 *houres a day) and ſuch need requireth that* 384 *Ioyners are ſet to the finiſhing of thoſe* 200 *ſtaues; in what time ſay you, will they make them vp?*

Scholar. I ſee here that I muſt turne my 2 dayes into houres. And ſo doing I ſet my numbers thus:

48 — 24
384

Saying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded vpon an old Prouerbe, *many hands make quicke ſpeed.*

I multiply 48 into 24, and it amounteth to 1152, which I diuide by 384, and my quotient is three houres, which is my deſire.

Note. *I take this for a note worthy the marking, either in the Rule of Three, forward or backward, when the two numbers are multiplied together, the Product is of the ſame nature and denomination that the ſecond number is of.*

The double Rule of proportion direct.

Maſter.

WEll, ſith you perceiue now the vſe of this Rule, I will ſhew other which enſue of the ſame, & firſt the *Donble Rule* which is called, becauſe there is in it double working, by which thing onely it differeth from this. The double Rule.

Scholar. Then by an example I ſhall vnderſtand it well enough.

Maſter. *So ſhall you, and let this be the example: If the carriage of* 100 *weight,* (*that is* 112 *pound*) *30 miles doe coſt* 12 *pence, how much will the carriage of* 500 *weight coſt, being carried* 100 *miles.* Queſtion of carriage

Scholar. I pray you ſhew me the working of it.

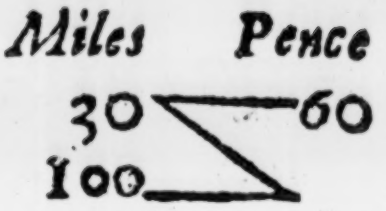
Maſter. You muſt make two workings of it: the firſt thus: If C weight coſt 12 pence how much will fiue hundred weight coſt? Set your figures thus:

C Weight.	*Pence*
1	12
5	

And multiply 5 by 12, and thereof amounteth 60, which if you diuide by one, the quotient will bee ſtill 60, that is the price of 500 weight for 30 miles.

Then begin the ſecond worke, ſaying: If

30 miles coſt 60 pence, how much will 100 miles coſt? Set your figures thus:



Then multiply 100 by 60, whereof amounteth 6000, which being diuided by 30, will yéeld 200 pence. Then you may ſay, that ſo many pence ſhall coſt the carriage of 500 pound weight 100 miles, after the rate of 12 pence for the 100 carried 30 miles.

Scholar. Now I perceiue it alſo.

¶ Maſter. Theſe and ſuch other like queſtions of the double Rule of Three, are to be anſwered much ſooner, at one onely working by the Rule of proportion compoſed of fiue numbers, which anon I will ſhew you, and then when you haue the vſe thereof, you may vſe which way you thinke good.

Scholar. Sir, I thanke you much for your courteſie, and I long now till this Rule be ended, that I may ſée how I ſhall behaue my ſelfe with that new rule of fiue numbers: for that I haue euer ſince you taught me hitherto in the Golden Rule both forward and backward, wrought but with three numbers onely.

Maſter. But yet a while wée will goe on forward with this Rule of Three, therefore anſwere to this queſtion.

Queſtion of ſowing.

Thirty buſhels of wheat ſowed, yeelded in one yeere 360, how many will 80 buſhels yeeld in 7 yeeres? I meane ſowing euery

yeere

yeere of thoſe ſeuen, ſtill 80 buſhels?

Scholar. First I say, that if 30 buſhels will yéeld 360 in one yéere, then 80 buſhels will yéeld 960 in one yéere. Then for the ſecond worke I say: If one yeere yeeld 960, then 7 yeeres will yeeld 6720; as theſe two figures doe ſhew.

Seede.	*Encreaſe*	*Yeere.*	*Encreaſe.*
30	360	1	960
80	960	7	6720

But now Sir, if I ſet forth 30 buſhels of Corne to another man for 7 yeeres, agreeing ſo that hee ſhall ſow euery yeere the whole increaſe of the Corne, and I at the end of theſe 7 yeeres to haue the halfe of the whole increaſe: I would know how many buſhels will there amount to my part, ſuppoſing the increaſe to be after the rate of the laſt queſtion, for 30 buſhels in one yeere to yeeld 360? Queſtion of Corne.

Maſter. In ſuch a queſtion you muſt haue ſo many ſeuerall worKings as there bee yeeres, as for example: in the firſt yeere 30 buſhels yéelds 360: then to know the yéelding of the ſecond yeere, I muſt ſay; If 30 yeeld 360, how many yeeldeth 360? Work by your Rule, and you ſhall finde 4320. Then ſay for the rhird yeere. If 30 yeeld 360, how many will 4320 yeeld? You ſhall haue 51840, and ſo euery yeere multiplying the whole increaſe by 360, and diuiding

it by 30. the increaſe of the next yeere will amount, as theſe 7 figures following doe orderly declare: where I haue ſet 7 letters for the 7 yeares, of which the firſt is ſet without Art, becauſe that is the increaſe which you do preſuppoſe: and the laſt number of each other doth ſhew the increaſe of that yeere that it ſtandeth for, which the letters doe declare, ſo that the increaſe of the ſeuenth yeere, is 1074954240 buſhels: how many quarters that is, and alſo how many waies you may by Reduction ſome finde.

a

30—360

b

30 360
360 4320

c

30 360
4320 51840

d

30 360
518406 22080

e

30 360
622080 7464960

f

30 360
7464960 89579520

g

30 360
89576520 1074954240

Queſtion of mowing.

Now with one queſtion more I will proue you. *If ſix Mowers do mow 45 acres in fiue daies, how many mowers will mow 300 Acres in ſix daies?*

Scholar.

Scholar. If 45 acres require 6 Mowers, then 300 Acres requires 40. Now againe: if fiue daies require 40 Mowers, then 6 daies neede but 33 Mowers.

Maſter. Why doe you not make mention of the 2 that remaineth in the laſt Diuiſion? for the laſt part of the queſtion is wrought by the Backer rule, where the firſt number 5 is multiplied into the ſecond, that is 40, whereof amounteth 200, which if you diuide by the third number 6, the Quotient will be 33, as you ſaid: but then will there remaine 2 which cannot well be diuided into 6 parts: howbeit you may vnderſtand by the 6 part of 2, the third part of one Mans worke, which you muſt put to the 33: or elſe you muſt ſay that 33 Workemen will end all the 300 acres in 6 daies ſaue 2 mens worke for one day or 2 daies worke for one man. But ſuch broken numbers called Fractions, you ſhall hereafter more better perceiue, when I ſhall wholly inſtruct you of them. Note this.

¶ Maſter. *Yet one queſtion more of field matters I will propone, and ſo I will make an end of this double Rule of Three.*

Scholar. With all my heart Sir I thanke you, and I will diſpatch it as ſoone as I can, becauſe I would faine ſée the order of the next Rule of 5 numbers,

Maſter. *If a Captaine ouer a band of men did ſet 300 Pioners a worke, which in eight houres did caſt a trench of 200 Rods: I demand how many labourers* Queſtion of entrenching.

Labourers will be able with a like trench in three houres, to intrench a campe of 3400 *Rods.*

Scholar. I thinke I am now in the Backhouse ditch: for I know not well which way to goe about it. And besides that, truely I thinke I shall neuer come to preferment that way, my groweth is so small.

Master. You know not how God may raise you hereafter by knowledge and seruice into the fauour of your Prince, for the auaile of your Country.

Example for nauigation: Sir *Francis Drake.* a man greatly honoured for his knowledge, is not the tallest man, and yet hath made as great an aduenture for the honour of his Prince and Country, as euer Englishman did.

Scholar. Sir, I thanke you for your good incouragement. My minde, though I be little, is as desirous of knowledge, as any other: I haue pondred now a little of it, and thus I set forth the worke.

Rod.	*Men.*
200	300
3400	Z

Saying, if 200 Rod require 300 men, what shall 3400 rods require? I multiply 3400 by 300, and it yeeldeth 1020000, which I diuide by 200, and my quotient is 5100 men.

Then must I say for the second worke, if in 8 houres 5100 men, be able to discharge it, how many shall performe the same in three houres? Now if I would worke by the Golden

Golden Rule of Proportion forward, I should finde a lesse number of men: because three houres is lesse then 8 houres: but because reason teacheth me, that the lesser the time is, wherein the trench must be made, the more Labourers I ought to haue, thereupon I vse now the Backer Rule, as in example. And I haue in my Quotient 13600. So many Pioners must I haue to intrench the Campe in three houres.

Master. You haue answered the question very artificially: And truely I commend you for your diligence and apt vnderstanding: and now according to my promise, I will (in whole numbers) giue you a little taste of the Rule of Porportion, compounded of fiue numbers.

The Rule of proportion, compoſed of 5 Numbers.

The firſt part of the rule of proportiō compoūd, direct.

THis Rule of Proportion compoſed is diſtinct for moſt needfull queſtions, into ſeuerall parts or workings: And there belongeth vnto it alwaies fiue numbers, whereof in this rule being the firſt part, the ſecond number and the fifth part, are alwaies of one nature and like denomination, which Rule is to be wrought thus: you muſt multiply the firſt number by the ſecond, and that ſhall be your Diuiſor: Then againe, multiply the other 3 numbers, the one by the other, and their product ſhall be your diuidend.

And now according to my promiſe, we will firſt worke the queſtion of weight and cartage, which I deliuered you in the double rule of three, to be abſolued by this rule, which was this.

If the carriage of 1 C weight 30 miles coſt 12 pence, what will the carriage of 5 C weight ſtand me in being carried 100. miles?

C. weight.	*Miles.*	*Pence.*	*C. weight.*	*Miles.*
1	30	12	5	100

Now marke well how theſe fiue numbers ſtand: Then multiply the firſt number by the ſecond, as 30 by 1, which maketh but

30.

30. that number kéepe for your Diuiſor. Then multiply the other three numbers, the one into the other: that is to wit, 12 by 5. which maketh 60: Laſtly 60 by 100 which as you ſée here in our Tables, ariſeth to 6000, which 6000 you ſhall diuide by the product of the two firſt numbers, which here is 30. And you ſée there is found 200 pence, which is the duety that you ought to pay for the carriage of 500 weight 100 miles, after the rate of 12 pence a hundred, and agréeth with the concluſion of the double Rule of three.

Scholar. Sir I thanke you, it is euen ſo.

Maſter. *Yet note this for a generality in this rule, looke what nature or denomination your middle number is of (which here are pence) and of the like denomination or nature is alwaies your quotient.* Note this

Scholar. Well now and if it pleaſe you, by your patience, I will ſée how I can end the queſtion, next following of 30 buſhels of wheat ſowed, which in one yeere yéelded 360: how many then will 80 buſhels yéeld in 7 yeere, ſowing euerie yeere of thoſe 7 ſtill 80 buſhels, and according to your reaſons I ſet my numbers thus:

Buſh. Yeere. Buſh. Buſh. Yeere.

30 — 1 — 360 — 80 — 7

80

28800

7

201600

Where

Where I multiply 30 by 1, and it maketh 30 my Diuisor: then multiplying the other three numbers the one into the other, as here appeareth in my tables, they make 201600, which I diuide by 30: & my quotient is 6720 bushels, my desire, for so much also it came to at two workings by the Rule of three.

Master. Yet one question more I will propound vnto you, and so leaue this Rule, till it please God hereafter, that I may make you worke it in broken numbers.

Question of Interest.

What comes the interest of 258 pound, *for fiue moneths to, after the rate of* 8 pound *taken in the* 100 pound, *for* 12 *moneths?*

Scholar. Sir, this is yet within the compasse of some reasonable blance. Therefore to minister equity in this case, I will see how I can worke the same,

which I set downe thus. praying you if I haue not done wel, to shew me mine errour.

li *moneths.* li li *mon.*
100—12—8—258—5

Master. Proceed, you haue done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yeelds 1200, and the three others numbers multiplied together produceth 10320, which I diuide by 1200: and my quotient is 8 pounds. Then according as you haue taught mee heretofore, I turne the 720 pound that I left, into shillings: and diuiding it by the first number, my Quotient is 12 shillings.

lings. So I answer, that the loane of 258 pounds for 5 moneths, after the rate of 8 pound in the 100 pound for a yeare, comes to 8 pound 12 shillings.

Master You say true, I commend your diligence: now behold the manner of the second part of this rule.

The

The backer Rule, or the second part of the Rule of Proportion compound.

Maſter.

IN *the ſecond part of this Rule of Proportion compoſed, the third number is like vnto the firſt. And the Rule is to be wrought thus: you ſhall now, contrary to the laſt rule, multiply the third number and the fourth together, and that Product ſhall be your Diuiſor. Then multiply the firſt by the ſecond, and the Product thereof by the firſt: and that is the number that ſhall be diuided.* For Example, I propound this queſtion, for a proofe of my laſt queſtion of intereſt.

The proof of the laſt queſtion.

A Merchant hath receiued 8 pound 12 ſhillings, *for intereſt of certaine money for* 5 *moneth tearme, which hee receiued after the rate of* 8 pound *in the* 100, *for a yeere. The queſtion is now, how much money was deliuered to raiſe this intereſt.*

Behold therefore the manner, how the queſtion is ſet forth.

li *moneths.*	li *moneths.*	li ſ.
100 — 12 — 8 — 5 —	— 8 — 12	

Scholar. Sir I perceiue it very well; and according

according to the doctrine which you prescribes for the working thereof: if please you now it is set downe, I thinke I can follow the worke.

Master. Nay stay a while, and before you worke, marke well how I deliuer a reason for the perfect vnderstanding of this Rule, which is thus: *If* 8 pound *in* 12 *moneths doe yeeld me* 100 pound, *to take* 8 pound 12 shillings *for* 5 *moneths, must needs yeeld a great deale more.* Note.

So vpon the knowledge that I haue in this Art, the first part of this rule is answerable to the rule of three forward; and this latter part accordeth to the rule of three backward.

Scholar. Sir, I yeeld you most heartie thanks for these your last instructions, they haue giuen me great light into these two rules, whereby I may the better by deliberation conceiue how to vse them hereafter when occasion shall require.

Master. You say well, goe to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turne all your money as you worke into shillings for your more ease in working. Note.

Scholar. If it please you to behold me a little, I will quickly end it: for I haue but my first, my second, and my last number to be multiplied together for my diuidend: And my third

into my fourth for my Diuisor.

li	*Moneths.*	li	*Moneths.*	li	s.
100	12	8	5	8	12
20		20		20	
2000		160		172	
12		5			
4000		800			
2000					
24000					
172					
48000		4128000 (			
168000		8 00			
24000					
4128000					

Which 4128000 I diuide by 802, and my Quotient is 5160 shillings, which in pounds yeeldeth 258, my desire:

Master. I will here for this time in whole numbers end this Rule, and I will instruct you in the Rules of fellowship. You may at your conuenient leasure for your exercise worke the same by the Rule of Three at twice. And for your aid and incouragement therein, I set downe héere a proffer how to apply it.

A		B	
Monethſ.	li, ſ,	*Pownd.*	li.
5	8-12	8	100
12	412 4/5	412 4/5	258 li.

The Rule of Fellowſhip.

The Rule of Fellowſhip without time.

BVt now will I ſhew you of the Rule of Fellowſhip or Company, which hath ſundry operations, according to the diuers number of the Company. This Rule is ſometime without difference of time, and ſometimes there is in it difference of time. Firſt I will ſpeake of that without difference of time, of which let this be an example.

A queſtion of company.

Foure Merchants of one Company made a banke of money diuerſly: for the firſt laid in 30 pound, *the ſecond* 50 pound, *the third* 60 pound, *and the fourth* 100 pound, *which ſtocke they occupy ſo long, till it was increaſed to* 3000 pound. *Now I demand of you what ſhould each receiue at the parting of this money.*

Scholar. I perceiue that this Rule is like the other, but yet there is a difference, which I perceiue not.

Maſter. Then will I ſhew it to you, Firſt by Addition you ſhall bring all the particular ſummes of the Merchants into one ſumme, which ſhal be the firſt ſumme in your working by the Golden Rule, and the whole ſumme of the gaines by that ſtocke ſhall be the ſecond ſumme. Now for the thrid ſumme

you

shall set the portion of each man one after another, and then worke by the Golden Rule, and the fourth summe will shew you each mans gaines: as in example.

30	
50	
60	
100	
240	

The parcels of these foure Merchants make in one summe 240 pounds: set that in the first place, the gaines in the second, and the first mans portion of stocke in the third place, thus:

240 — 3000
30

Now multiply the second by the third, and it will be 90000, which you shall diuide by 240, and there will appeare 375 pounds thus:

And that is the gains for the first man.

240 — 3000
30 — 375

Now for the second man, set the 50 pound that he brought, in the third place, and worke as before: and his part will be 625 pound: as this figure sheweth.

Likewise for the third man, set his money which was 60 pounds and his part of gaines will bee 750 pounds, as heere appeareth.

240 — 3000
50 — 625

And ſo for the fourth man; if you ſet his ſumme which is 100 pound his gaines will bee 1250 pound, as the worke will declare.

240 to 3000
60 to 750

Scholar. This I perceiue: but is there any way to examine whether I haue well done or no?

240 to 3000
100 to 1250

Note this common proofe.

Maſter. For the triall hereof, adde together all their foure portions, and if their addition make the whole ſumme of their gaines, then is the worke well done.

Scholar. That will I trie by and by, the foure parcels are theſe which added together make 3000. which is the iuſt ſumme of money that they gained, whereby I know the worke is well done.

375	
625	
750	
1250	
3000	

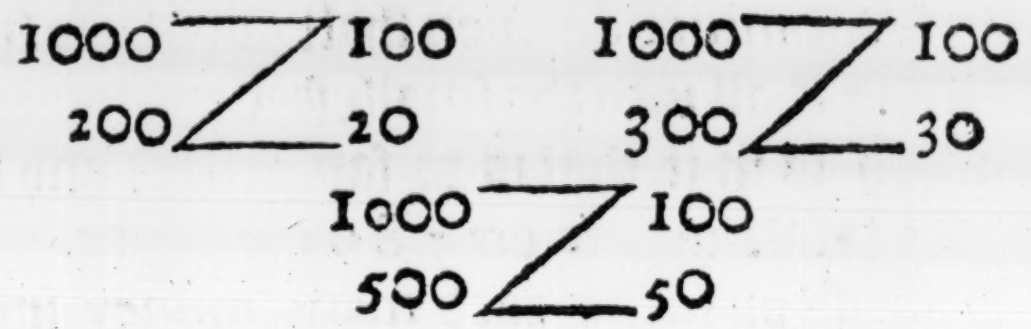
Maſter: Well, now another example will I put to you, not of gaines, but of loſſe: for one reaſon ſerueth for both.

A queſtion of loſſe.

If three Merchants in one ſhippe, and of one fellowſhippe, had bought Merchandize, ſo that the firſt had laid out 200 pound, *the ſecond* 300 *pound, the third* 500 *pound, and it chanced by tempeſt that they did caſt ouer board into Sea Merchandiſe of the value of* 100 pound, *how much ſhould each man beare in this loſſe?*

Scholar.

Scholar. If I shall doe in this, as you did in the other question, then must I ioyne their three portions together, 200. 300, 500, which maketh 1000. Then say I, if 1000 lose 100 then shall 200 lose 20, and 300 shall lose 30, and 500 shall lose 50, as by the three figures it doth appeare plaine.



Master. Well, sith now you haue done these, I willl propound a question of more importance, which shall make you not onely the abler to vnderstand this Rule, but also it will greatly aid you in the next Rule of Fellowship with time, if such need bee that your money be of diuers denominations.

For this may not be forgotten in all such questions: if the number be of diuers kinds, you must by Reduction bring it into one kind that is to say, to the least value that is named in the question. And likewise shall you doe, if the time be of diuers kinds, as some yeeres, some moneths, weekes, and daies, you shall make all moneths, weekes or daies, according as the least name of time in the question is, as for example.

First in diuersity of money. Three companions bought 2000 sheep & paid for them 241 pound 13-4 pence, of which summe one paide 101

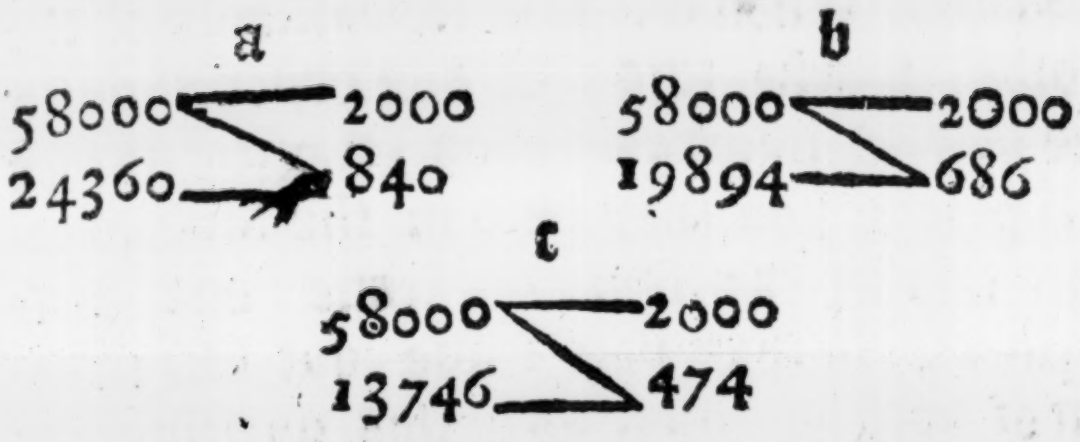
A question of sheepe.

pound, 10 ſhillings. *The ſecond* 82 pound, 17 ſhillings, 10 pence. *And the third paid* 57 pound 5 ſhillings 6 pence, *How many ſheepe muſt each of them haue?* Anſwer: *The firſt ſhall haue* 840. *The ſecond* 686. *And the third* 474. *And that muſt you worke thus.*

Solution. First, conſidering that your money is of diuers denominations, you ſhall (by Reduction) bring it all into the ſmalleſt denomination which is in it, that is to ſay, pence; and ſo will the totall ſumme bee 58000 pence.

Now if you turne each mans money into pence alſo, the firſt mans ſumme will bee 24360 pence: The ſecond mans money will bee 13746 pence.

Now to know how many ſheepe euery man ſhall haue, let the whole ſumme of money, that is, 58000 pence be ſet in the firſt place, and in the ſecond place ſet the number of ſheepe, and then orderly in the third place ſet each mans money, and then multiplying the third and the ſecond ſummes together, and diuiding that that amounteth by the firſt, there will appear the number of ſheepe that each man ought to haue: as theſe three figures doe ſhew.



Scholar. **Why doe you ſet the money in the firſt place, ſéeing in the queſtion you ſay 2000 ſheepe coſt 58000 peuce, and not thus: 58000 coſt 2000 ſheepe.**

Maſter. **You remember I taught you at the beginning of the Golden Rule, that the firſt and third numbers muſt bée of one name, and of like things: and euermoze the number that the queſtion is aſked of muſt be ſet in the third place.**

Now is the queſtion plainly this: If foure men bought 2000 ſheepe for 58000 pence, how many ſheepe ſhall each man haue.

But ſéeing in this queſtion, there ought moze reſpect to be had to the ſumme of money then to the ſumme of the perſons, (foz in the ſumme of money is their pzopoztion toward the ſheepe, and not in the number of perſons)

If 58000 *pence brought* 2000 *ſheepe, how many did* 24360 *buy? Againe, how many did* 19894 *pence buy? And how many bought* 13746 *pence?*

Scholar. **I perceiue it reaſonably, and ſo ſhall**

ſhall I doe in all like queſtions.

Maſter. Euen ſo. But for eaſineſſe of the worke, marke this: Whenſoeuer the firſt and ſecond numbers haue cyphers in the firſt places, you may both in the Multiplication and in the Diuiſion leaue out thoſe cyphers, ſo that you leaue out like many out of both ſummes, as in this queſtion, the firſt number 58000 hath three cyphers, and ſo hath the ſecond, that is 2000: therefore caſt away their cyphers, and ſo will the firſt number be 58, and the ſecond 2: ſet them in their places, and worke according to the Rule, and you ſhall perceiue that it will be all one, ſauing that this is the ſhorter and eaſier way, as theſe three figures doe ſhew.

Note.

a

58 — 2

24360 — 840

b

58 — 2

19894 — 686

c

58 — 2

13746 — 474

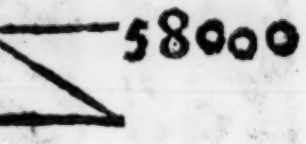
And this you ſée is both eaſier, and alſo the more certaine way to know the anſwer to this queſtion.

Scholar. Truth it is as you ſay: but ſir me ſéemeth I might aſke a further queſtion, here, not onely how many ſheepe each man ſhould haue, but alſo what euery ſheepe coſt.

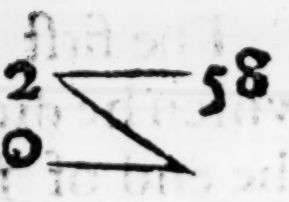
Maſter. That queſtion doth not onely be-

long to this Rule, but may also be discussed by Diuision, especially if the questions number be one only, as thus: Diuide the totall summe 58000 pence by 2000 (or 58 by 2, omitting the cyphers, and the quotient will be 29 pence that is 2 shillings 5 pence; Howbeit, by this Rule you may do it, and best when the number of the question doth exceede 1; as if I should aske this question,

2000 sheepe cost 58000 pence, how much doe 20 cost: Then shall I set my figure as before.



And doing after the Rule, there will amount 580 pence, that is 2 pound 8 shilling, 4 pence the price of one score: but if you will vse that easie way that I did teach you now, you may change the first and second number thus.



Thus doe you perceiue the vse of the Rule without Time.

Scholar. All this I vnderstand very well: I pray you now instruct mee in the Rule of Fellowship with time.

The Rule of Fellowſhip with time.

The Rule of Fellowſhip with time:

Maſter.

THe intent you may as well perceiue the ſame Rule with diuerſity of time, I propoſe this example.

Foure Merchants made a common ſtocke, which at the yeeres end was encreaſed to 35145 pound. *Now to know what ſhall be each mans portion of gaine, you muſt know each mans ſtocke, and time of continuance.*

Queſtion of a banke

The firſt man of theſe foure laid in 669 which he did take from the ſtocke againe at the end of 10 moneths. The ſecond man laid in 810 pound, for eight moneths. The third laid in 9000 pound, for ſeuen moneths. And the fourth laid in 1040 pound, for 12 moneths.

Note. A general Rule.

This queſtion ſhall you examine as you did the other before, ſauing that whereas in the third place of the figure you did ſet each mans ſumme alone, here you ſhall ſet the ſame being multiplyed by the number of their time: and likewiſe in the firſt place of the figure you ſhall ſet the number which amounteth of their whole ſummes ſo multiplied by their time, and added into one whole

whole ſumme, as thus.

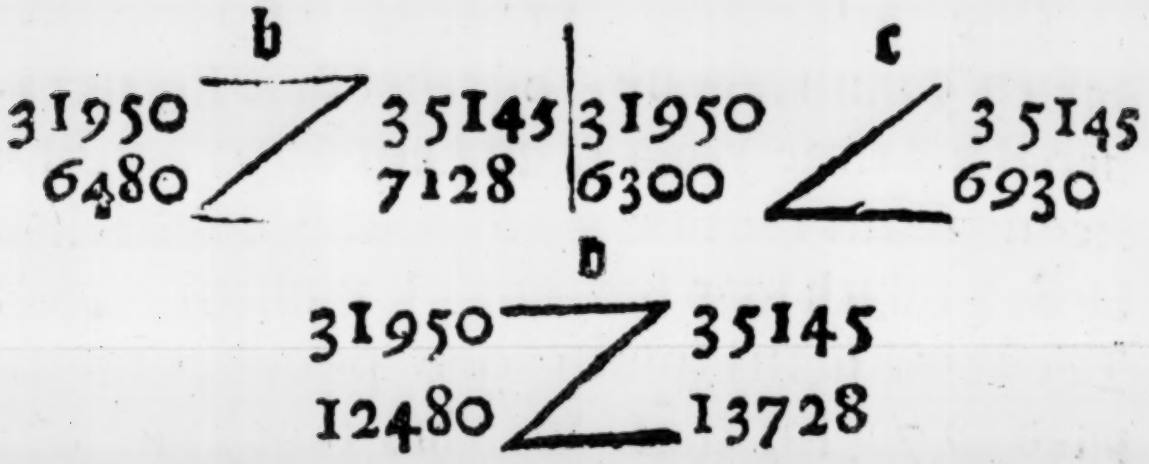
The firſt mans ſumme is 669 pounds, which I multiply by 10 (that was the number of his time) and it maketh 6690. The ſecond mans ſumme 810 pound, multiplied by 8, (which was his time) maketh 6480. The third mans ſumme 900 pound, multiplied by 7, (for that was his time) yéeldeth 6300. The fourth mans ſamme was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

The foure ſummes thus multiplied by their time, muſt be ſet orderly in the third place of the figure, and in the firſt place muſt be ſet the whole ſumme of all foure, which is 31950; and the gaine muſt be in the ſecond place, which is 35145. Now to end the queſtion, I ſay firſt; If

31950 did get 35145, what did 6690 get? Anſwer, 7359 pounds, as by this figure appeareth.

a	
31950	35145
6690	7359

Likewiſe, the ſecond man had to his part 7128 pound, the third muſt haue 6930 pounds. And the fourth man ſhall haue for his part 13728 pound, as theſe figures doe partly declare.



Scholar. This I like very well: but what proofe is there of this worke?

Another proofe.

Master. The same that I taught you for the other: howbeit, there is vsed both for this worke and the other also, this manner of proofe, to adde all the portions together, and if may agrée to the whole summe, then séemeth your worke well done: but this is no sure proofe.

Scholar. Yet will I proue in this example: The foure parcels are these, which if I adde together, there wil amount 35145, and that was the whole summe, whereby I perceiue the worke is well done.

	7359
	7128
	6930
	13728
	35145

Master. If it fall out otherwise, be sure it is not well.

Scholar. Then doe I vnderstand this worke also very well: but what haue I nexte to learne?

Master. There are many other excellent parts

parts behind, of which I will not as now make mention, because that without the knowledge of fractions they cannot bee duely taught, and much lesse vnderstood. Therefore will I propose to you two or three questions more, (that thereby you may better perceiue the vse of this Rule and all other the like) and so make an end for this time.

A question of losse.

¶ *Three partners, by some ill aduenture sustained the losse of* 160 pound, *whereof the first laid into the common stocke* 200 pound, *for ten moneths. The second laid in* 350 pounds, *and the third* 100 pound, *but for how long, the two latter is vnknowne. But breaking off their partnership, the first found himselfe a loser* 80 pound, *the second* 56 pound, *and the third* 24 pound. *The question is, for how long time was the money of the two latter in company.*

For the solution hereof, and of such other like, you must also multiply the first mans 200 pound, that he put into the stocke by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme, if his money that lost 80 pound, multiplied by his time make 2000: what shall his money make that lost 56 pound, and his that lost 24 pound, which two numbers I commit to the triall of the rule of Three, at two workings thus:

If 80 giue 2000, what giueth 56? And againe, if 80 giue 2000, what giueth 24?

80

80 Z 2000
50 Z 1400

80 Z 2000
24 Z 600

To conclude, if you now diuide 1400, the ſecond mans portion by 350, which was his ſtocke that hée laid into Company, you ſhall finde in your quotient 4 moneths, and for ſo long time did the ſecond man put his money into the common ſtocke.

Laſtly, if you diuide the third mans new laying in, which was 600 by 100, which was his ſtocke that hee put into Company: the Quotient declareth his time of continuance, which was 6 moneths. And thus is the queſtion reſolued.

Scholar. Sir, I haue attentiuely beheld your working, and the more we trauell herein, the more mée thinke I am in loue with this excellent Art.

Maſter. Then what ſay you to this Queſtion?

A queſtion of Canons

There is in a Cathedrall Church 20 *Canons and* 30 *Vicars, thoſe may ſpend by yeere* 2600 pound, *but euery Canon muſt haue to his part fiue times ſo much as euery Vicar hath: how much is euery mans portion, ſay you?*

Scholar. I pray you make the anſwere your ſelfe alſo, ſo ſhall I perceiue beſt the meanes to anſwer to ſuch other like.

Maſter: In this queſtion, you muſt doe as in thoſe before ſaid, that haue diuerſity of time,

time, for here is diuerſity of portions. Therfore ſhall you multiply the number of the perſons by their difference of portion: (as you did in the other by time:) Then muſt you multiply the 20, (which is the number of Canons) by 5, (for that is the number of their portion) ſo will it be 100: Then 30, (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put thoſe two ſummes together, and they make 130. Then ſay thus: If 130 ſpend 2600 pounds, what may 100 ſpend? The Rule ſheweth 1000 pounds.

Againe for Vicars: If 130 ſpend 2600 pound, what may 30 ſpend? Anſwer, 600 pound, as theſe figures ſhew.

130	2600	130	2600
100	2000	30	600

But if euery Canon ſhould haue ſo oftentimes 4 pound, as the Vicar ſhould haue 3 pound, then ſhould I multiply 20 by 4. (that were 80) and 30 by 3. (that were 90) and then both were 170. Then ſhould the figures be ſet as followeth.

	li	ſ	d		li	ſ	d
170	2600			170	26000		
80	1223	10	7	90	1376	9	5

But this ſort is ſo hard for you, by rea-

ſon of the fractions, therefore I will let it reſt to that place:

And by this Rule you ſée what the 20 Canons may ſpend, which ſumme if you diuide by 20, you ſhall ſée each Canons portion: and ſo of the Vicars, if you diuide their ſumme by 30, the quotient will declare euery Vicars portion.

The

The ſecond Dialogue.

The accompting by Counters.

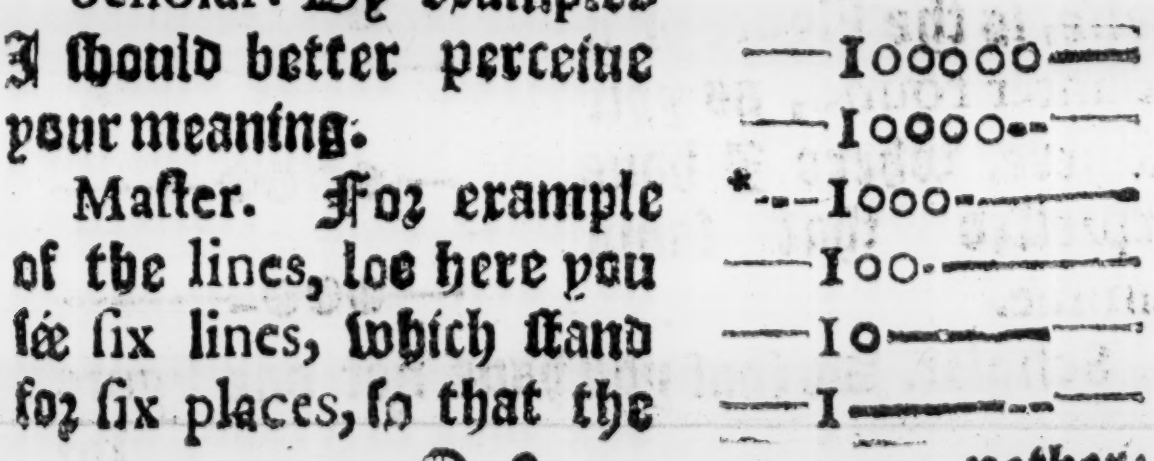
Maſter.

NOW *that you haue learned the common kindes of Arithmeticke with the pen, you ſhall ſee the ſame Art in* Counters: *which feat doth not onely ſerue for them that cannot write and reade, but alſo for them that can doe both, but haue not at ſometime their pen or tables ready with them.*

This ſort is in two formes commonly. The one by lines, and the other without lines. In that that hath lines, the lines doe ſtand for the order of places: and in that that hath no lines, there muſt be ſet in their ſtead ſo many Counters as ſhall need for each line one; and they ſhall ſupply the ſtead of lines.

Scholar. By examples I ſhould better perceiue your meaning.

Maſter. For example of the lines, loe here you ſée ſix lines, which ſtand for ſix places, ſo that the



Numeration by Counters,

nethermoſt ſtandeth for the firſt place, and the next aboue it for the ſecond, and ſo vpward till you come to the higheſt, which is the ſixt line, and ſtandeth for the ſixt place.

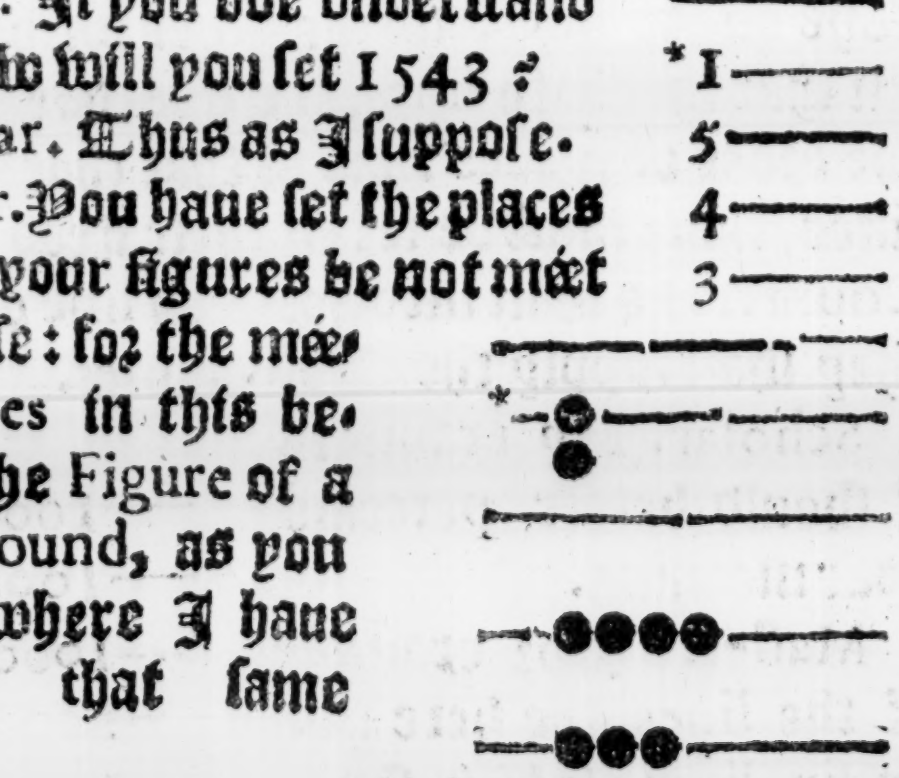
Now what is the value of euery place or line you may perceiue by the figure which I haue ſet on them, which is according as you learned before in Numeration of figures by the pen: for the firſt place is the place of vnites or ones, & euery counter ſet in that line, betokeneth but one: and the ſecond line is the place of 10, for euery counter there ſtandeth for 10: the third line the place of hundreds, the fourth of thouſands, and ſo forth,

Scholar. Sir, I doe perceiue that the ſame order is here of lines, as was in the other figures by places, ſo that you ſhall not need longer to ſtand about Numeration, except there be any other difference.

Maſter. If you doe vnderſtand it, then how will you ſet 1543?

Scholar. Thus as I ſuppoſe.

Maſter. You haue ſet the places truly, but your figures be not meet for this vſe: for the meeteſt Figures in this behalfe, is the Figure of a counter round, as you ſee here, where I haue expreſſed that ſame ſumme.

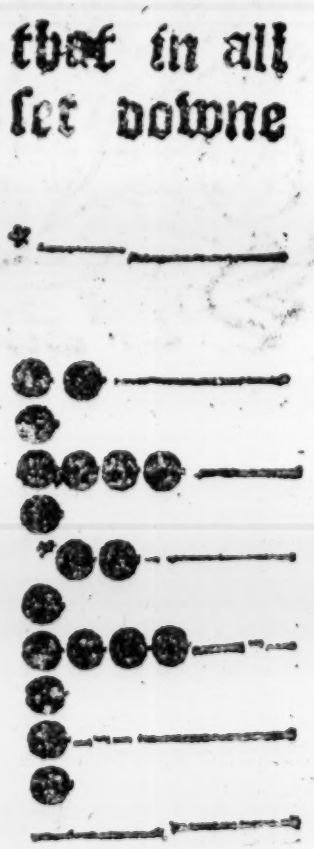


Scholar. So that you haue not one Figure for

for 2, nor 3, nor 4, & so forth, but as many Digits as you haue, so many Counters you set in the lowest line, and for euery 10 you set one in the second line, and so of other. But I know not by what reason you set that one Counter for 500 betweene two lines.

Master. You shall remember this, that whensoeuer you néed to set downe 5,50, or 500, or 5000, or set forth any number whose Numerator is 5, you shall set one counter for it in the next place aboue the line that it hath his denomination of: as in this example of that 500, because the numerator is 5, it must be set in a void space, and because the denomination is a hundred, I know that his place is the void space next aboue hundreds, that is to say, aboue the third line.

And further you shall marke, that in all working by this sort, if you shall set downe any summe betweene 4 and 10, for the first part of that number you shall set downe 5, and then so many counters more, as there rest numbers aboue 5. And this is true both of Digits and Articles. And for example, I will set downe this summe 297965, which summe if you marke well, you néed none other examples for to learne the numeration of this forme.



But this shall you marke, that as you did in other kindes of Arithmeticke, set a pricke in the places of thousands, in this worke you shall set a Starre, as you see before.

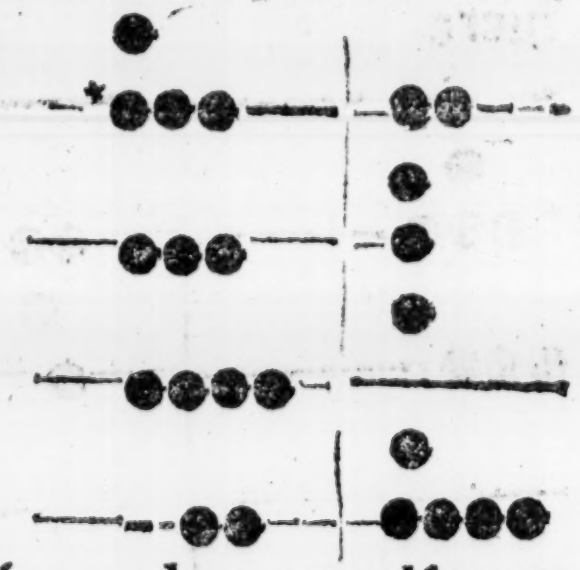
Scholar. Then I perceiue Numeration: But I pray you how shall I doe in this Art to adde two summes or more together.

Addition.

Addition.

Maſter.

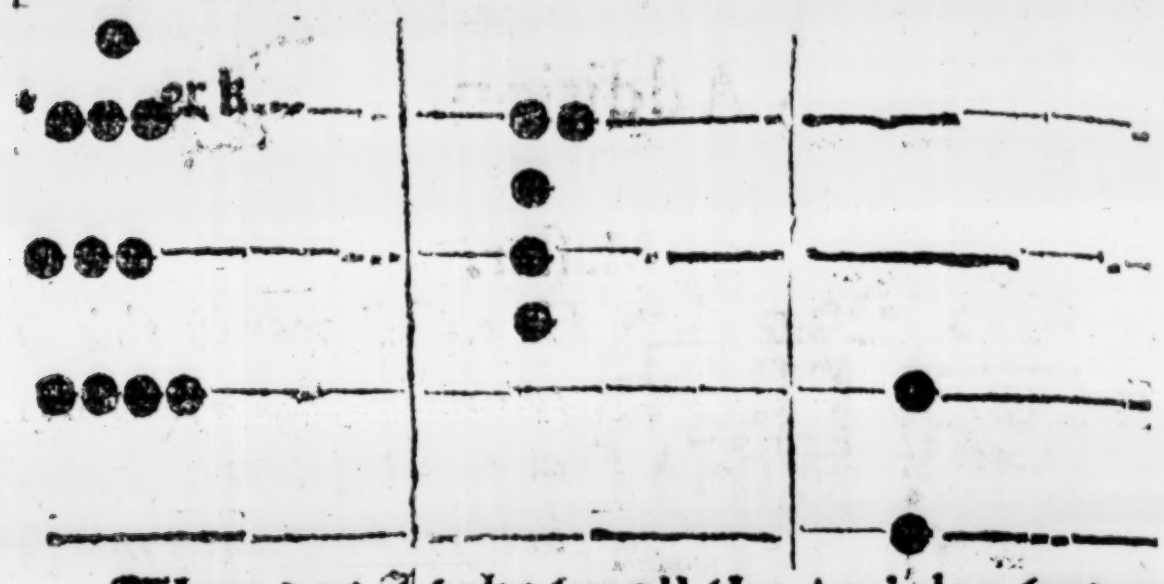
THe eaſieſt way in this is to add but two ſummes at once together: Howbeit you may adde more, as I will tell you anon. Therefore when you wil adde two ſummes you ſhall firſt ſet downe one of them, it forceth not which, and then by it draw a line croſſe the other lines. And afterward ſet downe the other ſumme, ſo that the line may be betweene them: as if you would adde 2659 to 8342, you muſt ſet your ſummes as you ſée héere.



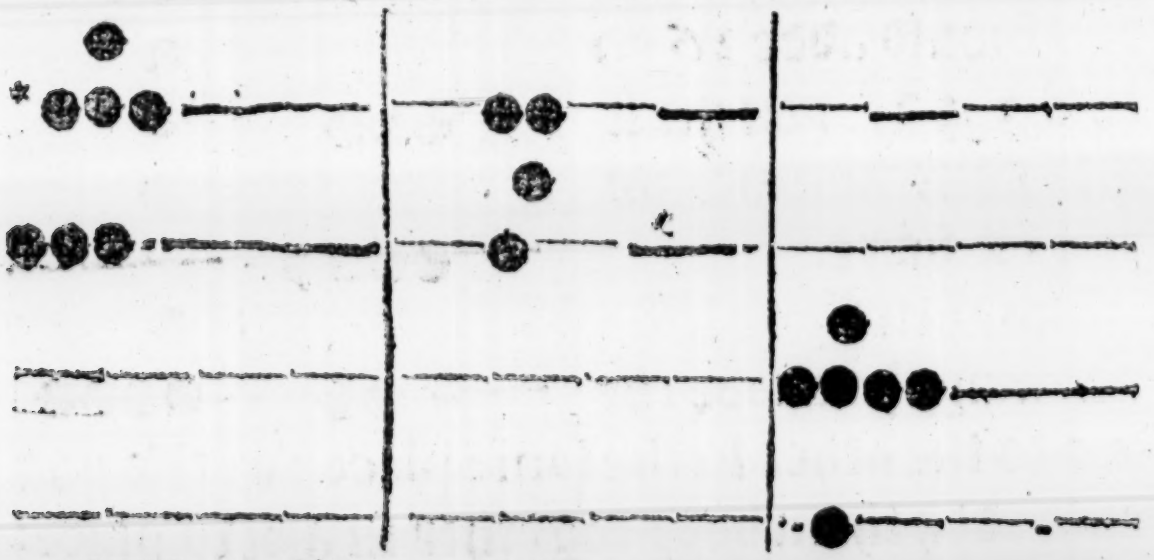
Addition of two ſummes.

And then if you liſt, you may then adde the one to the other in the ſame place: or elſe you may adde them both together in a new place: which way, becauſe it is moſt plaine, I will ſhew you firſt.

Therefore will I beginne at the vnites, which in the firſt ſumme is but 2, and in the ſecond ſumme 9, that maketh 11. Thoſe doe I take vp, and for them I ſet 11 in the new roome, thus:

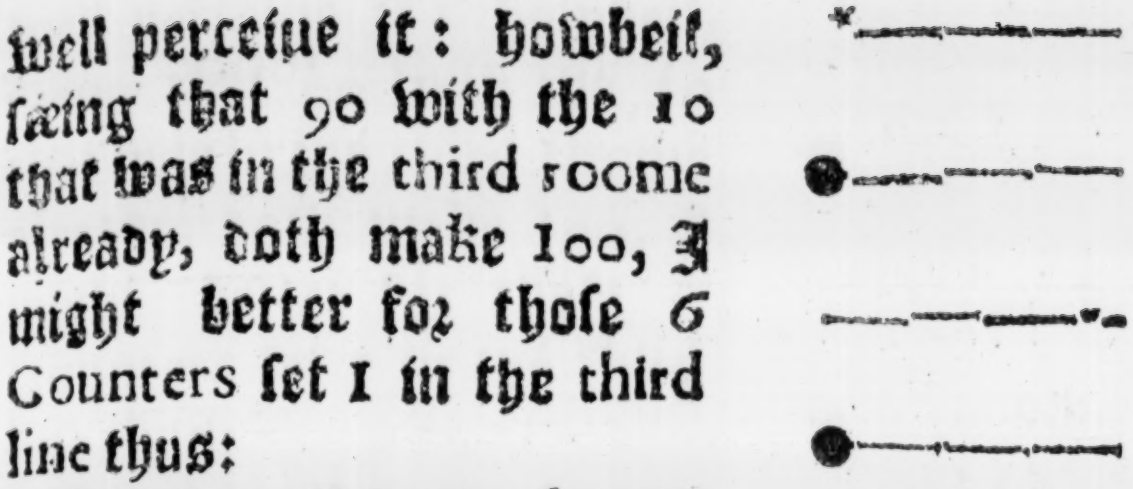


When doe I take vp all the Articles vnder a hundred, which in the first ſumme are 40, and in the ſecond ſumme 50, that maketh 90: or you may ſay better, that in the firſt ſumme there are foure Articles of 10, and in the ſecond ſumme 5, which maketh 9, but then take héed that you ſet them in their right lines, ſée héere.



Where I haue taken away 40 from the firſt ſumme, and 50 from the ſecond, and in their ſtead I haue ſet 90 in the third roome, which I haue ſet plainly, that you might well

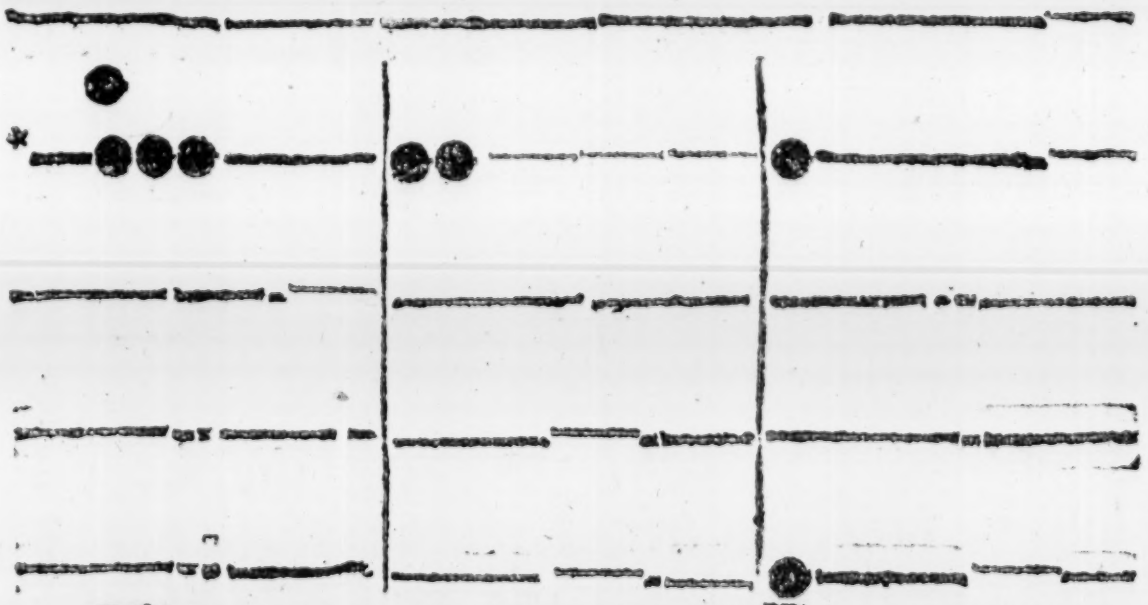
well perceiue it: howbeit, ſæing that 90 with the 10 that was in the third roome already, doth make 100, I might better for thoſe 6 Counters ſet 1 in the third line thus:



For it is all in one ſumme, as you may ſæ, but it is beſt neuer to ſet fiue Counters in any line, for that may be done with one Counter in a higher place.

Scholar. I iudge that good reaſon, for many are vnnædfull where one will ſerue.

Maſter. Well, then will I adde forth of hundreds: I finde 3 in the firſt ſumme, and 6 in the ſecond, which maketh 900, them do I take vp, and ſet in the third roome, where is 100 already, to which I put 900, and it will be 1000: therefore I ſet one Counter in the fourth line for them all, as you ſæ here.



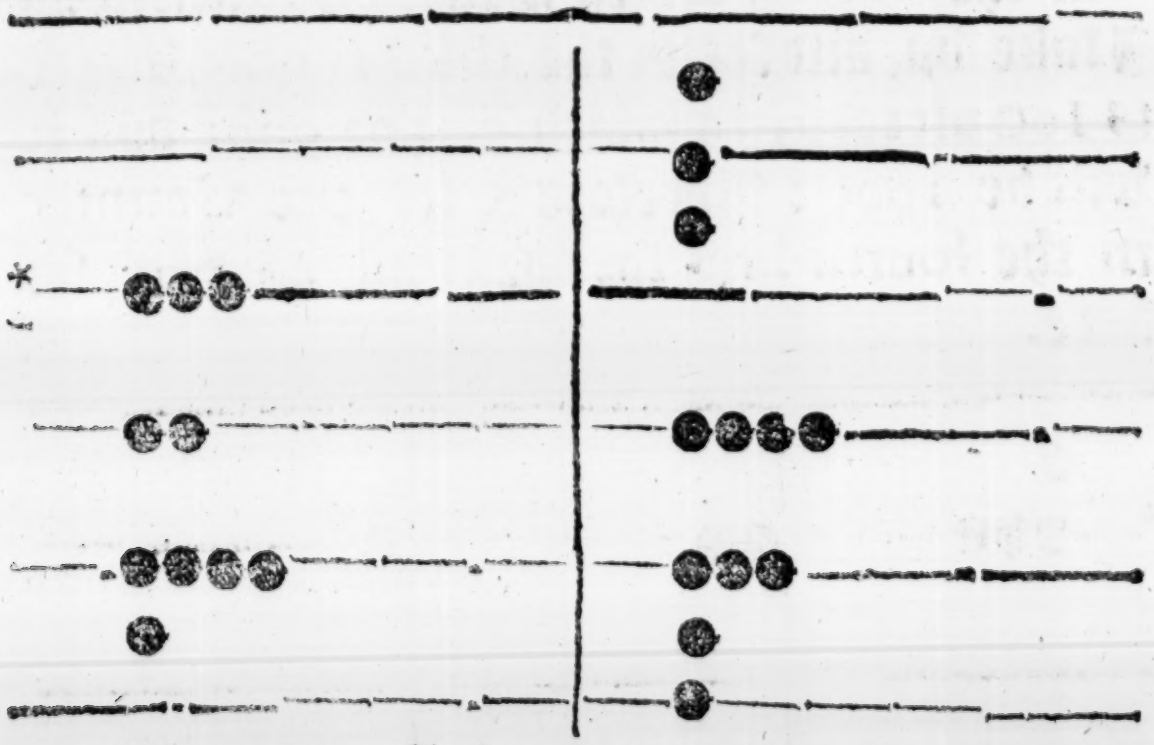
Then adde I the thouſands together, which in the firſt ſumme are 8000, and in the

the ſecond 2000, that maketh 10000: them doe I take vp for thoſe two places, and for them I ſet one Counter in the fift line, and then it appeareth as you ſée to be 11001, for ſo many doth amount of the Addition of 8342 to 2659.

To adde ſummes together.

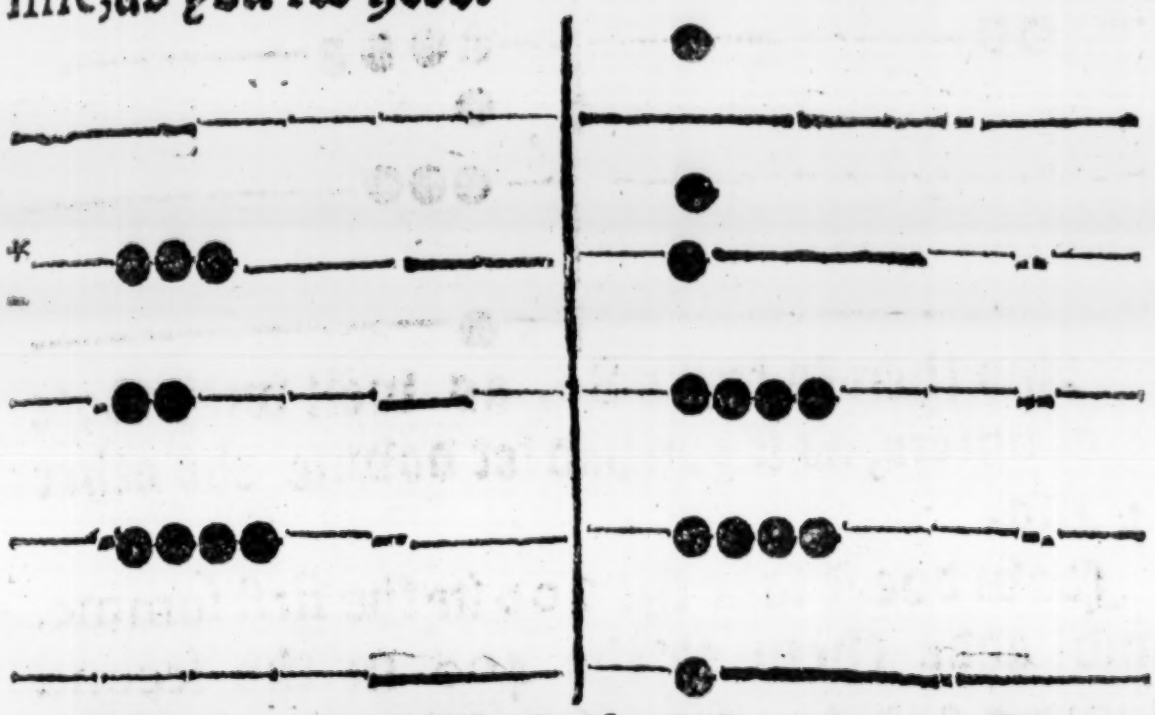
Scholar. Sir, this I doe perceiue: but how ſhall I ſet one ſumme to another, not changing them to a third place.

Maſter. Marke well how I do it: I will adde together 65436 and 32455, which firſt I ſet downe thus:



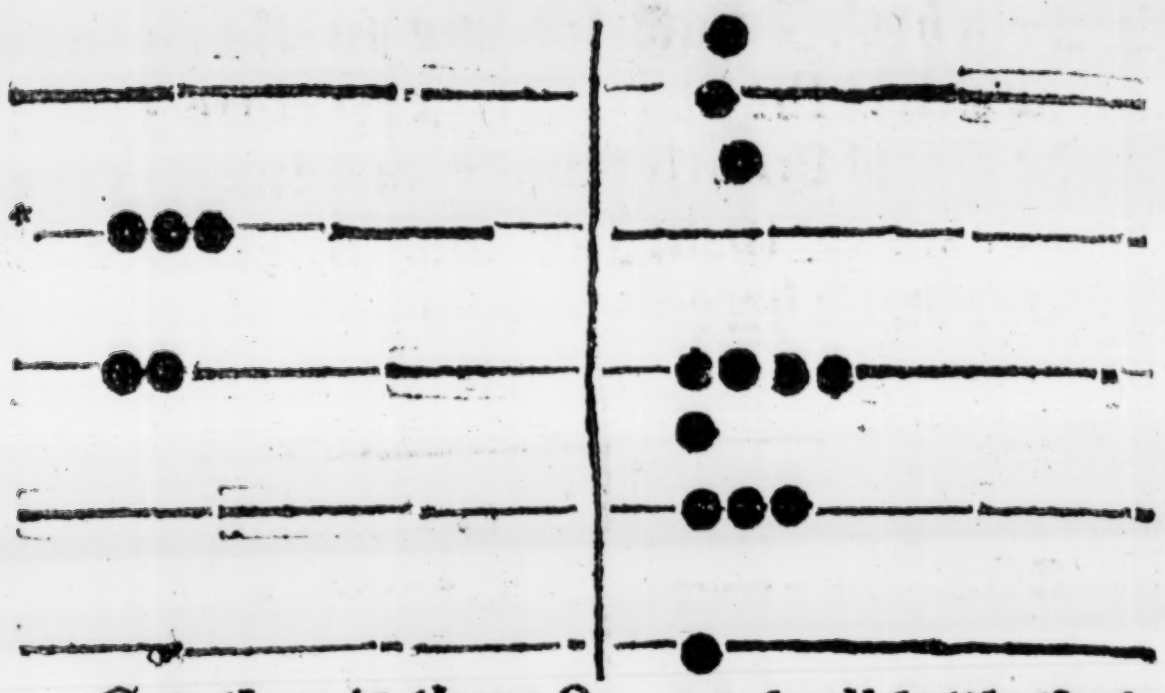
Then doe I beginne with the ſmalleſt denomination, which is 1 in the ſecond ſumme, and ſet it in his place: then doe I finde 5 in the firſt ſumme, and 5 in the ſecond, which I put together, ſaying the two counters cannot be ſet in a void place of 5, but for

for them both I muſt ſet one in the ſecond line, which is the place of 10, therefore I take vp the fiue of the firſt ſumme and the 5 of the ſecond, and for them I ſet one in the ſecond line, as you ſée here.



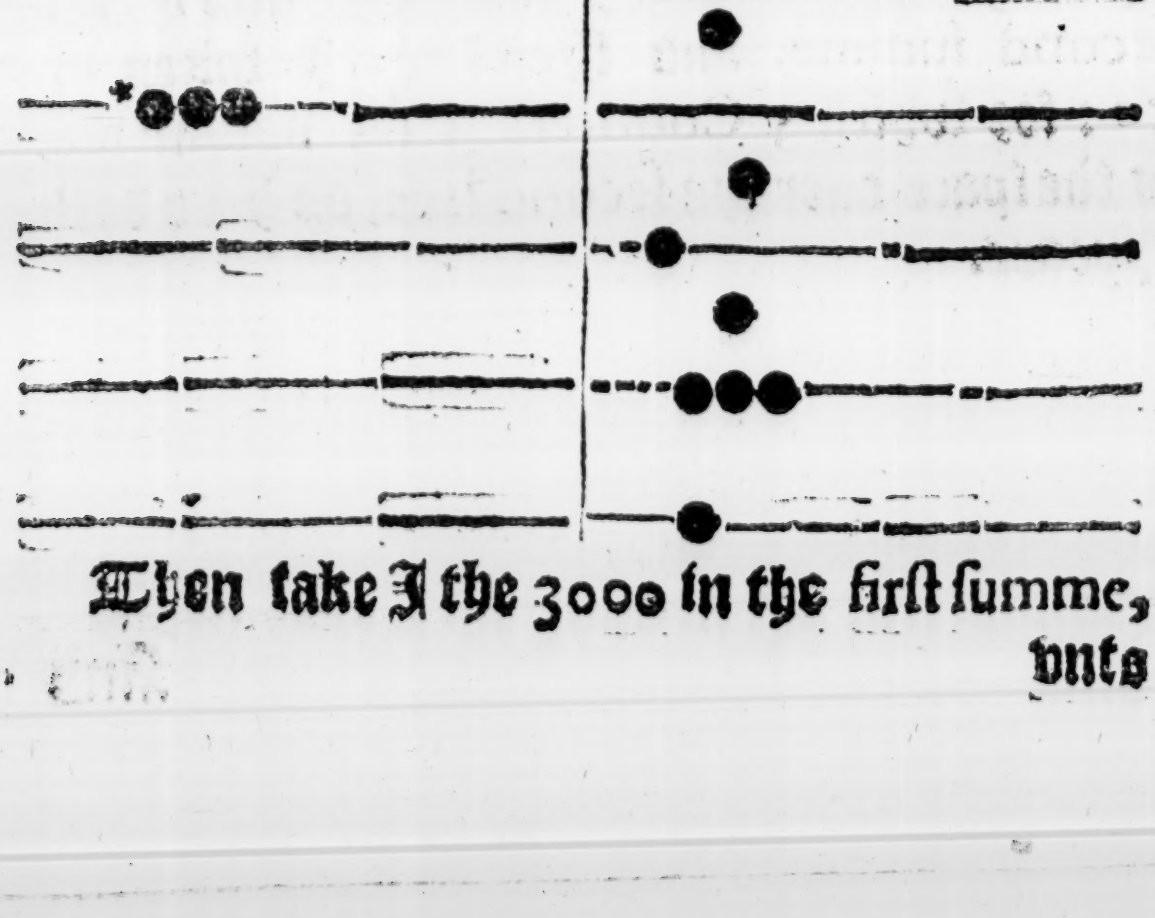
Then doe I likewiſe take the 4 Counters of the firſt ſumme and ſecond line, (which maketh 40) and adde them to the 4 Counters of the ſame line in the ſecond ſumme, and it maketh 80: but as I ſaid, I may not conueniently ſet aboue 4 counters in one line, therefore to thoſe 4 that I take vp in the firſt ſumme, I take one alſo of the ſecond ſumme, and then haue I taken vp 50: for which 5 Counters I ſet downe one in the ſpace ouer the ſecond line, as here doth appeare.

And



And then is there 80, as well with those 4 counters, as if you had set downe the other 4 also.

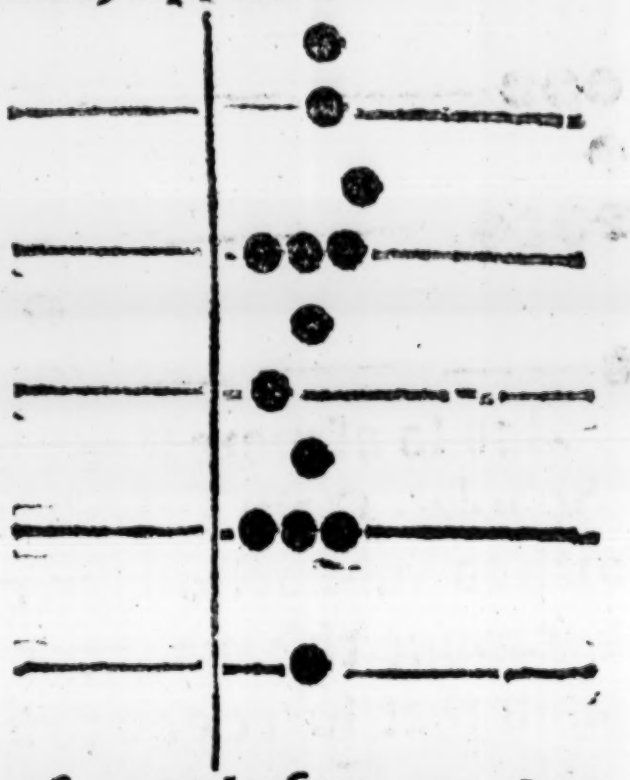
Now doe I take the 200 in the first summe, and adde them to the 400 in the second summe, and it maketh 600, therefore I take vp the two counters in the first summe, and three of them in the second summe, and for them 5, I set 1 in the space aboue, thus:



Then take I the 3000 in the first summe,

vnts

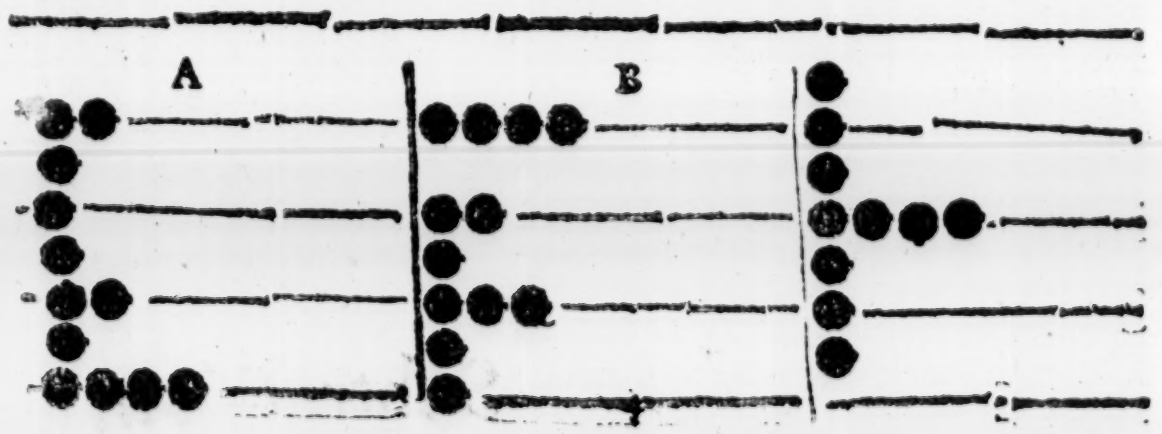
vnto which there are none in the second summe agréeing, therefore I doe only remoue those thrée counters from the first summe into the second, as here doth appeare.



And you sée the whole summe that amounteth of that Addition of 65436 with 3245, to bée 68681.

And if you haue marked these two examples well, you néed no further instruction in Addition of 2 onely summes: but if you haue moze then two summes to adde, you may adde them thus:

First adde two of them, and then adde the third and fourth, oz moze, if there bee so many: as if I would adde 2679, with 4286, and 1391. First I adde the two first summes thus:



And

And then I adde the third thereto, thus:

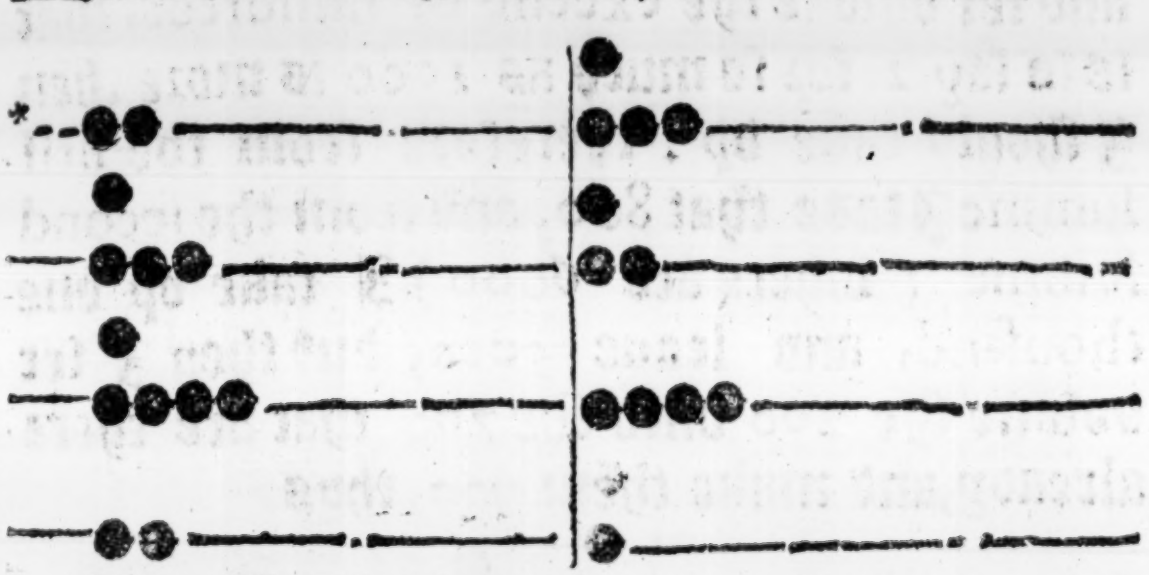
And so of more if you haue them.

Scholar. Now I thinke it best that you passe forth to Subtraction, except there be any way to examine this manner of Addition, then I thinke that were good to be knowne next.

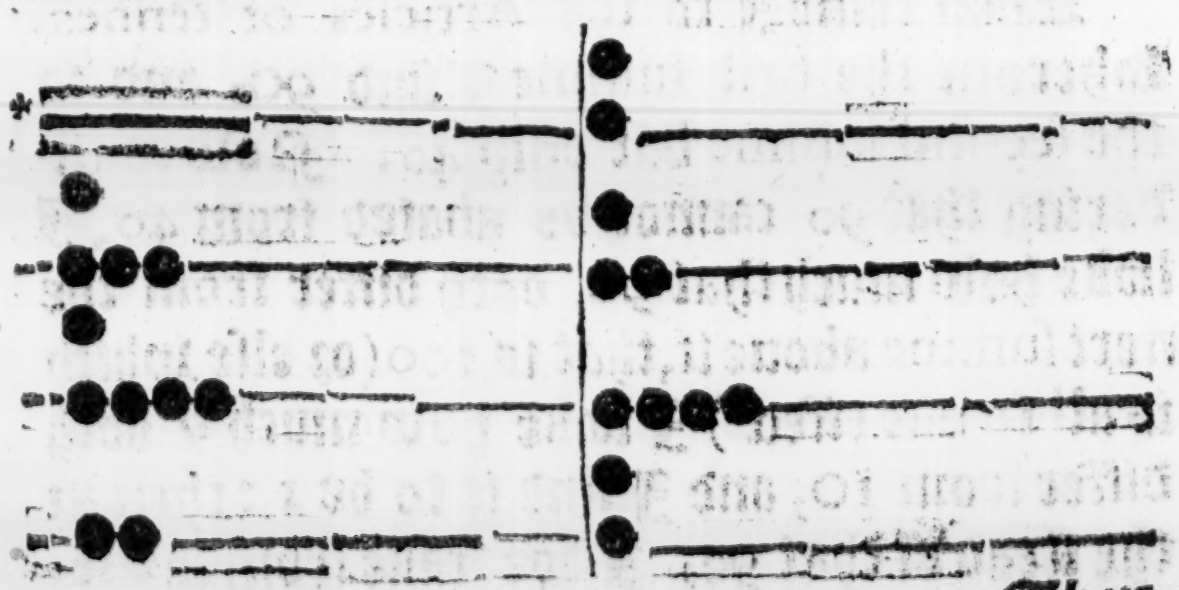
Master. There is the same proofe here that is in the other Addition by the pen, I meane Subtraction; for that onely is a sure way, but considering that Subtraction must bee first knowne, I will first teach you the Art of Subtraction, and that by this example.

Subtraction.

I Would Subtract 2892 out of 8746. These ſummes muſt I ſet downe as I did in Addition: but heere it is beſt to ſet the leſſer number firſt, thus:

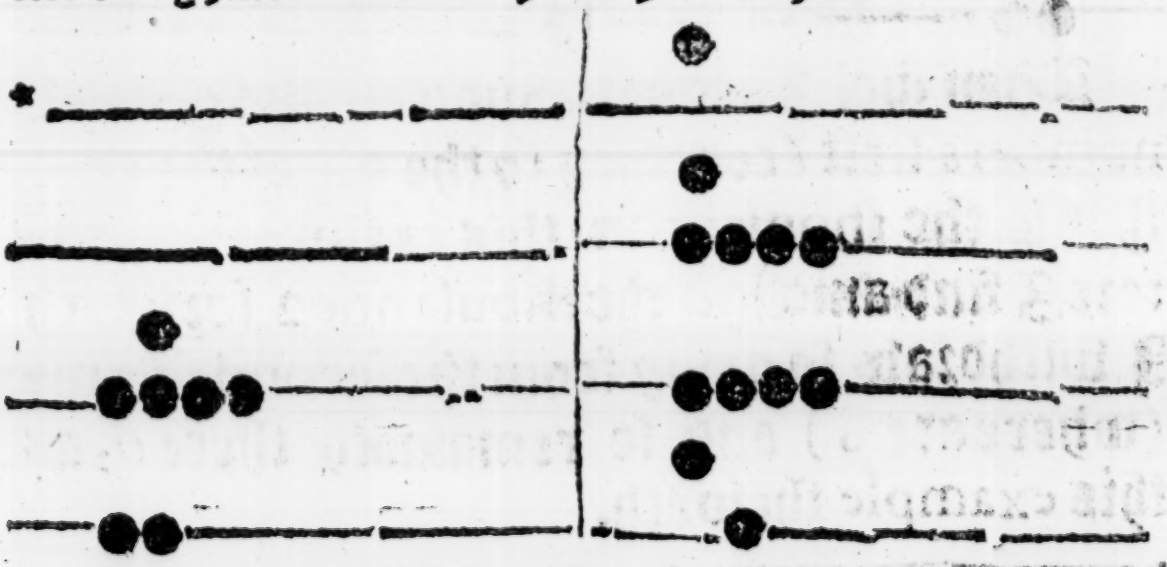


Then ſhal I begin to Subtract the greateſt numbers firſt (contrary to the vſe of the pen) that is, the thouſand in this example: therefore I find amongſt the thouſands 2, for which I withdraw ſo many from the ſecond ſumme (where are 8) and ſo remaineth there 6, as this example ſheweth.



Then

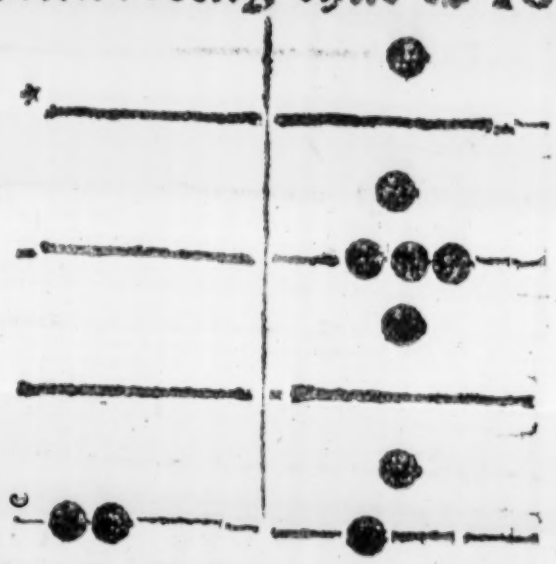
Then doe I likewise with the hundreds, of which in the first summe I finde 8, and in the second summe but 7, out of which I cannot take 8, therefore this must I doe: I must loke how much my summe differeth from 10. which I find here to be 2, then must I abate for my summe of 800, one thousand and set downe the excesse of hundreds, that is to say, 2, for so much as 1000 is more then I should take vp: therefore from the first summe I take that 800, and from the second summe (where are 6000) I take vp one thousand, and leaue 5000, but then I set downe the 200 vnto the 700 that are there already, and make them 900, thus.



Then come I to the Articles of tennes, where in the first summe I find 90, and in the second summe but only 40. Now considering that 90 cannot be abated from 40, I loke how much that 90 doth differ from the next summe aboue it, that is, 100 (or else which is all to one effect) I loke how much 9 doth differ from 10, and I find it to be 1: then in the stead of that 90, I doe take from the se-

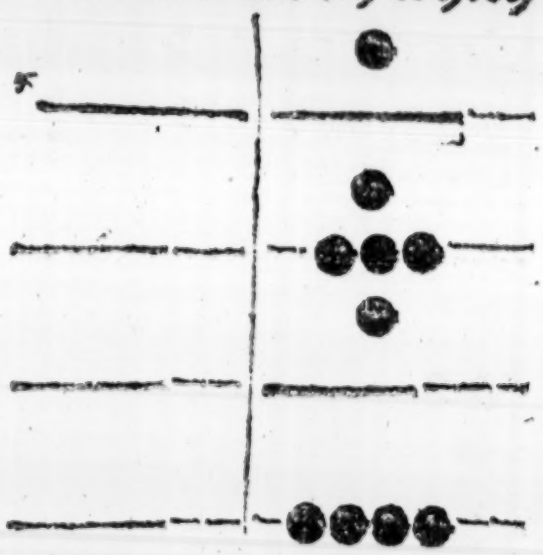
cond

cond ſumme 100: but conſidering that is 10 too much, I ſet downe 1 in the next line beneath for it, as you ſée here.



Sauing that here I haue ſet 1 Counter in the ſpace in ſtead of 5 in the next line.

And thus haue I ſubtract all ſaue 2, which I muſt abate from 6 in the ſecond ſumme, and there will remain 4, thus:

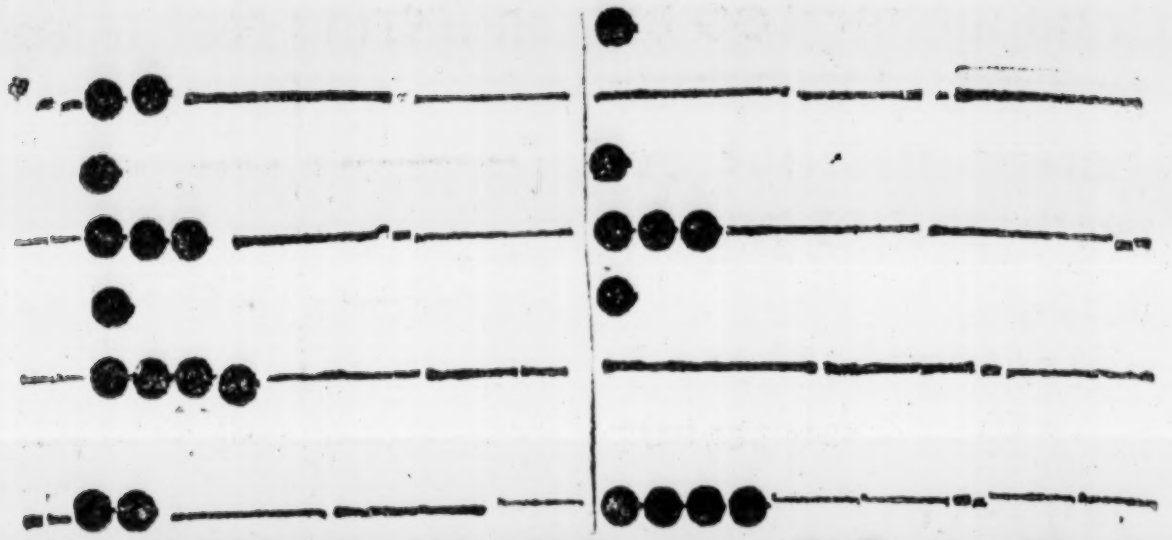


So that if I ſubtract 2892 from 8746 the remainer will bée 5854.

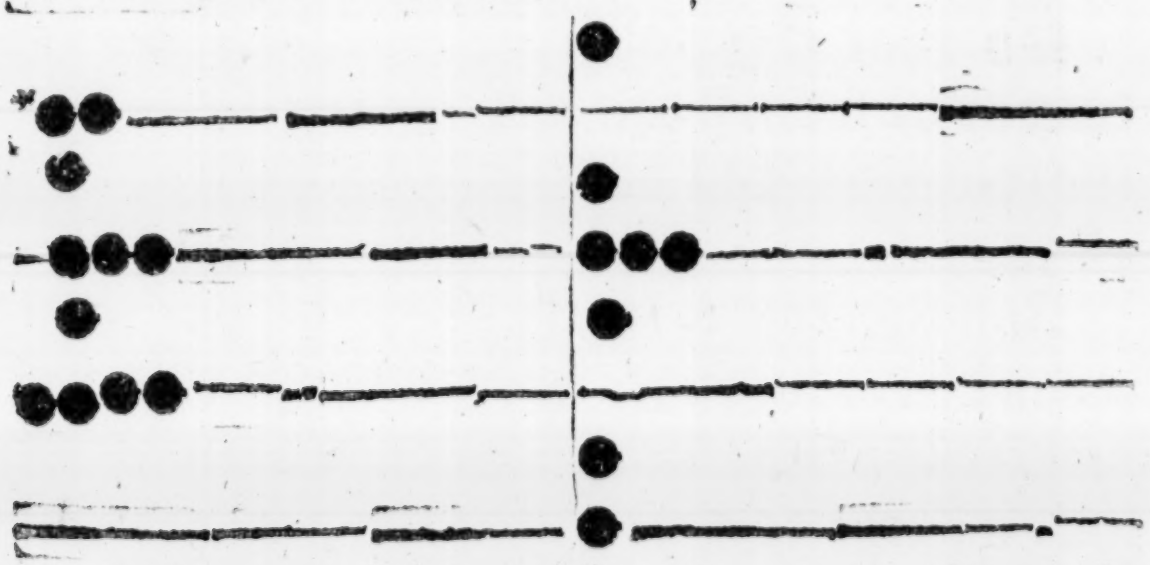
And that this is truely wrought, you may proue by Addition: for if you adde to this remainer the ſame ſumme that you did ſubtract then will the former ſumme 8746 amount againe.

Scholar. That will I proue, and firſt I ſet the ſumme that was ſubtracted, which was 2892, and then the remainer 5854, thus:

A proofe of Subtraction.

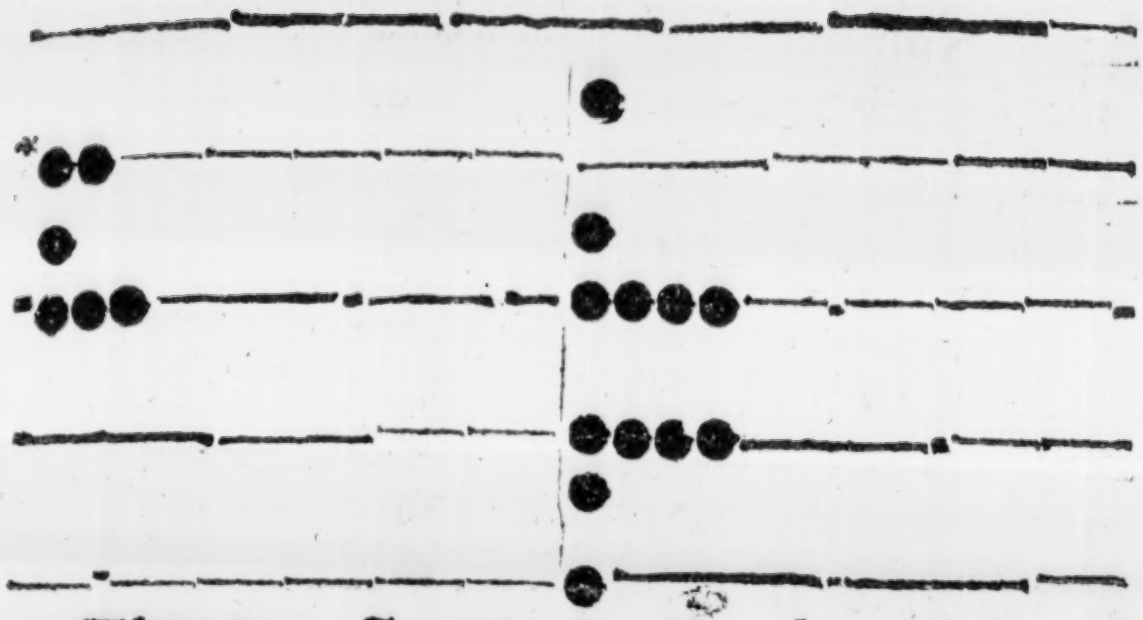


Then doe I adde the first 2 to 4, which maketh 6: so take I vp 5 of those Counters, and in their stead I set 1 in the space, and one in the lowest line, as here appeareth.

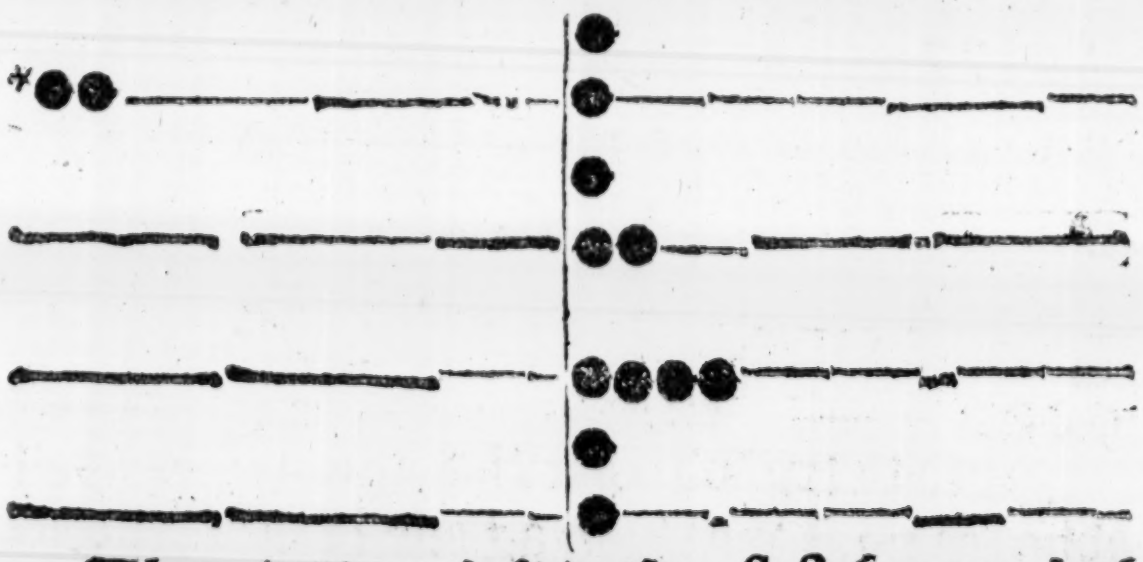


Then doe I adde the 90 next aboue to the 50, and it maketh 140. therefore I take vp those 6 counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus:

Then

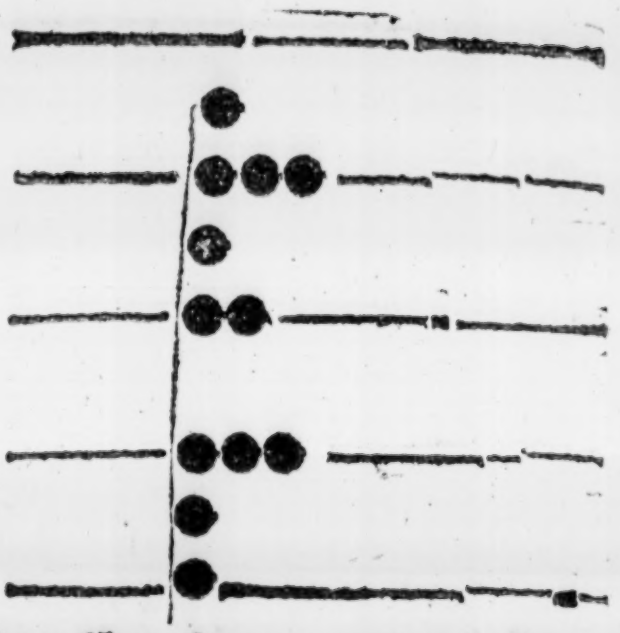


Then doe I come to the hundreds, of which I finde 8 in the first ſumme, and 8 in the ſecond, that maketh 1600, therefore I take vp thoſe 8 counters, and in their ſtead I ſet 1 in the fourth line, and 1 in the ſpace next beneath, and 2 in the third line, as you may ſée here.



Then is there left in the firſt ſumme but onely 2000, and in the ſecond 5000, which is 7000, which I ſhall take vp from thence, and ſet in the ſame line in the ſecond ſumme to the one that is there already: and then will the whole ſumme appeare as you may well ſée, to be 8746, which was the firſt

groſſe ſumme, and therfore I do perceiue that I had well ſubtracted before.



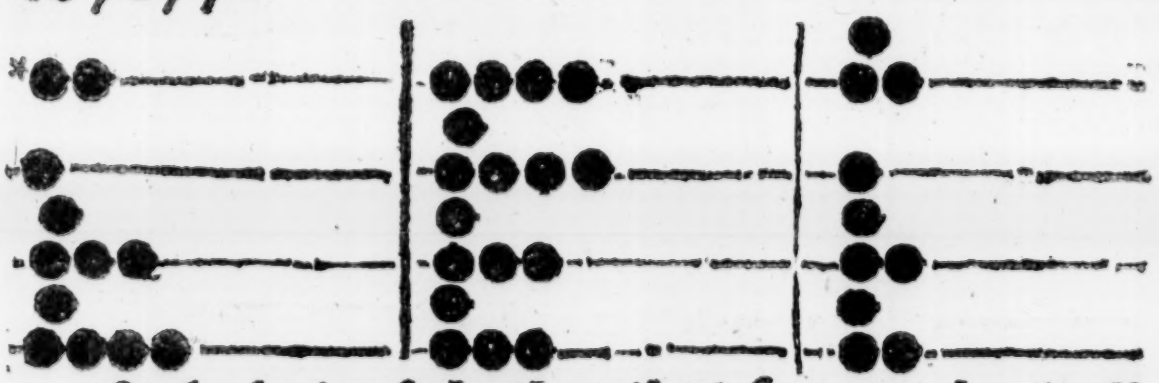
And thus may you ſée, how Subtraction may be tried by Addition.

Scholar. I perceiue the ſame order here with counters, that I learned before in figures.

Maſter. Then let me ſee how you can try Addition by Subtraction.

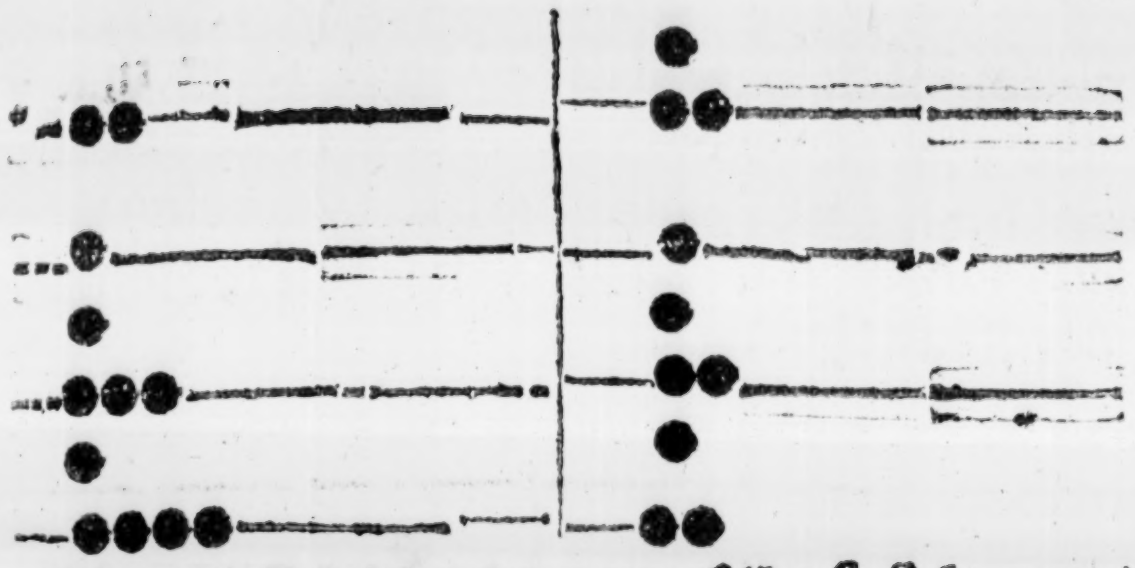
Scholar. Firſt I will ſet forth this example of Addition, where I haue added 2189, to 4988. And the whole ſumme appeareth to be 7177.

Proofe of Addition by ſubtraction.



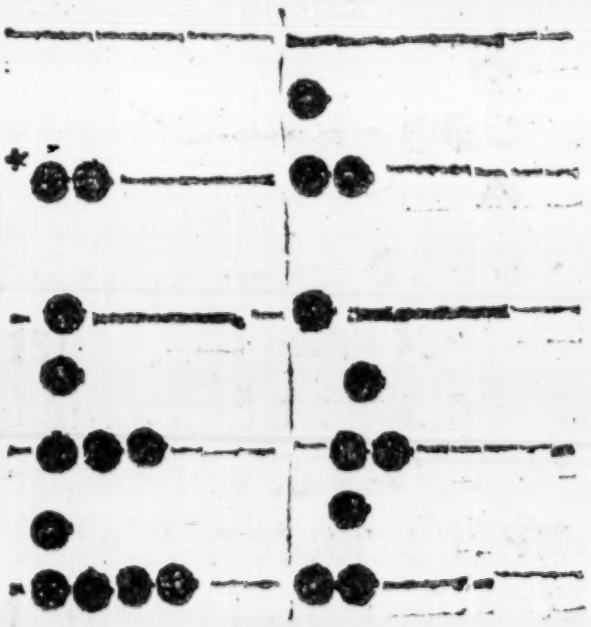
Now to try whether that ſumme be well added or no, I will ſubtract one of the firſt two ſummes from the third. And if I haue well done, the remainer will be like that other ſumme: as for example, I will ſubtract the firſt ſumme from the third, which ſet thus in order:

Then

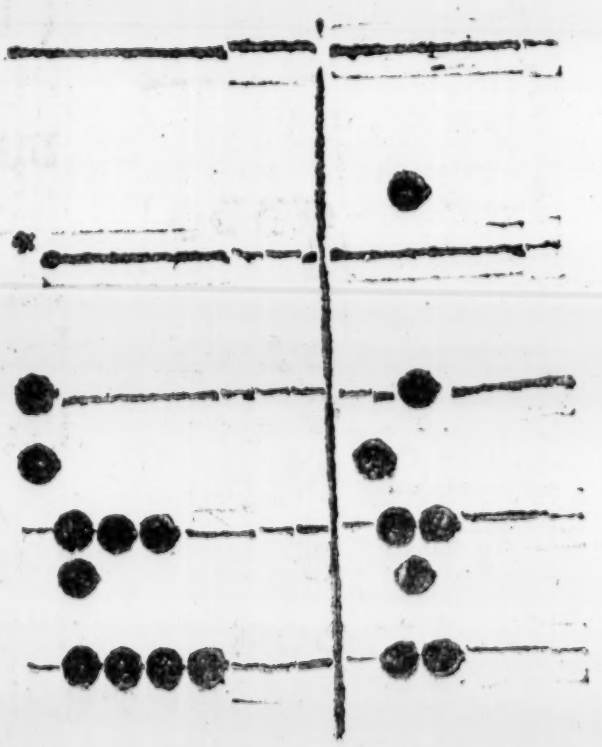


Then do I ſubtract 2000 of the firſt ſumme, from the ſecond ſumme, and then remaineth there 5000, thus.

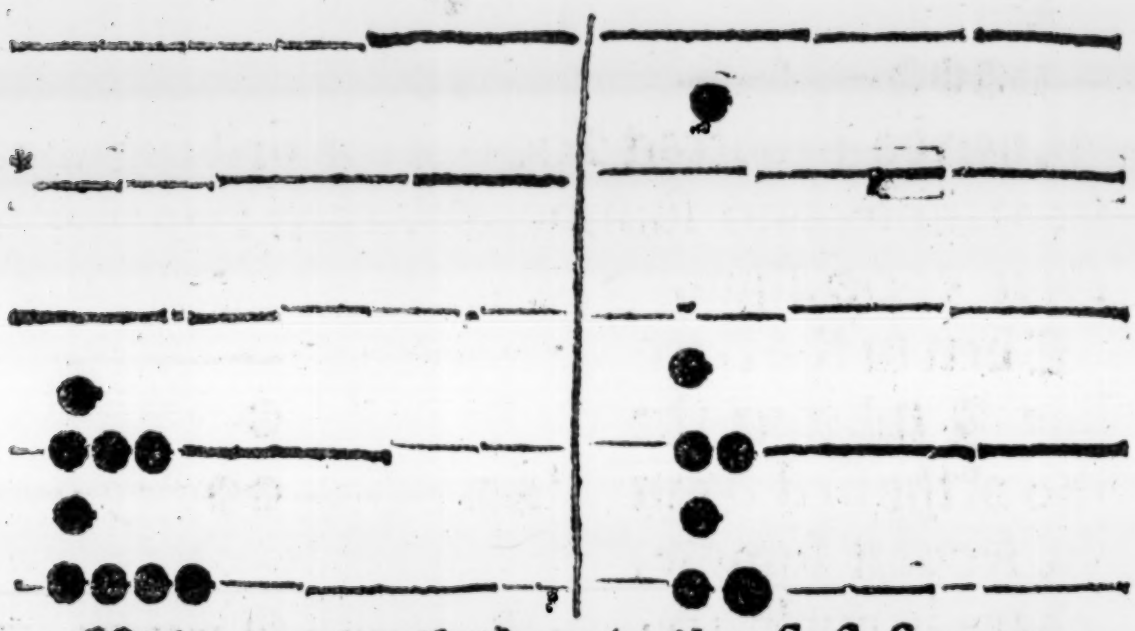
Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is onely 100 alſo: and then in the third line, reſteth nothing, as you may ſée in this example following.



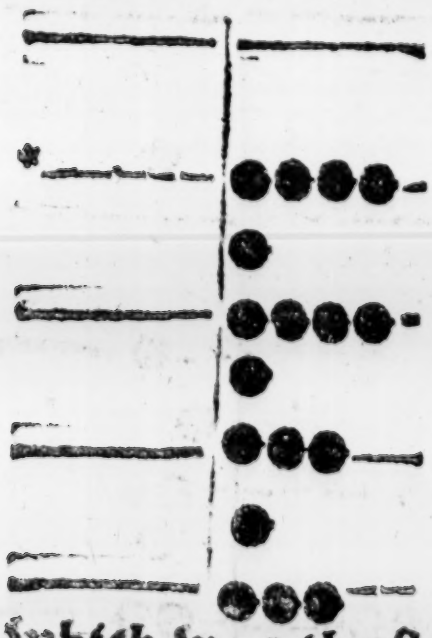
Then in the ſecond line with his ſpace ouer him I find 80, which I ſhould ſubtract from the other ſumme: then ſéeing there are but onely 70, I muſt take it out of ſome higher ſumme, which is here onely 5000:



therefore I take vp 5000: and ſéeing that is too much by 4920, I ſet downe ſo many in the ſecond roome; which with the 70 being there already, doe make 4990, and then the ſummes doe ſtand thus.



Yet remaineth therein the firſt ſumme 9, to be abated from the ſecond ſumme, wherein that place of vnites doth appeare onely 7: then muſt I abate a higher ſumme, that is to ſay 10, but ſéeing that 10 is more then 9, (which I ſhould abate) by 1, therefore ſhall I take vp one Counter from the ſecond, and ſet downe the ſame in the firſt line, or lowermoſt line, as you ſée héere.

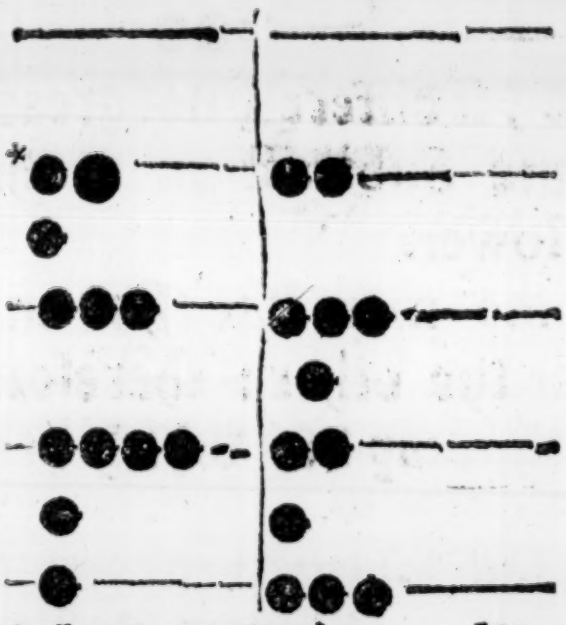


And ſo haue I ended this worke, and the ſumme appeareth to be the ſame which was the ſecond ſumme of mine Addition, and therefore I perceiue I haue wel done.

Maſter.

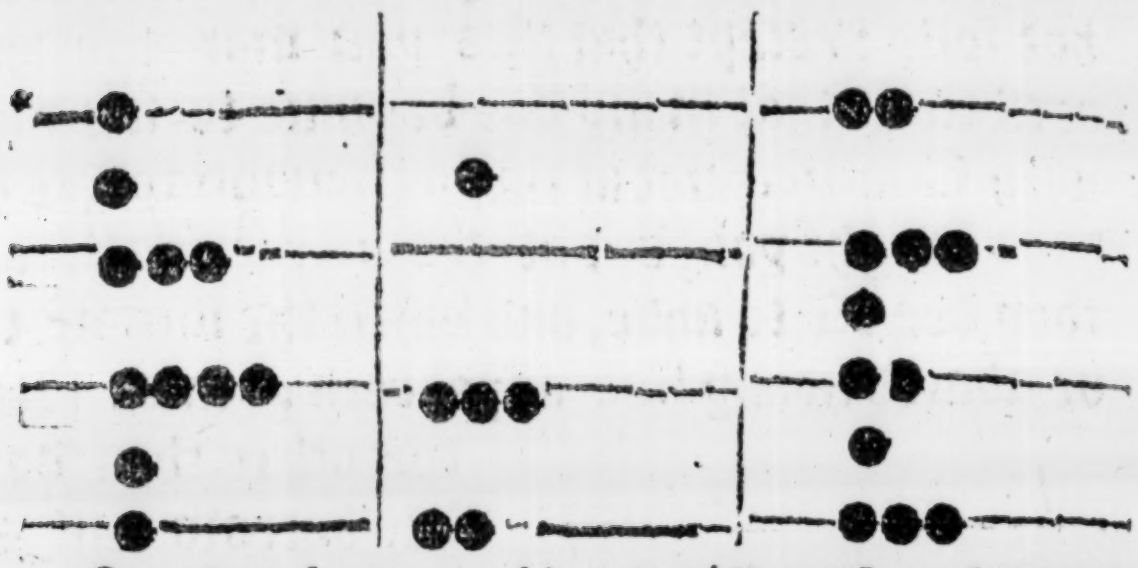
Maſter. To ſtand longer about this, it is but folly: except that this you may alſo vnderſtand, that many doe beginne to ſubtract with Counters, not at the higheſt ſumme, as I haue taught you, but at the nethermoſt, as they doe vſe to adde, and when the ſumme to be abated in any line appeareth greater then the other, then doe they borrow out of the next higher roome, as for example.

Another way of Addition.



I ſhould abate 1846 from 2378, they ſet the ſummes thus:

First they take 6, which is the lower line, and his ſpace from 8 in the ſame roomes in the ſecond ſumme, and yet there remaineth two Counters in the loweſt line. Then in the ſecond line muſt 4 be ſubtracted from 7, and ſo remaineth there 3. Then 800 in the third line, & his ſpace, from 300 of the ſecond ſumme cannot be, therefore do they abate it from a higher roome, that is, from 1000: & becauſe 1000 is too much by 200, therefore muſt I ſet downe 200 in the third line, after I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the firſt ſumme, which if I withdraw from the ſecond ſumme, then doe all Figures ſtand in order, thus: 532.



So that (as you ſée) it differeth not greatly whether you beginne Subtraction at the higher lines, or at the lower.

Howbeit, as ſome men like that one way beſt, ſo ſome like the other: therefore you now knowing both, may vſe which you liſt.

Multiplication.

Multiplication.

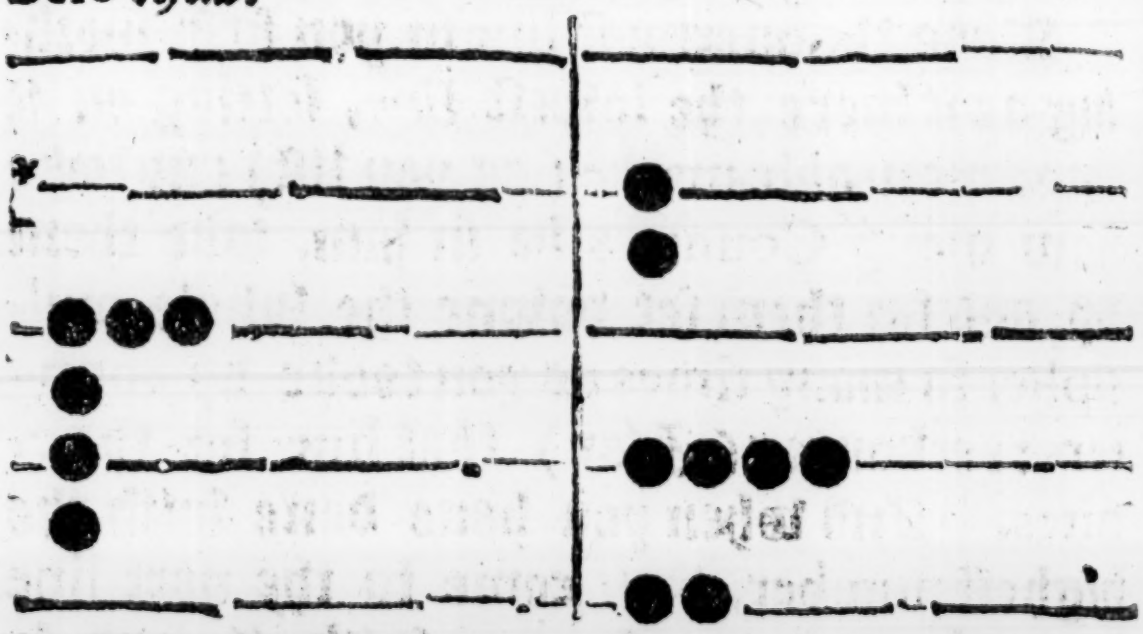
BVt now touching Multiplication: you ſhall ſet your numbers into two roomes (as you did in thoſe other kindes) but ſo that the multiplier be ſet in the firſt roome; then ſhall you begin with the higheſt numbers of the ſecond roome, and multiply them firſt after this ſort.

Take the ouermoſt line in you firſt working as it were the loweſt line, ſetting on it ſome moueable marke (as you liſt) and looke how many Counters be in him, take them vp, and for them ſet downe the whole multiplier ſo many times as you tooke vp counters: reckoning (I ſay) that line for the vnites. And when you haue done with the higheſt number, then come to the next line beneath, and doe ſo euen with it, and ſo with the next, till you haue done all. And if there be any number in a ſpace, then for it ſhall you take the Multiplier 5 times: and then muſt you reckon that line for the Vnites, which is next beneath that ſpace. Or elſe after a ſhorter way, yée ſhall take onely halfe the multiplier, but then ſhall you take the line next aboue the ſpace for the line of vnites. But in each working, if by chance your multiplier be an odde number, ſo that you

you cannot take the halfe of it iuſtly, then muſt you take the greater halfe, and ſet down that, as if that it were the iuſt halfe: and further, you ſhall ſet one Counter in the ſpace betweene that line, which you reckon for the line of vnites, or elſe onely remoue forward the ſame that is to be multiplied.

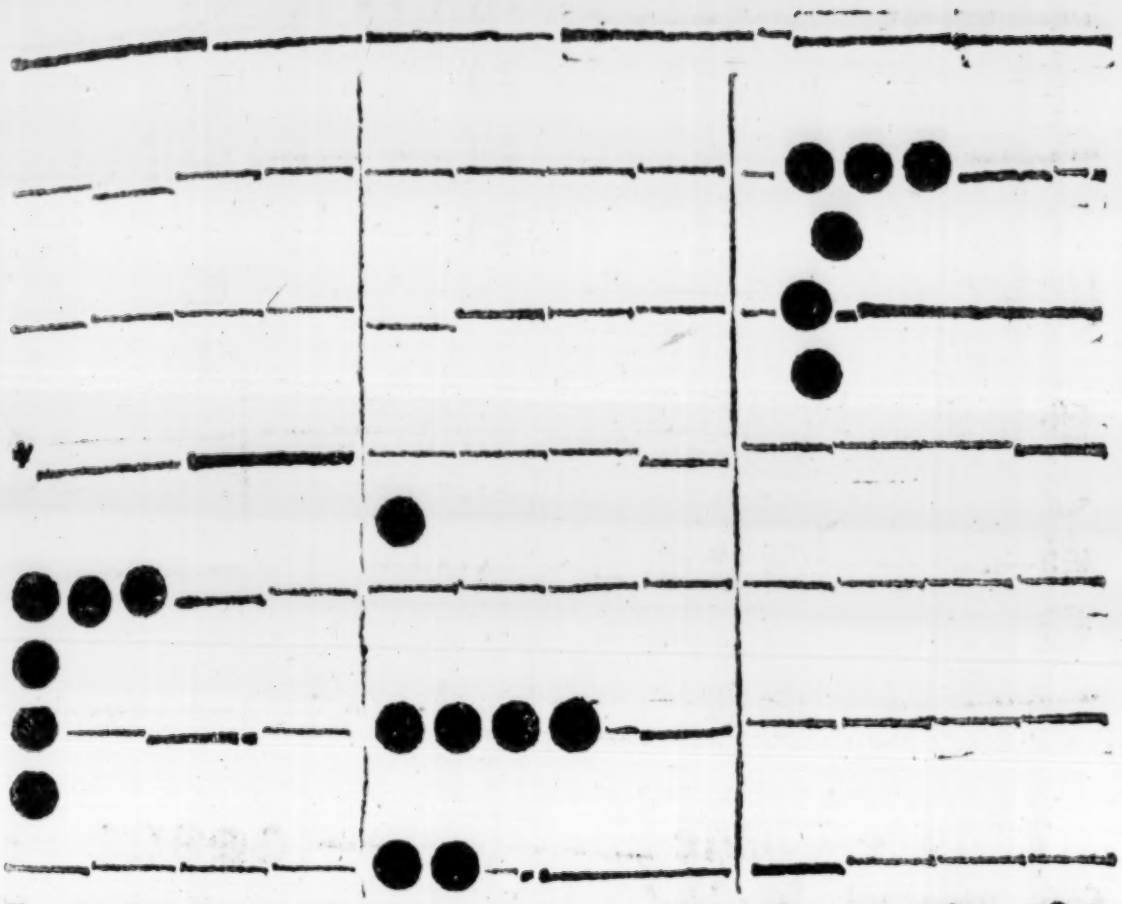
Scholar. If you ſet forth an example hereof, I thinke I ſhall perceiue you.

Maſter. Take this example: I would multiply 1542 by 365, therefore I ſet my numbers thus.



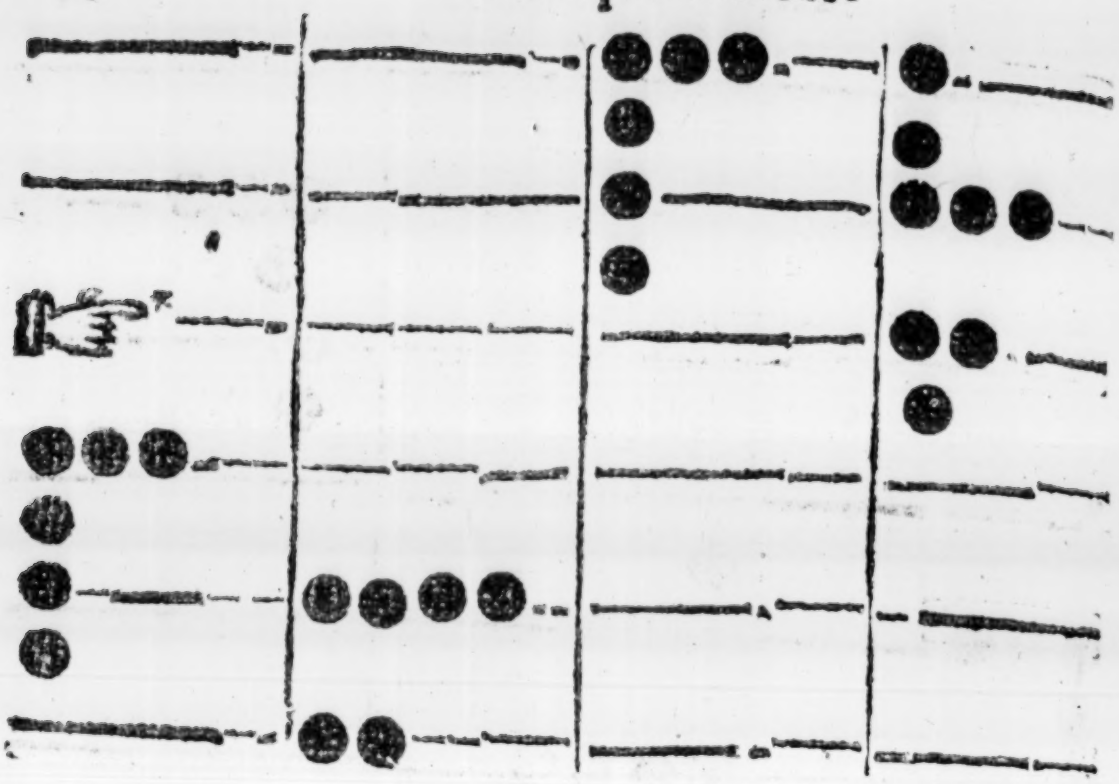
Then firſt I begin at the 1000 in the higheſt roome, as if it were the firſt place, and I take it vp ſetting downe for it ſo often (that is once) the Multiplier, which is 365, thus as you ſée here: where, for the one Counter taken vp from the fourth line, I haue ſet downe other fiue which make the ſumme of the multiplier, reckoning the fourth line, as if it were the firſt, which thing I haue marked by the ſtarre ſet at the beginning.

Scholar,

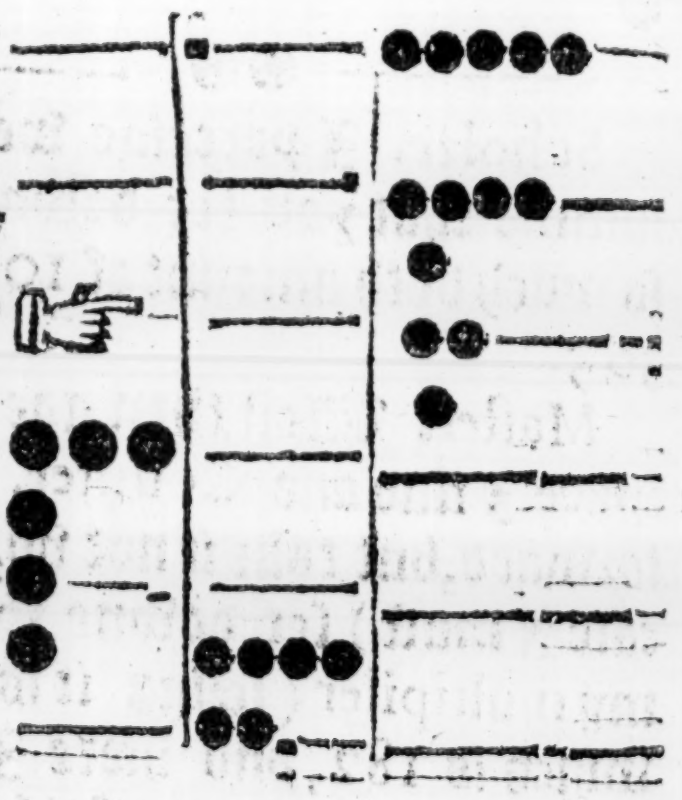


Scholar. I perceiue well, for indeed this ſumme that you ſet downe, is 365000: for ſo much doth amount of 1000, multiplyed by 365.

Maſter. Well then goe forth, in the next ſpace I find one Counter, which I remoue forward, but take it not vp, but (as in ſuch a caſe I muſt) ſet downe the greater halfe of my multipiler (ſéeing it is an odde number) which is 182, and héere I doe ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſée.

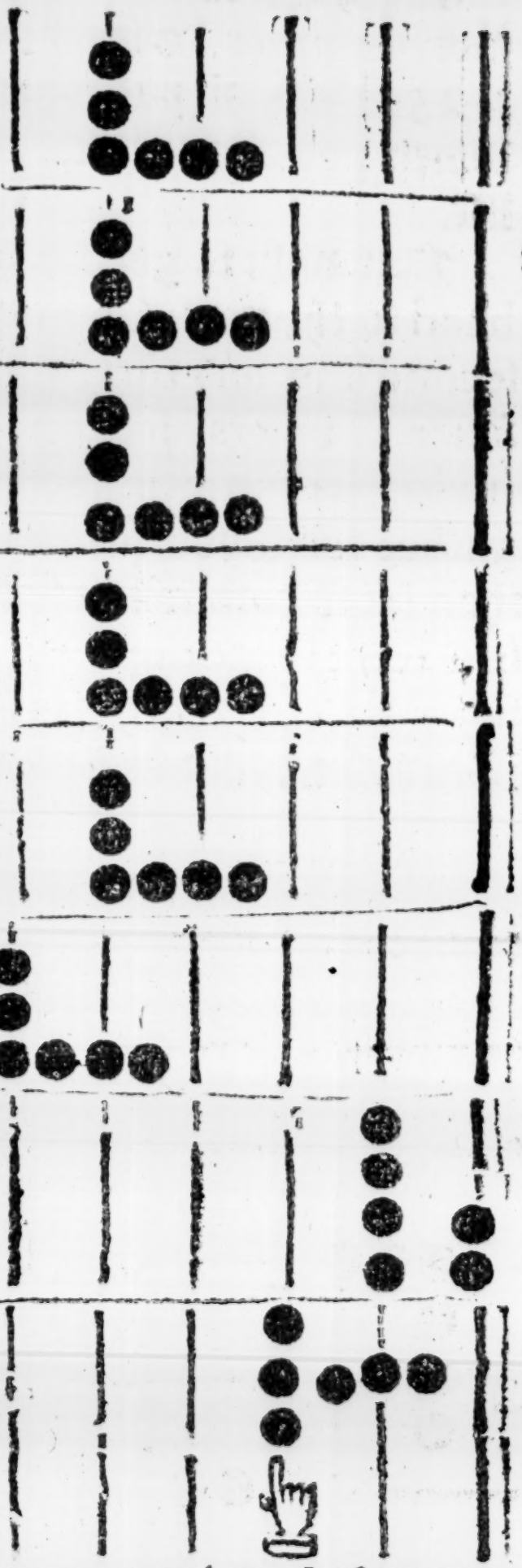


Where I haue set the Multiplicatio with other; but for the ease of your vnderstanding, I haue set a little line betwæn thē. Now should they both in one summe stand thus.



Howbeit

Yowbeit another ſorme to multiply ſuch counters in ſpace, is this: firſt to remoue the finger to the next line beneath the ſpace, and then to take vp the counter, and to ſet downe the Multiplier fiue times, as héere you ſée.



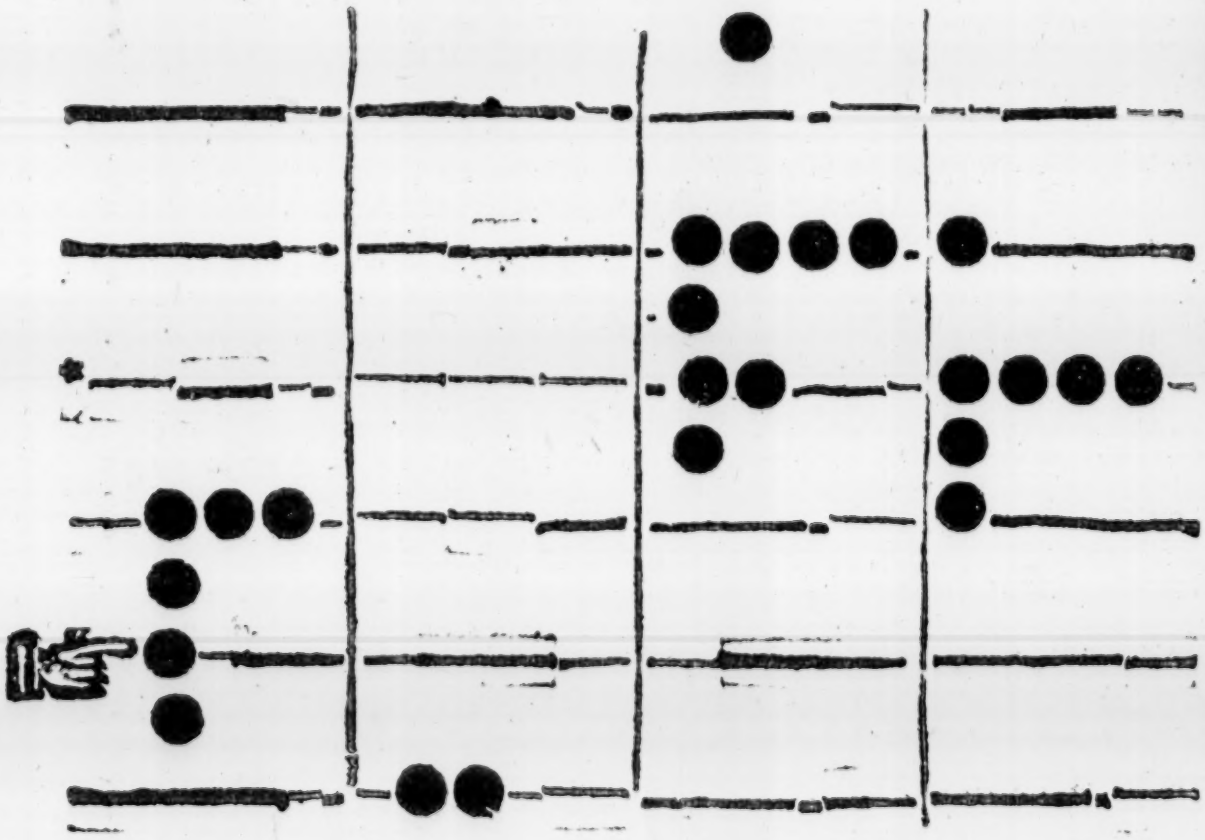
Another forme of Multiplication.

Which ſummes if you doe adde together into one ſumme you ſhall perceiue that it will bee the ſame that appeareth of the other working before, ſo that both ſorts are to one intent: but as the other is ſhorter, ſo this is plainer to reaſon, for ſuch as haue had ſmall exerciſe in this Art.

Notwithſtanding you may adde them in your minde before you ſet them downe: as in this example you might

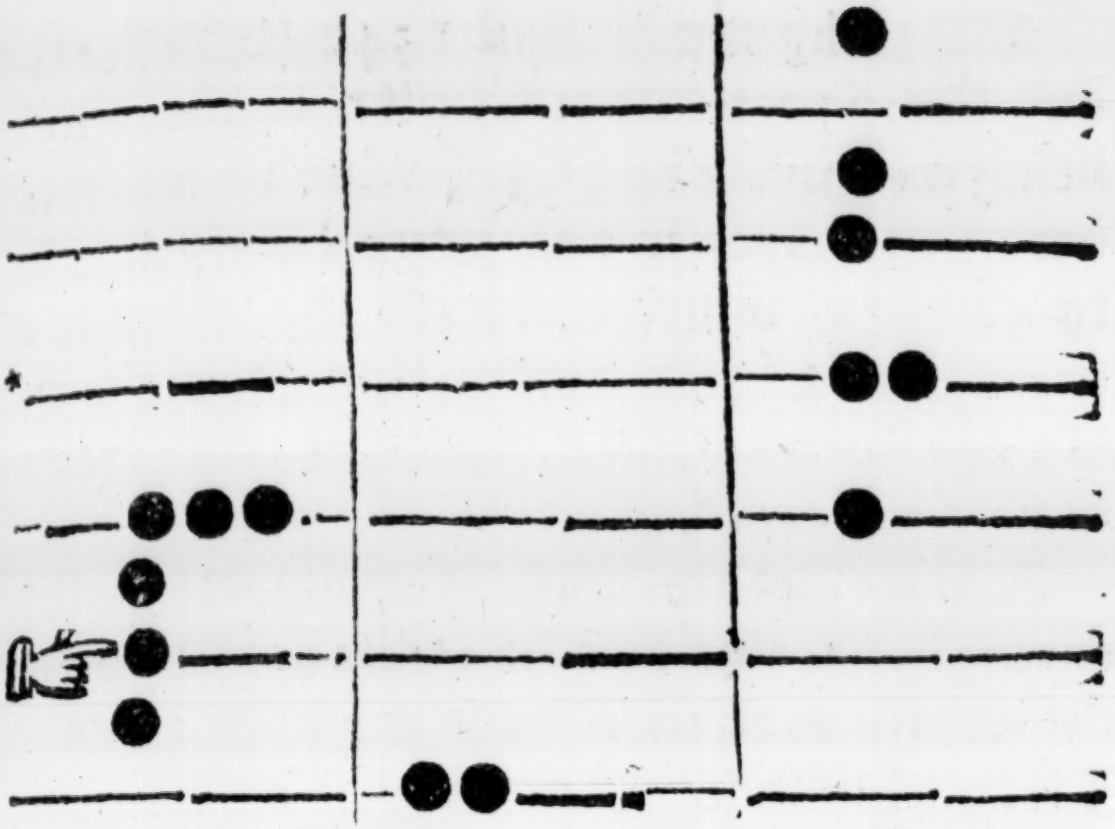
might haue ſaid, fiue times 3000 is 1500, and fiue times 60 is 300, alſo fiue times fiue is 25, which all put together, doe make 1825 which you may at one time ſet downe if you liſt.

But now to goe forth, I muſt remoue the hand to the next Counters which are in the ſecond line, and there muſt I take vp thoſe foure Counters, ſetting downe for them my multiplier foure times ſeuerally, or elſe I may gather the whole ſumme in my minde firſt; & then ſet it downe: as to ſay, foure times 300 is 1200: foure times 60 are 240; and foure times 5 make 20, that is in all 1460: that ſhall I ſet downe alſo, as here you ſée.

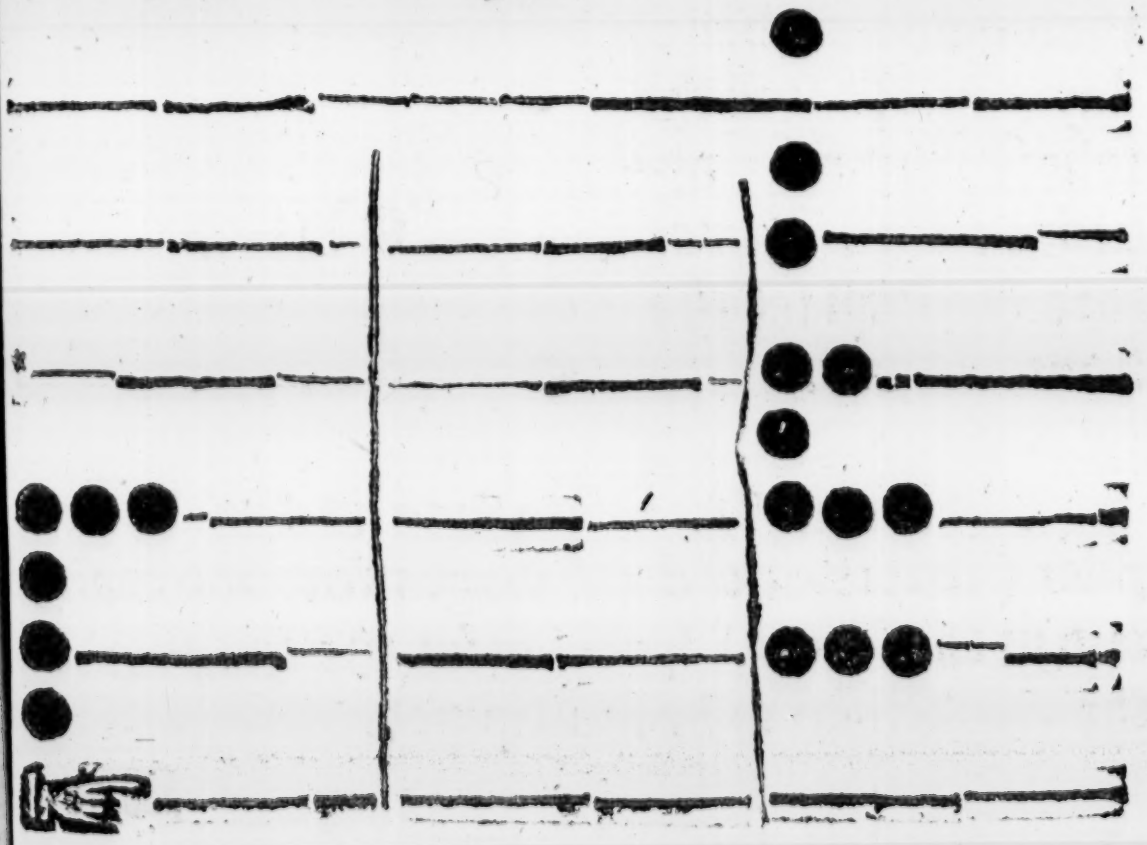


Which if I ioyne in one ſumme with the former numbers, it will appeare thus.

Then



Then to end this Multiplication, I remoue the finger to the loweſt line, where are onely 2, them doe I take vp, and in their ſtead doe I ſet downe twice 365, that is 730, for which I ſet one in the ſpace aboue the third line for 500, and 2 more in the third line with that one that is there already, and the reſt in their order, and ſo haue I well ended the whole ſumme thus:



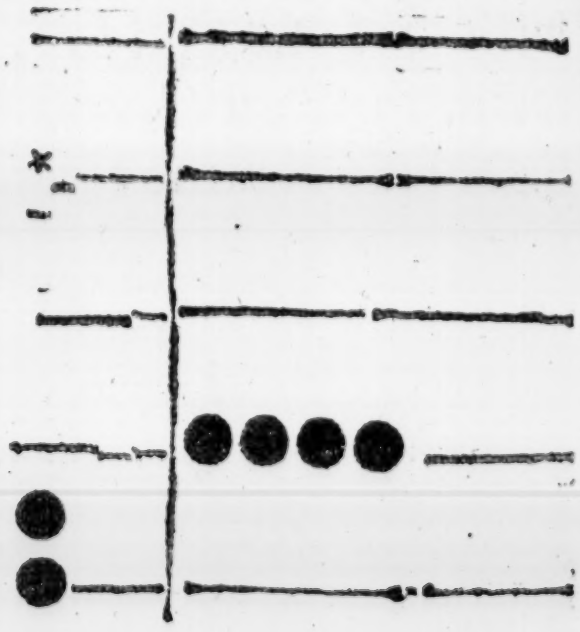
Whereby you ſée, that 1542 (which is the number of yeeres ſince Chriſt his incarnation) being multiplied by 365. (which is the number of the dayes in one yeere) doth amount to 562830, which declareth the number of dayes ſince Chriſts Incarnation vnto the end of 1542 yeeres, beſide 385 dayes, and twelue houres for leape yeeres.

Example of wages.

Scholar. Now will I proue by another example, as this: 40 Laborers (after 6 pence the day for each man) haue wrought 28 daies, I would know what their wages doth amount vnto.

In this caſe muſt I worke doubly: firſt I muſt multiply the number of the Laborers, by the wages of a man for one day, ſo will the charge of euery day amount.

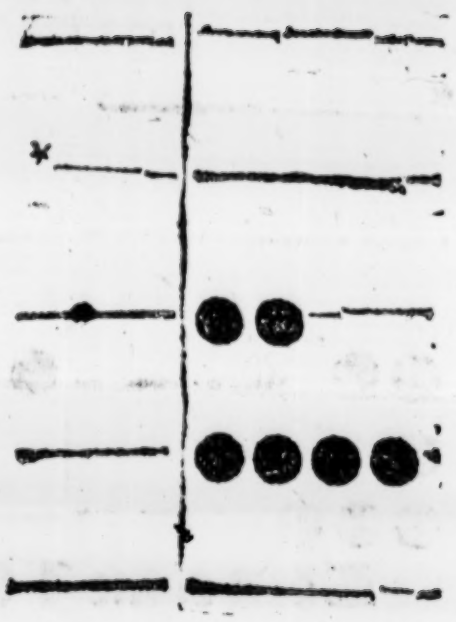
Then ſecondly ſhall I multiply the charge of one day by the whole number of dayes, and ſo will the whole ſumme appeare: Firſt therefore I ſhall ſet the ſummes thus.



Where in the firſt place is the Multiplier (that is one dayes wages for one man) and in the ſecond ſpace is ſet the number of the workemen to be multiplyed.

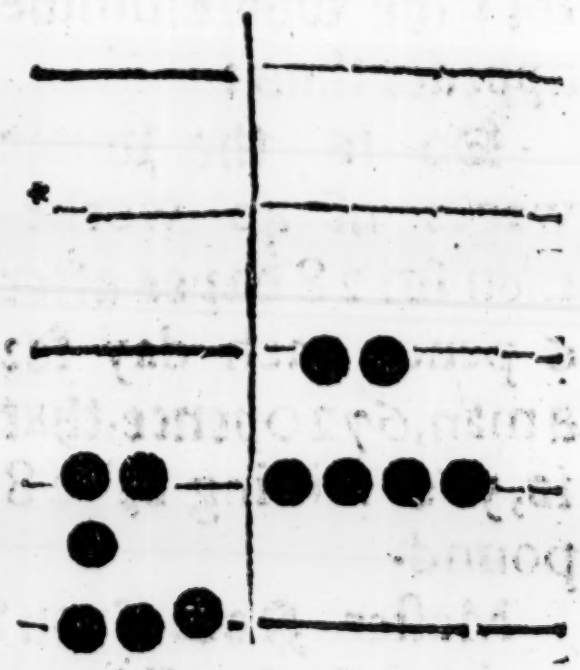
Then

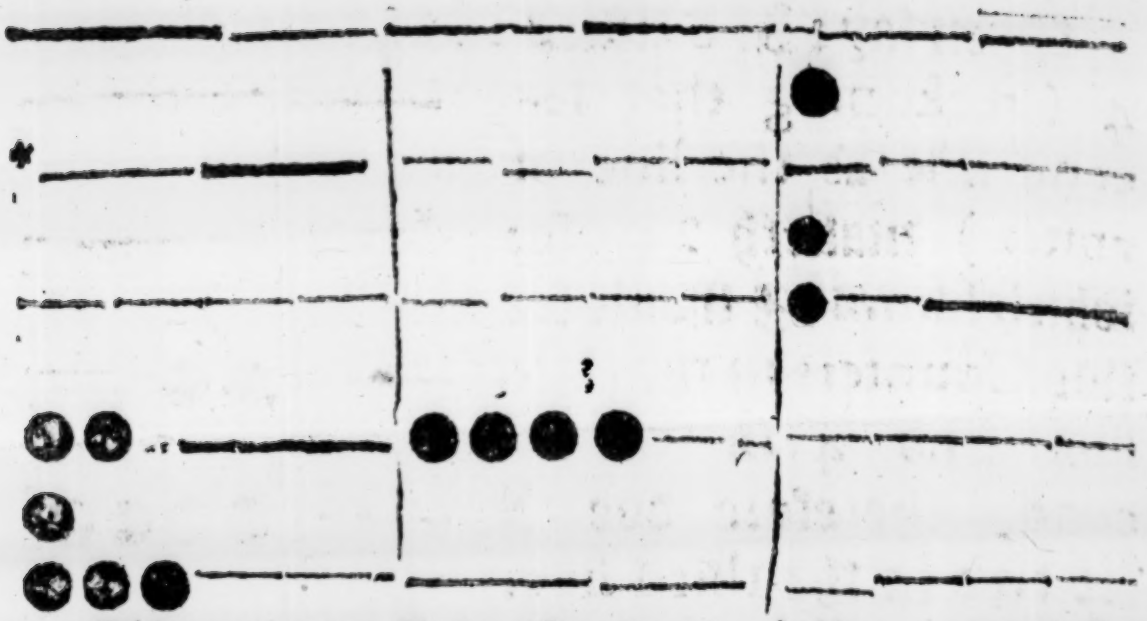
Then say: If 6 times 4 (reckoning that second line as the line of vnites) maketh 24, for which summe I should set two Counters in the third line and 4 in the second, therefore doe I set two in the third line, and let the foure stand stil in the second line thus.



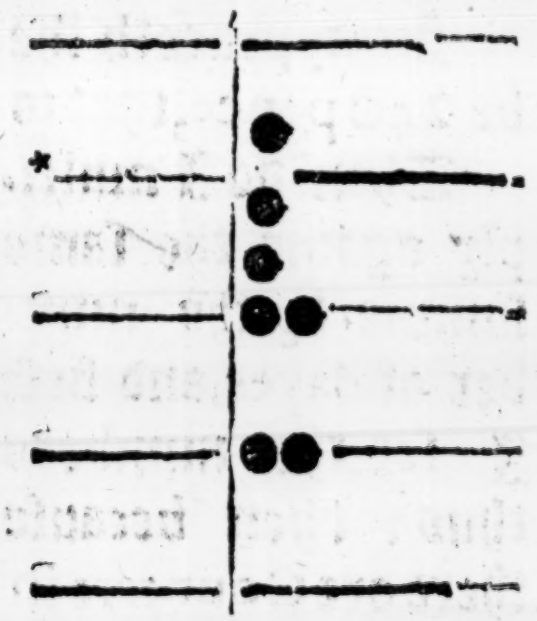
So appeareth the whole dayes wages to be 240 pence, that is 20 shilling.

Then do I multiply againe the same summe by the number of dayes, and first I set the numbers thus: then because there are Counters in diuers lines, I shal beginne with the highest, and take them vp, setting for them the Multiplier so many times as I toke vp Counters, that is twice, then will the summe stand thus.





Then come I to the second line, and take vp those 4 Counters, setting for them the multiplier 4 times, so will the whole summe appeare thus:



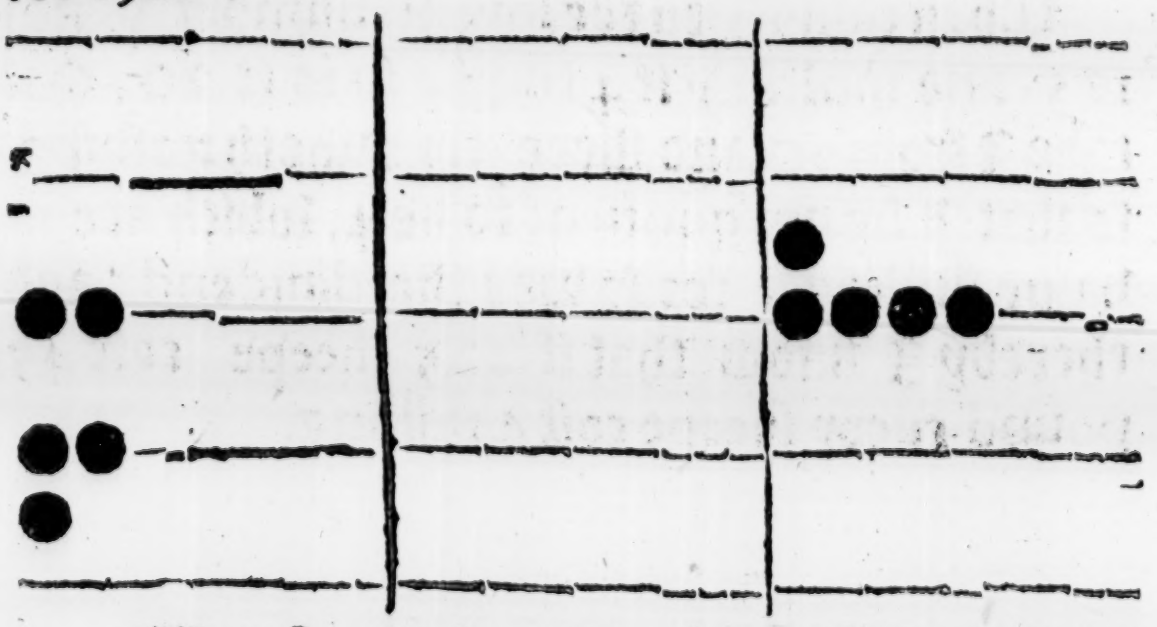
So is the whole wages of 40 workemen for 28 dayes after 6 pence each day for a man, 6720 pence, that is, 560 shilling or 28 pound.

Master. Now if you would proue Multiplication: the surest way is by Diuision: therefore will I ouerpasse it till I haue taught you the Art of Diuision, which you shall worke thus.

Diuiſion.

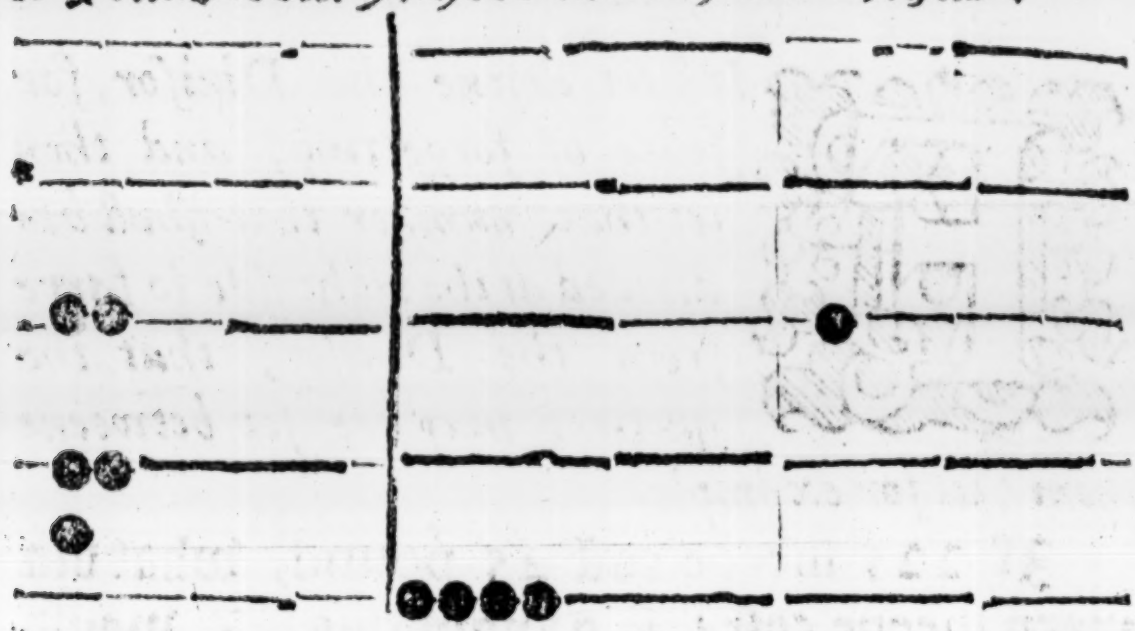
FIrſt ſet downe the Diuiſor, for feare of forgetting, and then ſet that number that ſhall bee diuided at the right ſide ſo farre from the Diuiſor, that the quotient may bee ſet betweene them: as for example.

If 225 ſheepe coſt 45 pound, what did euery ſheepe coſt? To know this, I would diuide the whole ſumme, that is 45 pound, by 225, but that cannot be: therefore muſt I firſt reduce that 45 pound, into a leſſer denomination, as into ſhillings, then I multiply 45 by 20, and it is 900: that ſumme ſhall I diuide by the number of ſheepe, which is 225, theſe two numbers therefore I ſet thus: An example of ſheepe.



Then begin I at the higheſt line of the diuidend, and ſeeke how oft I may haue the

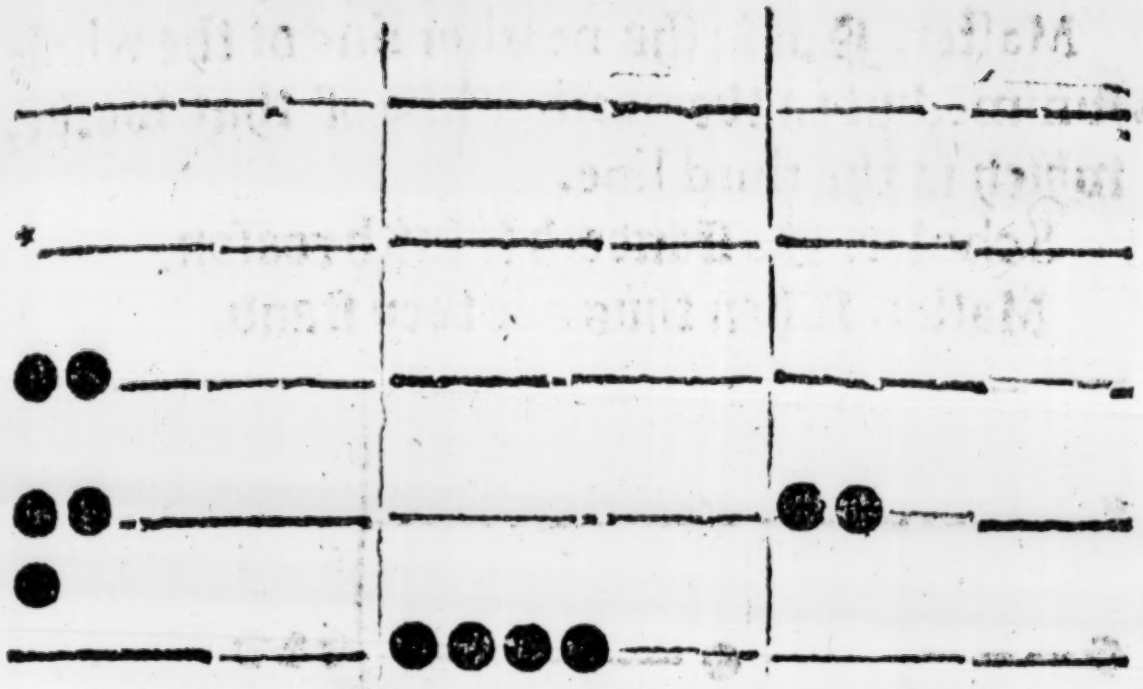
Diuiſor therein, and that I may doe foure times: then ſay I, foure times 2 are 8, which if I take from 9, there reſteth but 1, thus:



And becauſe I found the Diuiſor 4 times in the diuidend, I haue ſet, as you ſée, 4 in the middle roome, which is the place of the quotient: but now muſt I take the reſt of the Diuiſor as often out of the remainer: therefore come I to the ſecond line of the Diuiſor, ſaying two times 4 make 8, take 8 from 10, and there remaineth 2, thus:

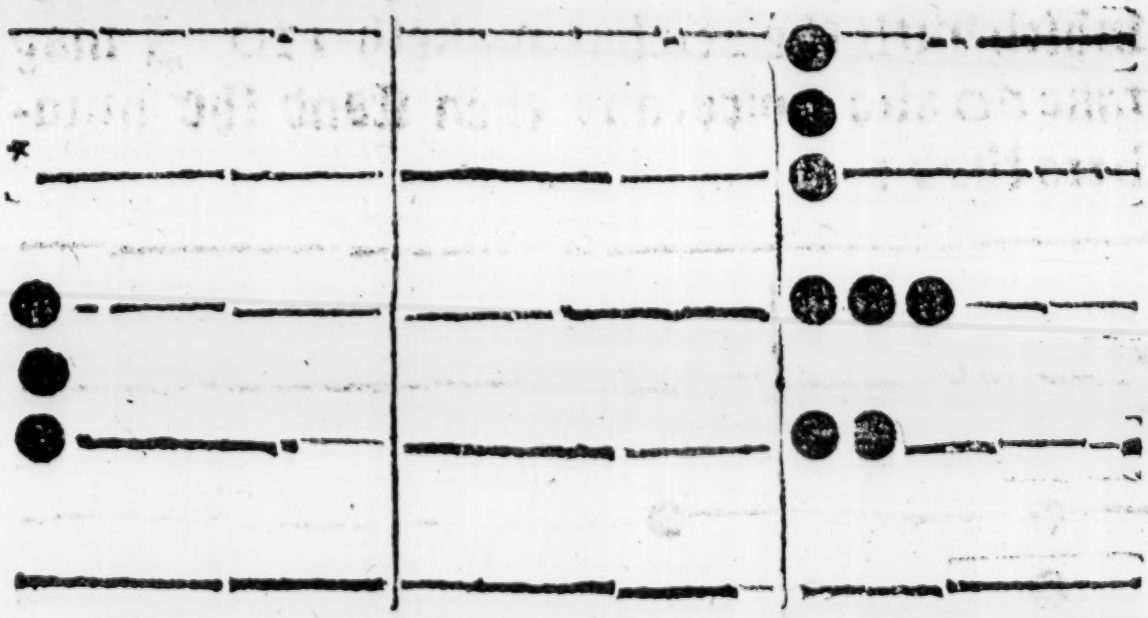
Then come I to the loweſt number which is 5, and multiply it 4 times, ſo is it 20, that take I from 20, and there remaineth nothing. ſo that I ſée my quotient to be 4, which are in value ſhillings, for ſo was the diuidend: and thereby I know that if 225 ſheepe coſt 45 pound euery ſheepe coſt 4 ſhilling.

Scholar



Scholar. This can I doe as you ſhall perceiue by the example. If 160 ſouldiers doe ſpend euery moneth 68 pound, what ſpendeth each man? Examples of ſouldiers wages.

Firſt, becauſe I cannot diuide the 68 by 160, therefore I will turne the pounds into pence by multiplication, ſo ſhall there bee 16320 pence: now muſt I diuide the ſomme by the number of ſouldiers, therefore I ſet them in order thus:

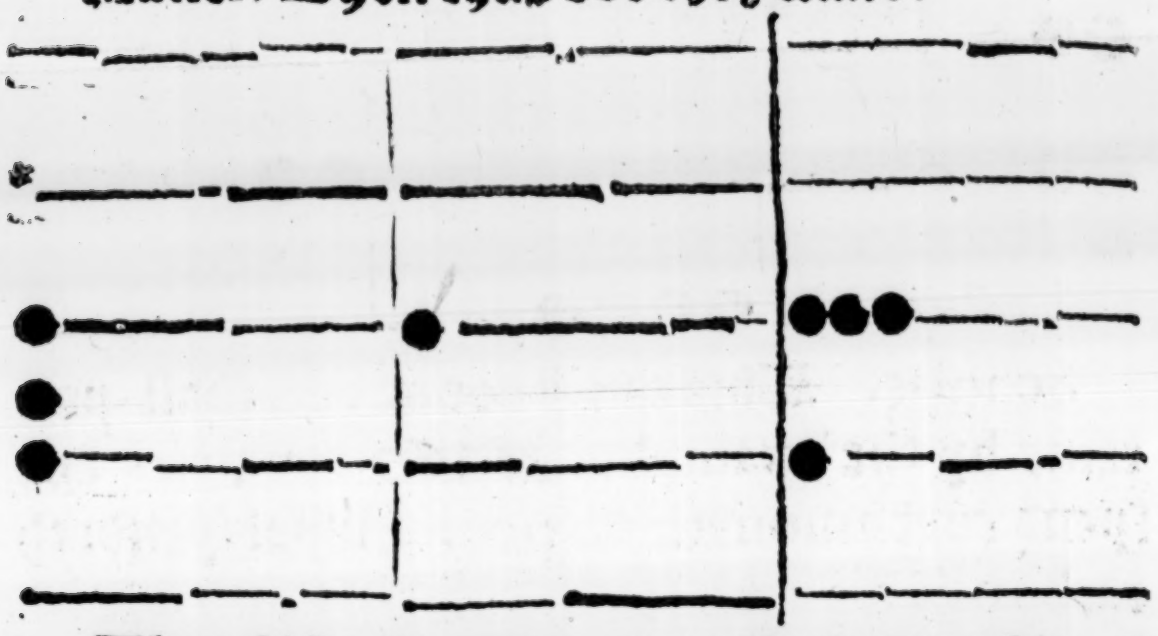


Then begin I at the higheſt place of the diuidend, ſeeking my Diuiſor there, which I find once, therefore I ſet 1 in the nether line.

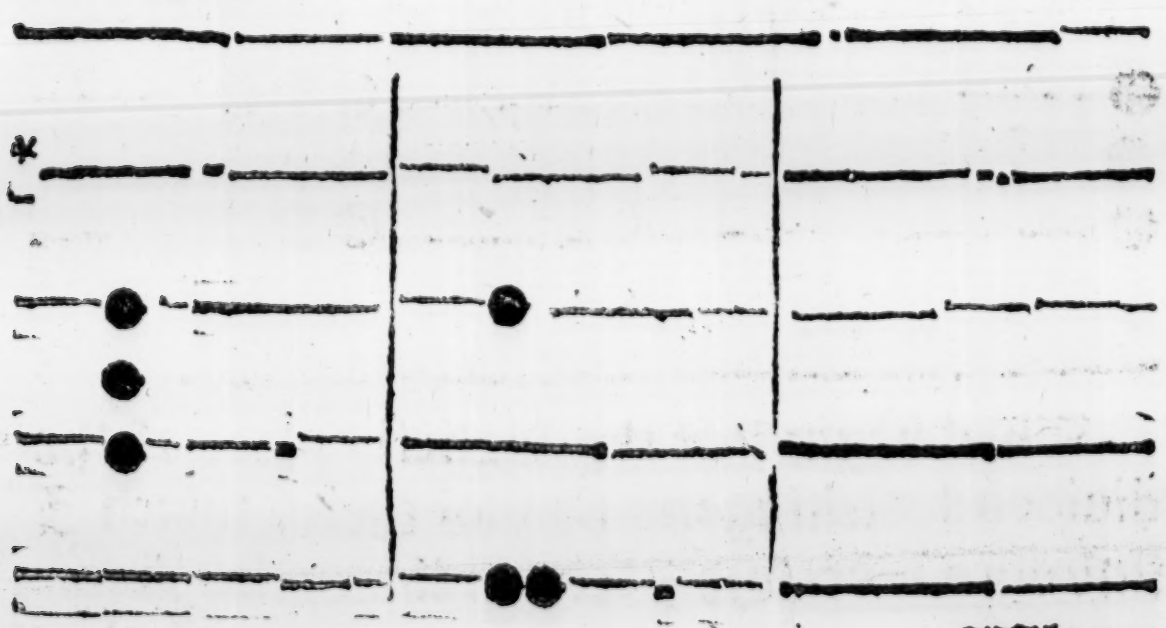
Maſter. Not in the neather line of the whole ſumme, but in the neather line of that worke, which is the third line.

Scholar. So ſtandeth it with reaſon.

Maſter. Then thus doe they ſtand.



Then ſéeke I againe the reſt, how often I may find my diuiſor: and I ſée that in 300 I might find 100 thrée times: but then the 60 will not be ſo often found in 20, therefore I take 2 for my Quotient; then take I 100 twice from 300, and there reſteth 100, out of which with the 20 that maketh 120, I may take 60 alſo twice, and then ſtand the numbers thus:



Where

Where I haue ſet the quotient 2 in the loweſt line: So is euery ſouldiers poztion 102 pence, that is 8 ſhillings 0 pence.

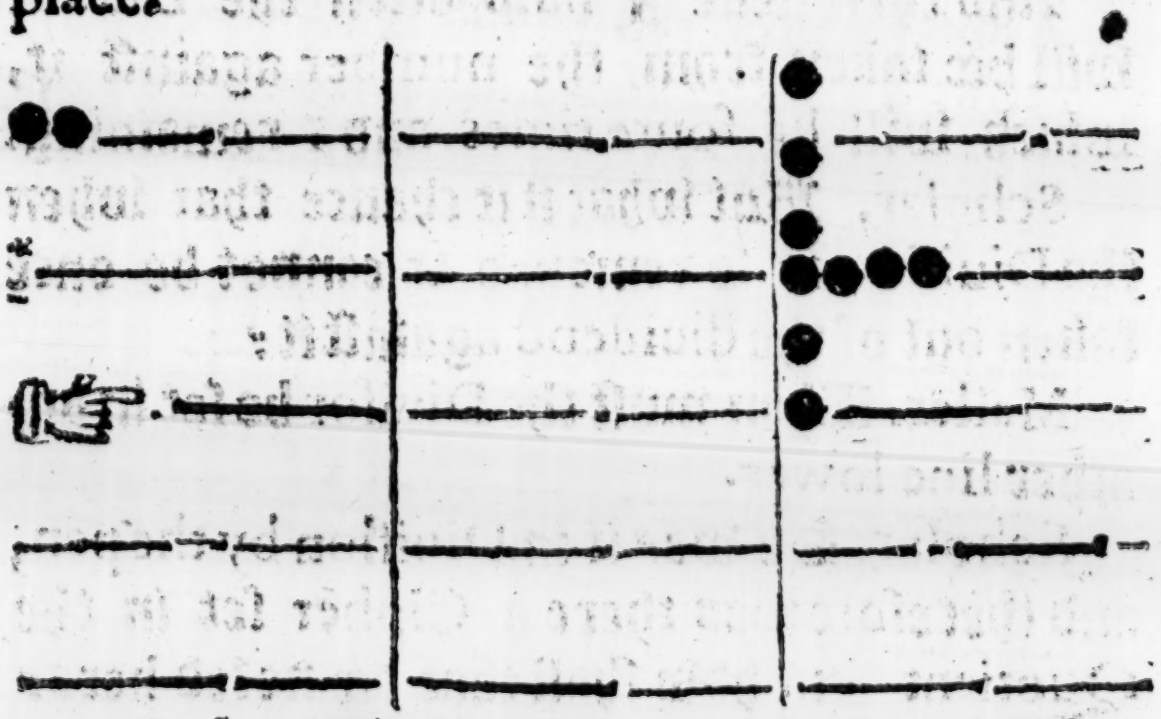
Maſter. But yet becauſe you may iuſtly perceiue the reaſon of diuiſion, it ſhall bee good that you ſet your diuiſor ſtil againſt thoſe numbers from which you doe take it, as by this example I will declare:

An example of purchaſe.

If the purchaſe of 200 acres of ground did coſt 260 pound, what did one acre coſt?

Firſt will I turne the pounds into pence, ſo will there be 69600 pence. Then in ſetting downe theſe numbers, I ſhall doe thus:

Firſt ſet the diuidend on the right hand as it ought, and then the diuiſor on the left hand againſt thoſe numbers from which I intend to take him firſt, as here you ſée where I haue ſet the diuiſor two lines higher then his owne place.

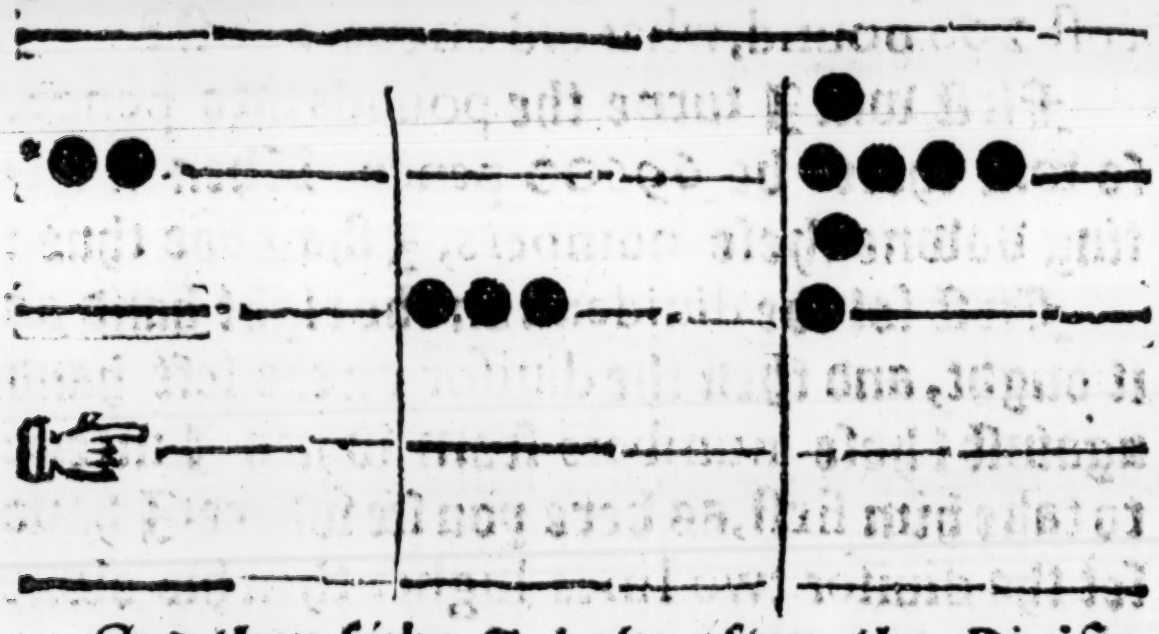


Scholar. This is like the ozder of diuiſion by the pen.

Maſter. Truth you ſay, and now muſt I

ſet the quotient of this worke in the third line, for that is the line of vnites in reſpect of the Diuiſor in this worke.

Then I ſée how often the Diuiſor may be found in the diuidend, and that I find 3 times, then ſet I 3 in the third line for the quotient: and take away that 60000 from the diuidend and further I ſet the Diuiſor one line lower: as you ſée here.



And then ſéeke I how often the Diuiſor will bée taken from the number againſt it, which will be foure times, and 1 remaining.

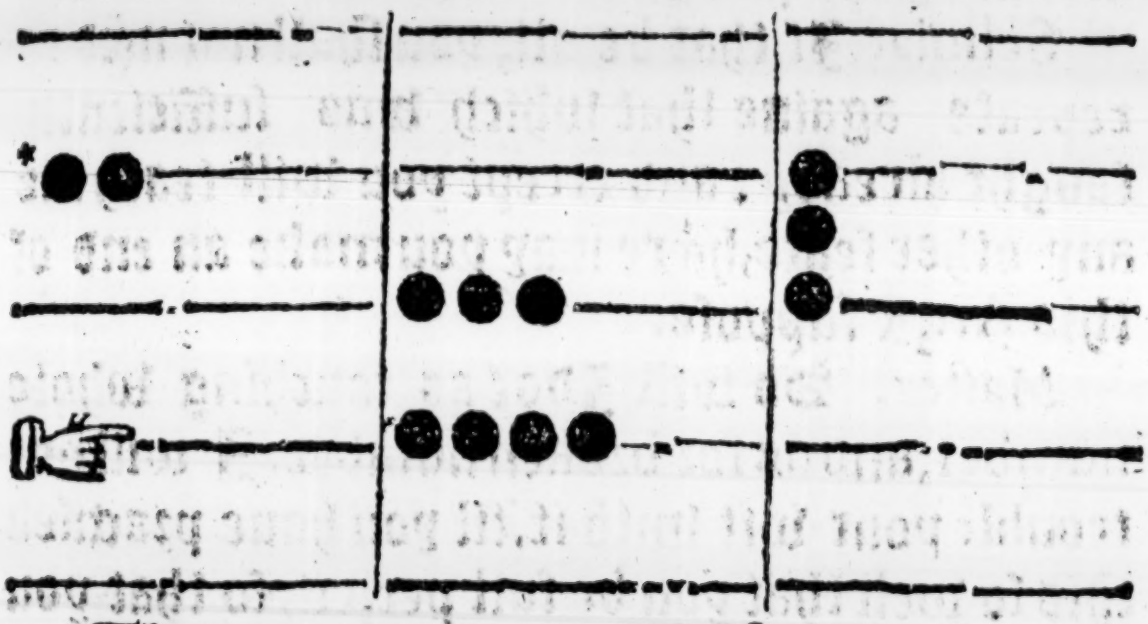
Scholar. But what if it chance that when the Diuiſor is ſo remoued, it cannot be once taken out of the diuidend againſt it?

Maſter. Then muſt the Diuiſor be ſet in another line lower.

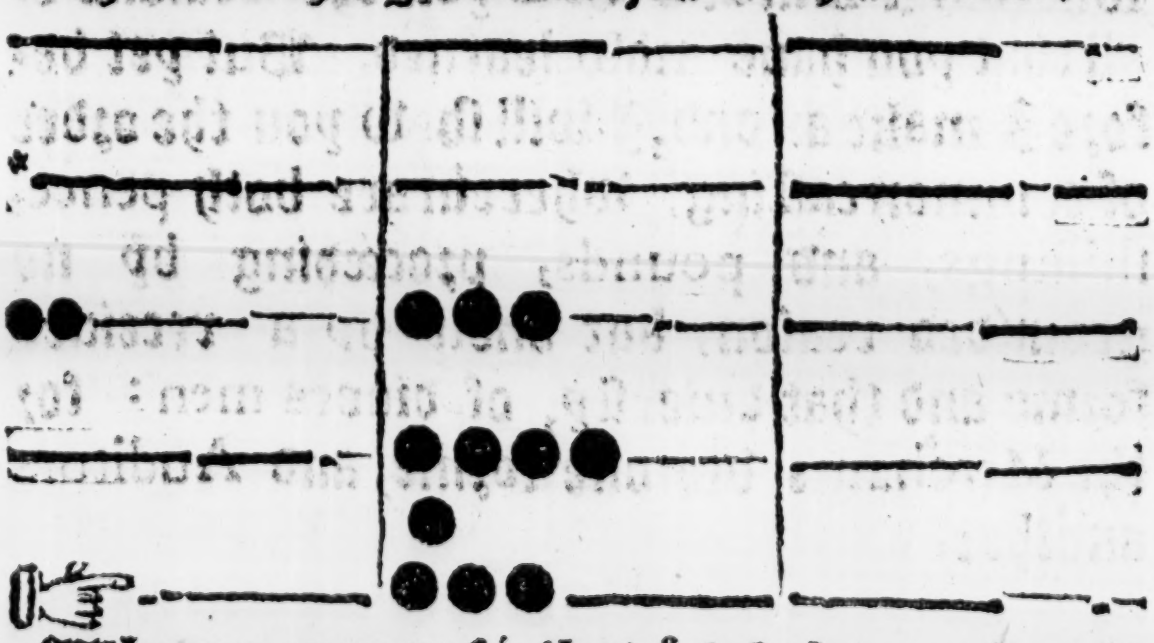
Scholar. So was it in Diuiſion by the pen, and therefore was there a Cipher ſet in the Quotient: but how ſhall that bée noted here?

Maſter. Héere nédeth no token, for the lines doe repreſent the places, onely loke that you ſet your Quotient in that place which

ſtandeth for vnites in reſpect of the Diuiſor. But now to returne to the example: I finde the Diuiſor foure times in the diuidend, and 1 remaining: for 4 times 2 make 8, which I take from 9, & there reſteth 1, as this figure following ſheweth: and in the middle ſpace for the Quotient, I ſet 4 in the ſecond line, which is in this worke the place of vnites.



Then remoue I the Diuiſor to the next lower line, and ſéeke how often I may haue it in the Diuidend, which I may do here 8 times iuſt, and nothing remaine, as in this forme.



Where you may ſée that ỹ whole quotient is 348 pence, that is, 29 ſhillings, wherby I know that

that ſo much coſt the purchaſe of one Acre.

Scholar. Now reſteth the prooſes of Multiplication, and alſo diuiſion.

Maſter. Their beſt prooſes are each one by the other, for Multiplication is prooued by diuiſion, and diuiſion by Multiplication, as in the worke by the pen you learned.

Scholar. If that be all, you ſhall not néed to repeate againe that which was ſufficiently taught already: and except you will teach me any other feate, héere may you make an end of this Art, I ſuppoſe.

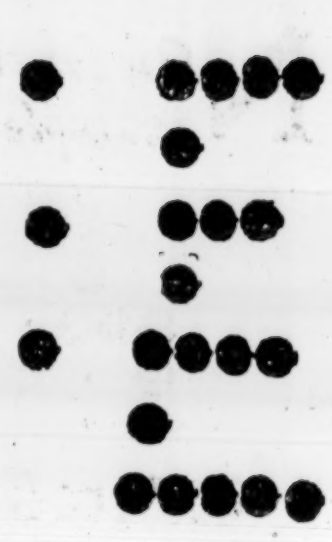
The reaſon of all the former rules.

Maſter. So will I doe as touching whole number, and as for broken number, I will not trouble your wit with it, til you haue practiſed this ſo well that you be full perfect, ſo that you néed not to doubt in any point that I haue taught you, & then may I boldly inſtruct you in the Art of fractions or broken numbers: wherein I will alſo ſhew you the reaſons of all that you haue now learned. But yet before I make an end, I will ſhew you the order of common caſting, wherein are both pence, ſhillings, and pounds, proceéding by no grounded reaſon, but onely by a receiued forme, and that diuerſly, of diuers men: for the Merchants vſe one forme, and Auditors another.

Merchants

Merchants vſe.

BVt firſt for Merchants forme, marke this example here, in which I haue expreſſed this ſumme 198 pounds, 19 ſhillings, 11 pence. *So that you may ſee that the loweſt line ſerueth for pence, the next aboue for ſhillings, the third for pounds, & the fourth for ſcores of pounds.*



And further you may ſée that the ſpace betwéene pence and ſhillings may receiue but one counter (as all other ſpaces likewiſe doe) and that one ſtandeth in that place for 6 pence.

Likewiſe betwéene the ſhillings and the pounds one counter ſtands for 10 ſhillings.

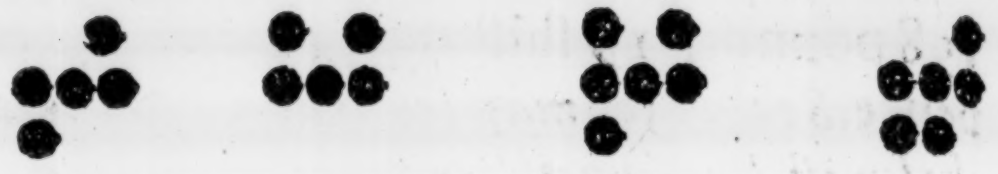
And betwæne the pounds and 20 pounds, one Counter ſtandeth for 10 pound.

But beſide thoſe, you may ſée at the left ſide of ſhillings, that one number ſtandeth alone, and betokeneth 5 ſhillings.

So againſt the pounds, that one Counter ſtandeth for 5 pound. And againſt the 20 pounds, the one Counter ſtandeth for fiue ſcore pounds, that is, 100 pounds: ſo that euery ſide Counter is fiue times ſo much as one of them againſt which he ſtandeth.

Auditors Accompt.

Auditors Accompt. NOw for the *Accompt of Auditors, take this example.*



Where I haue expreſſed the ſame ſumme 198 pound——19 ſhillings——11 pence.

But here you ſée the pence ſtand towards the right hand, and the other increaſing orderly toward the left hand.

Againe you may ſée, that Auditors will make two lines (yea and moze) foz pence, ſhillings, and all other values, if their ſummes extend thereto. Alſo you ſée that they ſet one Counter at the right end of each row, which ſo ſet there, ſtandeth foz fiue of that roome, and on the left cozner of the row it ſtandeth foz 10 of the ſame row.

But now, if you would adde, oz ſubtract after any of both thoſe ſozts, if you marke the ozder of the other feate which I taught you, you may eaſily doe the ſame here without much teaching: foz in Addition, you muſt firſt ſet downe one ſumme, and to the ſame ſet the other ozderly, and in like manner, if you haue many: but in Subtraction, you muſt ſet downe firſt the greateſt ſumme, and from it muſt you

abate

abate the other, euery Denomination from his due place.

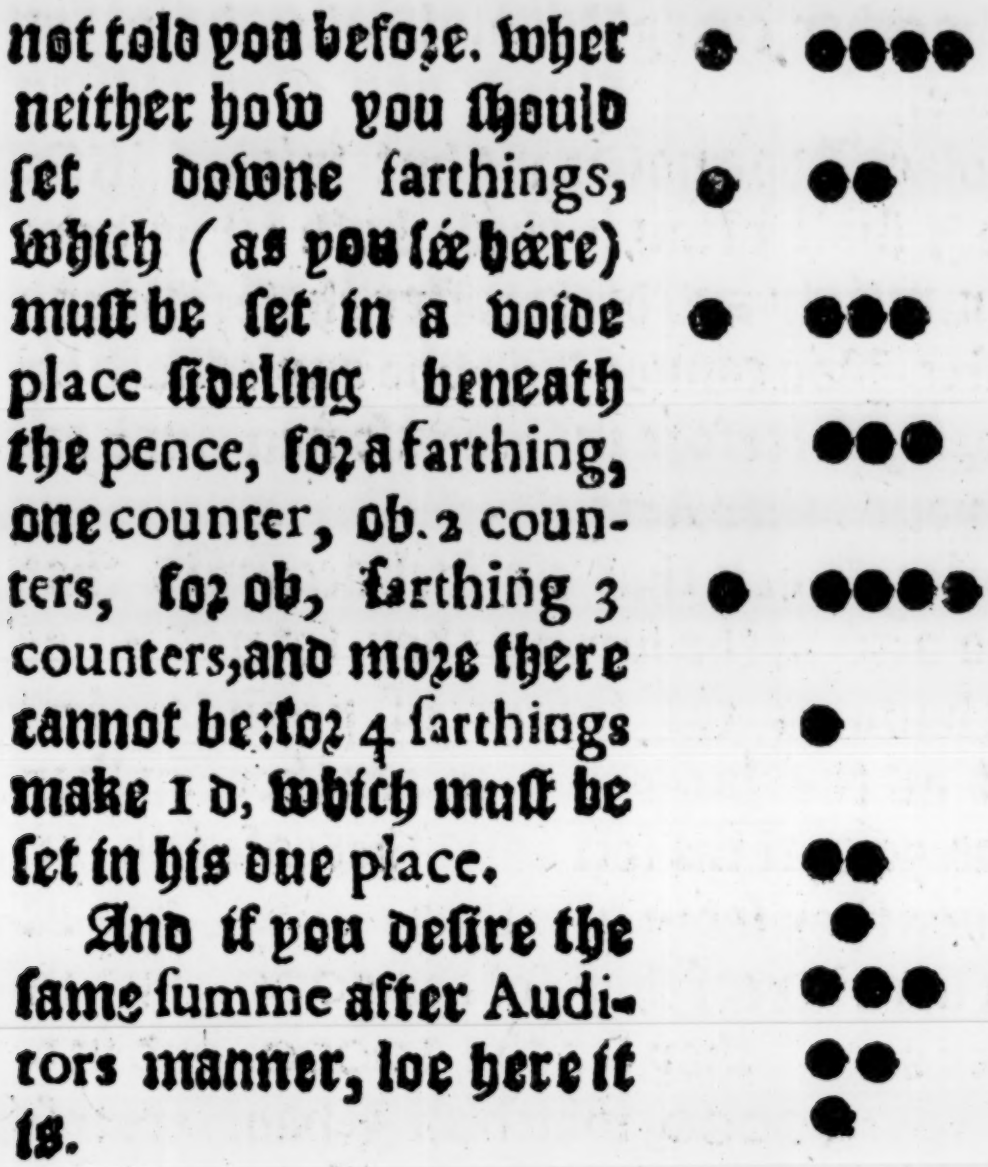
Scholar. I doe not doubt but with a little practise I shall attaine these both: but how shall I multiply and diuide after these formes?

Master. You cannot duly doe any of both by these sorts, therefore in such case you must resort to your other Arts.

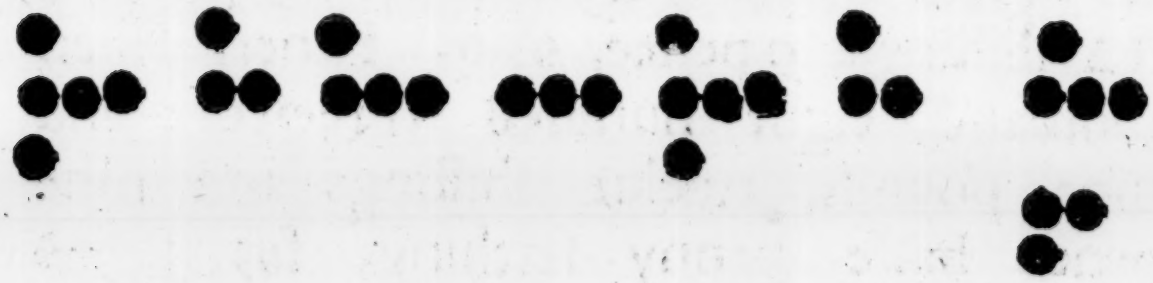
Scholar. They that vse such accounts that it exceed 200 in the summe they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in another farther row, and 500 at the left hand thereof, and the thousands they set in a farther row yet, and at the left side thereof they set the 5000, and in the space ouer, they set the 10000, and in a higher row 20000, which all I haue expressed in this example, which is 97869 pounds, 12 shillings, 9 pence, ob,q. Ninety seuen thousand, eight hundred threescore and nine pounds, twelue shillings and nine pence halfe penny farthing, for I had

not

not told you before. wher neither how you ſhould ſet downe farthings, which (as you ſée héere) muſt be ſet in a voide place ſtanding beneath the pence, for a farthing, one counter, ob. 2 counters, for ob, farthing 3 counters, and more there cannot be: for 4 farthings make 1 d, which muſt be ſet in his due place.



And if you deſire the ſame ſumme after Auditors manner, loe here it is.



But in this thing you ſhall take this for ſufficient, and the reſt you ſhall obſerue as you may ſée by the working of each ſort, for the diuers wits of men haue inuented diuers and ſundry waies, almoſt innumerable:

THE

THE SECOND PART OF ARITHMETICKE, touching Fractions, briefely set forth.

Scholar.

ALbeit I perceiue your manifold businesses doth so occupy or rather oppresse you, that you cannot as yet compleatly end the Treatise of Fractions Arithmeticall, which you haue prepared, wherein not onely sundry workes of Geometry, Musicke, and Astronomy be largely set forth, but also diuers conclusions and naturall workes, touching mixtures

Arithmeticall fractions.

mixtures of Mettals and compoſitions of medicines, with other ſtrange examples. Yet in the meane ſeaſon, I cannot ſtay my moſt earneſt deſire, but importunely craue of you ſome briefe preparation toward the vſe of Fractions, whereby at the leaſt I may bee able perfectly to vnderſtand the common workes of them, and the vulgar vſe of thoſe rules, which without them cannot well bee wrought.

Maſter. If my leaſure were as great as my will is good, you ſhould not néed to vſe any importunate crauing, for the attaining of that thing, whereby I may bée perſwaded that I ſhall any wayes profit the commonwealth, or helpe the honeſt ſtudies of any good members in the ſame: wherefore while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

what a fraction is.

Therefore firſt to beginne with the explication of this name Fraction, what take you it to be?

Scholar. Mary ſir, I thinke a Fraction (as I haue heard it often named) to be a broken number, that is to ſay, to be no whole number but part of a number.

Maſter. A Fraction indéed is a broken number, and ſo conſequently the part of another number: but that muſt be vnderſtood of ſuch another number as cannot bée diuided into any other parts then Fractions: for although I may take the third part of 60, or

the

the fourth part of it, and so of other parts diuerslie, yet these parts bee not properly, nor ought not to be called Fractions, because they may be expressed by whole numbers, for the third part of it is 20, the fourth part is 15, the twelfth part is 5, and so forth of other parts, all which bee whole numbers.

Wherefore properly a Fraction expresseth the parts or part onely of a vnite, that is to say, that the number which is the whole or entire summe of any Fraction, may not be greater then one: and therefore it followeth, that no one Fraction alone can bee so great, that it shall make 1, as by example I will declare, as soone as I haue taught you to know the forme how a Fraction is expressed or representeth in writing. What a fraction is properly.

Numeration.

Numeration.

The expreſſing of fractions.

But firſt to beginne with expreſſing of a Fraction, *which is the numeration of it: you muſt vnderſtand that a* Fraction *is repreſented by two numbers ſet one ouer the other, and a line drawne betweene them as thus,* $\frac{1}{3}\ \frac{3}{4}\ \frac{2}{5}\ \frac{10}{17}$, *which foure fractions you muſt pronounce thus:* $\frac{1}{3}$ *one third part,* $\frac{3}{4}$ *three quarters,* $\frac{2}{5}$ *two fift parts,* $\frac{10}{17}$ *ten ſeuenteene parts.*

Scholar. I vnderſtand the forme of their expreſſion and pronunciation: but their meaning or valuation ſeemeth more obſcure. Yet I think that by the two firſt fractions I vnderſtand the valuation of the two latter fractions, and conſequently of other.

Maſter. Value them then, that I may perceiue your taking of them.

Scholar. $\frac{2}{5}$ betokeneth two fift parts, that is to ſay, if one be diuided into 5 parts, that fraction doth expreſſe two of thoſe 5 parts: $\frac{10}{17}$ doth ſignifie, that if one be diuided into 17 parts, I muſt take ten of them. And this I gather of the two firſt examples: for $\frac{1}{3}$, that is, one third part, doth eaſily declare, that if one thing be be diuided into three parts, I muſt take out one of them: ſo $\frac{3}{4}$, that is three quarters, doth declare that one being diuided into foure quarters, I muſt take (for this Fraction) three

of

of those quarters.

If there be no more difficulty in their Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kinds of workes. For I vnderstand that the same kindes of workes bee in fractions, that be in whole numbers.

Master. There are the same kindes of workes in both, albeit the order of them is diuers, as I will anone declare: but yet more in Numeration before wee leaue it. You must vnderstand, that those two numbers which expresse a Fraction, haue seuerall names, the ouermost, which is aboue the line, is called the Numerator, and the other beneath the line, is called the Denominator. Numerator. Denominator.

Scholar. And what is the reason of their diuers names? For (in mine opinion) both be Numerators, seeing both they doe expresse the numeration of the Fraction:

Master. You are deceiued: for one onely (which is the ouermost) doth expresse the Numeration, and the Denominator doth declare the number of parts, into which the vnite is diuided, as in this example, when I say: diuide a pound weight of Gold betweene foure men, so that the first man shall haue $\frac{2}{15}$, the second $\frac{3}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$:

Now doe you perceiue that by the Denominator (which is one in all foure fractions) it is intended that the pound waights should be diuided into so many parts, I meane 15.

and by the foure ſeuerall Numerators is limited the diuers portion that each man ſhould haue, that is, that when the whole is parted into 15, the firſt man ſhall haue two of thoſe 15 parts: the ſecond man three of them: the third man foure: and the fourth man ſixe. And ſo may you ſée the ſeuerall offices (as it were) of thoſe two numbers, I meane of the Nnmerator and the Denominator.

And hereby you perceiue that a man can haue no more parts of any thing then it was diuided into, neither yet aptly ſo many: ſo that it were vnaptly ſaid: You ſhall haue $\frac{15}{15}$ that is 15, fiftéene parts of any thing, ſéeing it were better ſaid, you ſhall haue the whole thing.

Scholar. So doth it appeare reaſonably, for the labour is vaine to diuide any thing, and then to apply the Diuiſion to no vſe. And much leſſe reaſonable were it to ſay $\frac{16}{15}$: for if the whole be diuided into 15 parts onely, it is not poſſible to take 16 of them, that is to ſay, more then all together.

Maſter. This is true touching the proper and apt vſe of the name of a Fraction: yet improperly, (and after a vulgar acceptation for eaſineſſe in worke) both thoſe formes be called fractions, becauſe they be written like fractions, although they be none indéed: for $\frac{15}{15}$, and generally all ſuch other, where the Numerator and Denominator be equall, are not fractions, but the whole thing with all his parts. And ſo $\frac{14}{13}$ is not to be called a fracti-

on,

on, but a mixt number, of a whole number & a fraction, for it is as $1\frac{2}{12}$ that is, one whole and $\frac{2}{12}$ parts, as shall be declared in Reduction. Therefore they doe abuse the names that call them fractions, where the Numerator is either equall, or greater then the Denominator. An improper fraction, or a mixt number.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in worke write some numbers after that sort like fractions: but they needed not to call them fractions, but (as they be) whole numbers or mixt numbers, (that is, whole numbers with fractions) expressed like fractions, or as improper fractions.

Now must you understand, that as no fraction properly can be greater then one, so in smalnesse vnder one the nature of fractions doth extend infinitely, as the nature of whole numbers is to increase aboue one infinitely, so that not onely one may bee diuided into infinite fractions or parts, but also euery fraction may be diuided into infinite fractions or parts, which commonly be called fractions of fractions, and they bee expressed diuersly: as for example, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is three quarters, of two third parts, of one halfe part. Whereby is signified, that if one bee diuided into two halfes, and the one halfe into three parts, and two of those Fractions of Fractions.

three parts bée diuided ioyntly into foure quar-
ters, this fraction of fractions doth repzesent
three of those quarters.

Scholar. I pray you let me pzoue by an ex-
ample in common money, whether I doe
rightly vnderstand you oz no. One Crowne
which I take foz an vnite, doth containe 60
pence; therefoze the halfe of it is 30 pence,
$\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fifteen
pence, so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a
Crowne: and so is 3 pence $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a
shilling.

Master. You perceiue this well enough: yet
this note I giue you by the way, that the
fozme of expzessing these fractions is volunta-
ry, and hath no other reason than the will of
the Diuisor, which fozme many follow: foz
some expzesse them thus $\frac{3}{4}\,\frac{2}{3}\,\frac{1}{2}$ without any
figure of distinction betweene them, which
fozme also many follow. Some other do make
lines betweene euery fraction, & adde wozds
of distinction, after this sozt, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, which
fozme is best.

Some other expzesse them thus in slope fozme, to distinct them from Fractions of one whole number, foz if they were set in one right line thus, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$ then ought it to be pzonounced, three quarters,
and two third parts, and a halfe, which maketh
almost two whole vnites, lacking but one
tweluth part. And so is it nothing agreeable
with

with the other fraction of fractions: wherefore it is a great ouerſight in certaine learned men, which doe expreſſe them ſo confuſedly with ſuch ſeuerall tractions, that a man cannot know the one from the other.

Therefore ſome men (as Stifelius) *doe expreſſe without a line numbers of proportion, being applied to* Addition *or* Subtraction, *becauſe they muſt be taken as two, where the line in* fractions *maketh them to be taken for one: For of the* Numerator & Denominator *is made one number.*

Scholar. When I perceiue there be three ſeuerall varieties in fractions: firſt, when one onely fraction is ſet for one number, as $\frac{4}{5}$, that is, foure fift parts. The ſecond is, when there be ſet two or more ſeuerall fractions of one number, as $\frac{4}{9}$, $\frac{2}{5}$, that is, foure ninth parts, and two fift parts. The third ſort is fractions, of fractions as $\frac{4}{9}$, of $\frac{2}{5}$, that is, foure ninth parts of two fift parts. Three ſeuerall varieties.

Maſter. You haue ſaid well, if you vnderſtand well your owne words.

Scholar. If it ſhall pleaſe you I will by an example in the parts of an old Engliſh Angell, expreſſe my meaning.

Maſter. Let mée heare you.

Scholar, The old Engliſh Angell did containe 7 ſhillings 6 pence, that is 90 pence: Now $\frac{4}{5}$ of it is 72 pence And of the ſame 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is, foure nine parts, and two fift parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that

is, foure nine parts of two fift parts, seeing is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16. for $\frac{1}{9}$ of 36, is but 4, and that taken foure times, maketh 16.

Master. This is plainely expressed, and truly, and hereby (I doubt not) but you doe perceiue that as great a difference, as is betweene 16 and 76. so much difference is betweene those two fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that you vnderstand these varieties, I will proceede to the rest of the workes: first, admonishing you, that there is another order to bee followed in fractions, then there was in whole numbers: for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuision, and Reduction: but in fractions (to follow the same aptnesse in proceeding from the easiest workes to the harder) we must vse this order of works, Numeration, Reduction, Addition, Subtraction, Multiplication and Diuision.

The order of workes in fractions.

Scholar. That Addition and Subtraction should goe together, and Diuision to follow Multiplication, naturall order doth perswade; but why Reduction should be first in order heere next to Numeration, and Addition and Subtraction in the middle, I desire to vnderstand the reason.

Master. As in the Art of whole numbers, Order would reasonably begin with the easiest, and so goe forward by degrees to the hardest:

hardeſt: euen reaſon teacheth in fractions the like order And conſider that Addition or Subtraction of fractions can very ſeldome bee wrought without Reductiō and contrariwiſe, Reduction may be wrought without this forme of Addition or Subtraction: therefore was it orderly required, that Reductiō ſhould goe before Addition and Subtraction, and this reaſon ſerueth for the placing of Reduction before the other.

Scholar. Then, if Reduction be the eaſieſt, I pray you declare the forme of it firſt by Rule, and then by example.

Maſter. Your requeſt is good.

Reduction

Reduction of Fractions.

Of Reduction of Fractions, there are fiue varieties.

THerefore will I now declare the diuerſities of Reduction of Fractions, which commonly hath fiue varieties, or formes.

1 *Firſt, when there be ſundry Fractions of one intire vnite, they muſt be reduced to one denomination, and alſo into one Fraction.*

2 *Secondly, when there be proponed fractions of fractions, they muſt bee reduced likewiſe into one fraction: for otherwiſe they cannot bee brought into one denomination.*

3 *Thirdly, when an improper fraction is proponed, that is to ſay, a fraction in forme, which indeed, is greater then an vnite: it muſt bee reduced into apt forme, expreſſing the vnite or vnites of it, and the proper fraction diſtinctly. And ſometimes alſo it ſhall be needful to conuert ſuch a mixt number of vnites with fractions, into the forme of a fraction, that is, into an improper fraction: which two formes I eſteeme but as one, becauſe they worke on one kind of number.*

4 *Fourthly, there happeneth ſometimes fractions to be written in great numbers, which might be written in leſſer numbers: therefore is there a meane to reduce ſuch great numbers into their ſmalleſt termes.*

5 *Fiftly, when any fraction betokeneth the parts of a whole thing, which hath by common partition*

certaine

certaine parts, but none of like denomination with that Fraction, then may you reduce the ſaid fractions into another, whoſe denomination ſhall expreſſe the common parts of that whole thing.

Scholar. This diſtinction in doctrine delighteth me much, but more with hope then preſent fruit: for as yet I doe not vnderſtand ſcarcely the varieties, & much leſſe the practiſe and vſe of their workes.

Maſter. Reduction is an orderly alteration of numbers out of one forme into another, which is neuer done orderly but for ſome néedfull vſe, as in euery of the ſaid fiue ſeuerall formes, I will diſtinctly declare.

The firſt forme of Reduction

Firſt therefore, when two, or more ſeuerall fractions of any vnite be proponed; as for example $\frac{3}{16}$ and $\frac{4}{6}$, becauſe it is hard to tell what proportion of the intire number thoſe two Fractions doe expreſſe; therfore was Reduction diuiſed, to be a meane whereby theſe ſeuerall fractions might be brought into one denomination and fraction.

And in theſe fractions, this is the Art for bringing them to one denomination.

How to reduce fractions of diuers denominations into one denomination.

Multiply firſt the denominators together, and the totall thereof you ſhall ſet twice down vnder two ſeuerall lines for two new denominators, or rather for one common Denominator. Then multiply the Numerator of the firſt fraction, by the Denominator of the ſecond, and ſet the totall thereof for the Numerator ouer the firſt line. Likewiſe multiply the Numerator of the ſecond fraction by the Denomina-

tor

tor of the first, and set that totall ouer the second line for the Numerator of that fraction: and so are those two first fractions of seuerall denominations, brought to one denomination.

Scholar. If I vnderstand you, as I thinke I doe, my example shall declare the same. The fractions which you proponed were these, $\frac{3}{16}$, and $\frac{4}{6}$, whose denominators (being 16 and 6) I multiply together, and there amounteth 96, which I set vnder two lines thus: $\frac{}{96}$, $\frac{}{96}$.

Then I multiply the Numerator of the first fraction by the Denominator of the second saying, 3 into 6 maketh 18, that I set ouer the first line for a new Numerator, and it will be thus $\frac{18}{96}$.

Likewise I multiply the Numerator of the second fraction by the Denominator of the first saying, 4 times 16 maketh 64, that I set for the second Numerator, and the fraction will appeare thus, $\frac{64}{96}$.

So that both fractions brought to one denomination, must stand thus, $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You haue done well.

Scholar. I beseech you let me examine it after my accustomed forme, by common parts of coyne or other measure.

Master. Goe to.

Scholar. I haue a peece of Gold which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is, the sixteenth part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that

is $\frac{18}{96}$. Againe $\frac{1}{6}$ of the same peece of gold is 16 pence, so that $\frac{4}{6}$ parts maketh 64 pence, that is $\frac{64}{96}$. And so I finde the summes to agree with the other before.

Master. *So haue you now the Art to bring two such fractions into one denomination. And if there bee more then two, then must you multiply all the Denominators together, and set the totall thereof so many times downe as there be fractions; and then to get for each one a new Numerator, multiply the Numerator of the* first, *by the Denominator of the* second, *and the totall thereof multiply by the Denominator of the* third, *and so forth, if there be more. Likewise multiply the Numerator of the* second, *by the Denominator of the* first, *and the totall thereof by the Denominator of the* third. *And in the same sort multiply the Numerator of the third into the Denominator of the* first, *and the totall thereof into the Denominator of the* second, *and so forth, if there were moe. So these three fractions* $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$ *doe make by reduction these other three fractions of Denomination* $\frac{24}{60}$, $\frac{45}{60}$, $\frac{40}{60}$. *All which you may bring into one fraction by adding the Numerators together, and putting that totall for the totall Numerator, reserving still that same common Denominator. And those three fractions make one improper fraction, thus* $\frac{109}{60}$.

Note the Reduction of three Fractions or more, into one.

Scholar. All this I perceiue, and also that this last fraction is more then an vnite, and therefore you did call it an improper fraction.

Master.

Maſter. There be certaine other formes of working in this Reduction, which I will briefly touch alſo, to giue you an occaſion to exercile your wit therein.

The firſt variety of Reduction

The firſt variety is this: When you haue made and written downe your common Denominator (as I haue taught before) then to get a Numerator for the firſt, doe thus: Diuide the common denominator by the denominator of the firſt fraction, & the quotient multiplied by the Numerator of the ſame, yeeldeth a new Numerator for the firſt new fraction. So likewiſe doe with the ſecond and the third, and with all the reſidue, if there be more.

Scholar. That will I proue in your laſt example of theſe three fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$. When the denominators be multiplyed, they make 60: for 5 into 4 maketh 20, & 20 by three yeeldeth 60, that I ſet down three times thus: 60, 60, 60: then to haue a Numerator, for the firſt, I muſt diuide 60, by 5, (the denominator of the firſt) and the quotient is 12, which I muſt multiply by 2, (the Numerator firſt) and that maketh 24, and ſo haue I for the firſt fraction $\frac{24}{60}$.

Likewiſe for the ſecond fraction: I diuide 60 by 4, and there commeth 15, which I multiply by 3, and ſo haue I 45, for the ſecond fraction $\frac{45}{60}$. Then for the third in like ſort will come $\frac{40}{60}$.

The ſecond variety.

Maſter. *Another way is this: If it happen ſo, that the leſſer denominator can by any multiplication make the greater, then note the multiplier, and by it multiply the Numerator ouer that leſſer denomina-*

ter,

tor, & for the leſſer denominator put the greater, as thus in theſe two fractions $\frac{3}{12}$ *and* $\frac{2}{3}$, *three being the leſſer denominator multiplied by* 4. *will make* 12, *which is the greater denominator: therefore by the ſame* 4 *I doe multiply* 2 *which is the Numerator ouer* 3, *and that maketh* 8: *vnder which I doe put* 12, *being the greater Denominator, which is alſo made by multiplication of* 4 *into* 3, *and ſo haue I theſe two fractions* $\frac{3}{12}$, $\frac{8}{12}$, *thus ſhortly reduced, without altering the one fraction.*

Scholar. This I vnderſtand.

Maſter. *Then marke this third way: If the denominators doe not happen ſo, that one by multiplication may make the other, then looke whether they both may be parts of any other one number, as in* $\frac{5}{12}$, *and* $\frac{7}{18}$, *although the leſſer taken but twice, be too great to make* 18, *yet they both may be parts vnto* 36: *therefore looke how many times twelue is in* 36, *and that quotient being multiplyed by the Numerator ouer* 12, *the totall ſhall be put inſtead of the Numerator ouer* 12, *and for* 5 *put* 15, *thus* $\frac{15}{36}$ *So likewiſe looke how often is* 18 *in* 36, *becauſe it is twice, therefore by* 2 *multiply* 7, *which is ouer* 18, *and it will be* 14: *ſet that for the Numerator, and inſtead of* 18 *put* 36; *and then your Fractions reduced ſtand thus,* $\frac{15}{36}$, $\frac{14}{36}$ *inſtead of* $\frac{5}{12}$ *and* $\frac{7}{18}$. The third varietie.

And if you will proue whether you haue wrought well or no, that may be proued by Reduction of them againe to their former denominations, which Art ſhall be taught in the fourth kinde of Reduction, where greater termes of Fractions be reduced into ſmaller in Proofe.

number

number, but no ſmaller in proportion. And if in ſuch Reduction the ſame termes or numbers come againe that were before, then is the worke good, elſe not.

Scholar. Sir, I heare your words, but I doe not vnderſtand many of them: which if it pleaſe you, declare.

Maſter. With a good will, when conuenient place ſerueth, but that muſt bee in the ſaid fourth kinde of Reduction, which teacheth how to reduce fractions of fractions into one fraction, and ſo to one Denomination.

The ſecond forme of Reduction of fractions of Fractions into one Fraction and Denomination.

When Fractions of Fractions be proponed, you ſhall multiply the Numerators of each into other, and ſet the totall for the new Numerator, and then multiply all the denominators likewiſe, and take their totall for the new Denominator, and ſo are they ſpeedily reduced.

Scholar. If that be all, then I vnderſtand it already, as by this example I will declare. These be the fractions, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{6}{7}$ of $\frac{7}{9}$, which I would reduce to one Denomination, and proper ſimple fraction.

Therefore begin I with the Numerators, and multiply them together, ſaying, 3 into 2, maketh 6: and 6 by 6, maketh 36, which multiplyed by 7, yeldeth 252: that I ſet ouer a line for the Numerator, thus, 252

Then I multiply the denominator, 4 by 3, maketh 12, and that by 7 bringth 84, which multiplyed by 9, yeldeth 756, the new

new Denominator. And so the whole reduced fraction is this, $\frac{252}{756}$ which is too hard a fraction for me to vnderstand yet.

Master. You thinke so, and no maruell, but anon you shall learne to iudge it easily: for this Fraction is no more indeed than $\frac{1}{3}$, although it be in greater termes, and therefore more straunger, and more obscure.

And this sufficeth for this Reduction, saue that I will shew you by a figure of measure, the iust rate and reason of this kinde of fractions, and also the due vnderstanding of their Reduction.

The entire measure parted into 9.

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	$\frac{7}{9}$	
1		2		3		$\frac{6}{7}$		
1	2	3	4	$\frac{2}{3}$				
1	2	3	$\frac{3}{4}$					

Here you see the longest measure, (which standeth for the whole and entire quantity) first parted into 9 diuisions, whereof 7 are seuered by the second measure; and thereof againe are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being diuided into foure parts, the lowest

measure doth containe $\frac{3}{4}$. so that the same must bée named, not $\frac{3}{4}$ of the whole measure, but indéed is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar. This Example is so sensible, that I cannot chuse but sée it. And furthermore I sée also, that the same fraction is equall to $\frac{3}{9}$ of the entire measure, as the lines which runne vp and downe doe expresly set forth. Also I sée héere that $\frac{2}{3}$ of $\frac{6}{7}$ is equall to $\frac{4}{9}$. And further yet, that $\frac{2}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$.

Master. I am glad that you sée it so well, not doubting but you will gather greater light of knowledge hereby.

The third forme of Reduction of improper Fractions.

But now it is time that wee come to the third forme of Reduction, which teacheth of improper Fractions, that is to say, mixt numbers of vnites and Fractions, although they appeare like Fractions, as this $2\frac{1}{5}$, *which doth include 5 vnites wholly, and* $\frac{1}{5}$ *ouer. Wherefore first you shall know them, by that the Numerator is greater then the Denominator.*

Scholar. Indéed Sir, that appeareth reasonable, that if the Numerator doe expresse more parts to be taken of any vnite, then the Denominator doth signifie that vnite to bée diuided into, it must néeds follow that such a fraction importeth more then the whole, that is to say, the whole with certaine parts ouer: but what Reduction is there in it?

Two seuerall wayes in this Reduction.

Master. There be two seuerall kindes of Reduction, concerning such Fractions. Some-

Sometimes it ſhall bee needfull to conuert theſe fractions into vnites, and the proper fractions that will remaine. And ſometimes contrariwiſe, it ſhall be meete to reduce mixt numbers, that is, vnites written with fractions, into the forme of one ſimple fraction, and ſo be there two wayes.

Scholar. **What is the meane of the firſt way to turne** improper fractions **into** vnites **with their** proper fractions?

Maſter. *That is thus. Your Numerator being greater then the Denominator, muſt be diuided by the ſame Denominator, and the quotient thereof expreſſeth the vnites: the Remainer ſhall be put for the Numerator of the fraction that reſteth, and the Denominator muſt bee the ſame that was before.*

The firſt way.

Reduction of improper fractions into vnites, with their proper fractions.

Scholar. For example, I take $\frac{17}{5}$. And diuiding 17 by 5, the quotient will be 3, and there will remaine 2.

Maſter. That muſt you write thus, $3\frac{2}{5}$ where (you ſée) I haue written 3 without any line, as entire numbers ought to bée written, and the 2 that remained I haue ſet ouer the former Denominator with a line as a proper fraction. And this number doth ſignifie now three vnites, and $\frac{2}{5}$ of one.

Scholar. Then if I would by vnites here vnderſtand crownes, ſo it were 3 Crownes, and $\frac{2}{5}$ that is 2 s.

Maſter. Euen ſo, and therefore $\frac{17}{5}$ did ſignifie the ſame. But this hapneth ſometimes

that when the reduction is so wrought, there remaineth nothing. And then it is not a mixt number, but a simple entire number, represented like a fraction.

The second way.

Scholar. As $\frac{15}{5}$ will make 3 iust and $\frac{18}{3}$ will make euen 6. This I will remember. But now, what is the second forme of Reduction that you speake of, for these sorts of fractions?

Reduction of whole numbers either alone, or ioyned with fractions into improper fractions.

Master. *Whensoeuer you haue any of these two sorts of numbers, that is to say, whole numbers without fractions, or whole numbers with fractions, and you would turne them into the forme of a fraction, you must multiply the whole number by that denominator which you will haue to remains still, and to the totall thereof adde the Numerator, which you haue already, and all that shall you set for the new Numerator, keeping still the former Denominator: as if you haue* $6\frac{3}{4}$ *which you would conuert into an improper fraction, you must multiply* 6 *by* 4, *whereof commeth* 24, *and thereto adde the numerator which is* 3, *and so haue you* 27 *for the numerator, &* 4 *still for the denominator.*

Scholar. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Note.

Master. Euen iust, and so backward, (as appeareth by the former Reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proofe of the other worke.

Scholar. This I perceiue: But now if you would turne whole numbers without fractions into any fractions, I see not how that may be done, because there is no Denominator to make the multiplication by.

Master.

Maſter. That is well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that Denominator by which the multiplication muſt be made: for the proofe whereof I ſet downe 7, which is a whole number. And if you will haue this number conuerted into any certaine fraction, will me to doe it.

Scholar. I pray you reduce 7 into a Fraction.

Maſter. Then you care not what the Fraction be, ſo it be ſome Fraction.

Scholar. No, I paſſe not for the ſort of the Fraction.

Maſter. Then how can you think that you require me to doe any thing certaine, when you leaue me to doe as I liſt: And ſeeing you ſtand at that ſtay, whether thinke you that I muſt firſt intend in minde what fraction I will make of it, before I can doe it indeed?

Scholar. Elſe you ſhould doe ignorantly.

Maſter. Then will I limit my ſelfe (ſeeing you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the Denomination of quarters.) and there amounteth 28 to be ſet for the Numerator, and the 4 muſt be ſet for the Denominator, and the fraction will be thus $^{28}/_{4}$.

Scholar. Indeed I perceiue this to be reaſonable, for without much triall I vnderſtand that $^{28}/_{4}$ of any thing doth make 7. And ſo then

if I would turne 8 into 5 parts, it will make $4\frac{0}{5}$, which is all one with 8: for 8 crownes turned into 5 parts, (that is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a Crowne.

The fourth forme of Reduction

Master. *Seeing you understand now these three kindes of Reduction, I will declare vnto you the fourth kinde, that is, when fractions be written in greater termes then they need, how they may bee brought to lesser termes.*

Scholar. To write any thing in greater termes then néedeth, séemeth to be a fault, and so this Rule séemeth to amend that fault.

Master. It were a fault to doe any thing without néed, which after must be redressed: but in this case it is not so, neither did I say absolutely (as you doe) that it néedeth not to expresse those fractions in so great termes, but that the fractions doe not néed, I meane for their value to be vnderstood: but yet it may be néedfull for the ease of these workes whereto they be applied, as for example: In the first kinde of Reduction this was your owne example: $\frac{3}{16}$ and $\frac{5}{6}$, which when you would reduce, you were faine to turne them first into one denomination, and so appeared they thus: $\frac{18}{96}$, and $\frac{64}{96}$, where the fractions (for their owne vnderstanding) néeded not to be turned out of smaller termes into greater, but yet the easinesse of working néeded it.

Scholar. Sir, I vnderstand now, not onely the difference of this néed (for the fractions might better be vnderstood as fractions

seue-

ſeuerall, each in his value, when they were in leſſer termes, although they could not ſo well be reduced) but alſo I vnderſtand what you meane by greater termes and leſſer termes, whereof before I was in doubt: for I ſée you call the Numerator and Denominator, the termes of the fraction.

Termes of Fractions.

Maſter. *I am glad you vnderſtand it ſo well: Now then when you would value any fractions (becauſe they may beſt be done when the termes are ſmalleſt) you ſhall reduce them to the ſmalleſt that you can, which thing you may doe thus: Diuide the greateſt of any ſuch two termes by the leſſer, and if any thing remaine by that remainer, diuide the laſt diuiſor: and if any thing remaine now, by that diuide the firſt diuiſor (which was before the remainer of the laſt diuiſion) and ſo continue ſtill, till nothing doe remaine in the diuiſion: and then marke your laſt diuiſor, for it is the number that will eaſily reduce your fractions, if you diuide both the numerator and the denominator by the ſame number, and put for the numerator the quotient of his diuiſion, and for the denominator alſo his quotient, that riſeth by his diuiſion.*

Reduction of Fraction, into their ſmalleſt termes or leaſt value,

Scholar. I take for example $\frac{18}{96}$, and becauſe 96 is the greateſt number, I diuide it by 18, and the quotient is 5, and there reſteth 6: what ſhall I doe with this Quotient?

Maſter. Nothing in this worke, but now ſéeing there remaineth ſomewhat, by that remainer muſt you diuide the laſt Diuiſor.

Scholar. If I ſhall diuide 18, (which was

the last diuisor) by 6, that was the remainer, so is the quotient 3, and nothing resteth.

Maister. As for the Quotient I omit him yet: but because there doth remaine nothing, therefore is 6 (which was your last Diuisor) that number by which you may reduce the Fraction prepared.

Scholar. Then as you taught me, I must diuide the Numerator 18 by 6, and the quotient is 3, which I must put for the Numerator ouer a line, thus: $\frac{3}{}$ And then by the said 6 must I di- $\frac{}{16}$ uide also the Denominator 96, and the Quotient will be 16, which I must take for the Denominator, and so is the Fraction $\frac{3}{16}$. And so me thinketh this Rule doth proue the worke of the first Reduction.

Maister. That is true, if the first Reduction were made of fractions into their least termes, and else not, without some helpe, as the second number in that place will declare.

Scholar. The second number was $\frac{4}{6}$ which was turned into $\frac{64}{96}$ by that Rule. Now if I shall by this Rule reduce it againe into the least termes, I must diuide 96 by 64, and there remaineth 32, wherefore I must take that 32 for the Diuisor, to reduce the said fraction. Then doe you diuide 64 by 32, and the quotient is 2, which I set for my numerator. Againe, I diuide 96 by 32, and the quotient will be 3, and so I haue but $\frac{2}{3}$.

Maister. Rule not at the matter, for you haue

haue done well enough: but you thinke you haue not the fraction that you looked for, that is, $\frac{4}{6}$, yet haue you one equall to it, as by the parts of a shilling you may proue.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$, and $\frac{2}{3}$, be all three equall. And now I perceiue that because $\frac{4}{6}$ was not written in the least termes that it might be, therefore this Reduction brought forth not it, but that other which is written in the least termes. Now vnderstand I this rule well. But is there any other way to worke this Reduction?

Another way to worke this Reduction

Master. *Yes, but first note this, that if you finde no such Diuiser to reduce the Fraction till you come to* 1, *because one doth make no Diuision: therefore that Fraction is already in his least termes, as by* $\frac{21}{100}$ *you may proue, and so of* $\frac{85}{98}$, *and many other like.*

Note that to mediate any number is to diuide by two.

But now for your better aid to finde the due proportion in least termes, with more ease for a yong learner, you shall mediate or take the halfe of the Numerator, and also of the Denominator as long as you may vpon a line, alwayes parting them with a right downe dash of your pen as you worke, which may easily be done, if the numbers be euen: as 2, 4, 6, 8, *or* 10, *but if they be odde (though it be but one of them) them must you abreuiate them by* 3, 5, 7, *or* 9 *&c.*

And because examples doe most instruct, I haue here set downe the manner of two or three, whose last number at the end of the line shew-

ſheweth the leaſt terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his leaſt terme or value, whereupon I ſet forth $\frac{288}{576}$ with a long line drawne from it thus:

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And becauſe both the Numerator, and the Denominator end in euen numbers, I ſée this may be abreuiated by 2, or 4, or 6, &c. Therefore on the other ſide of the right downe dalh toward the right hand, I firſt take the halfe of the Numerator: ſaying, the halfe of 2 is 1, the halfe of 8 is 4: and againe, the halfe of 8 is 4: which 144 is now a new Numerator, and therefore I part it with a right downe dalh, as before.

Then doe I alſo take the halfe of 576, in ſaying, the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and ſo haue I 288 for a new Denominator.

Then beginning againe: ſaying the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or diuiſion by 2 vntill you come to the laſt worke, as appeareth here in the example, where the ſame is reduced to $\frac{1}{2}$, which is equall to $\frac{288}{576}$.

So this ſecond example $\frac{28}{112}$ firſt abreuiated by 2, and againe by 2, and laſt by 7, is reduced to $\frac{1}{4}$, which is equall to $\frac{28}{112}$.

28	14	7	1
112	56	28	4

Againe,

Againe, $\frac{1465}{4391}$ Abbreuiated first by 5, then by 293.

1465.	293	I
4395	879	3

Scholar. Sir I thanke you much, this is very easie and good for a young learner.

Master. So it is, but yet notwithstanding, if you can without that diuision by memory espie the greatest number that may diuide exactly both termes of your fraction proponed, then need you not to vse that diuision, as in this fraction $\frac{60}{96}$, I see that 12 is the greatest number that can diuide them both: and therefore without any worke, by memory onely, I turne that into $\frac{5}{8}$, but this ability in knowledge is got by exercise.

Yet one other way of easie reduction in this kind there is, when your fraction hath any cyphers in the first places of both termes, then may you by casting away the cyphers, make a briefe reduction as thus: $\frac{300}{400}$ here take away the cyphers, and it will be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholar. And so if I haue $\frac{400}{650}$, it will be $\frac{4}{65}$.

Master. You are deceiued, for you take away more cyphers from the Numerator then you doe from the Denominator, which you may not doe,

Scholar. I confesse my fault, which came of too much hast, I was more gladder of the Rule,

Rule then wise in using it: but now I understand it I trust.

Master. Then may I goe in hand with the fift or last kind of Reduction, which teacheth how to turne any fraction proponed into any other Denomination that you list, or into any parts of common coynes, weights or measures, or such like.

The fift kinde of Reduction.

For declaration whereof, first you shall marke whether your fraction be a simple Fraction, either else a Fraction of sundry parts, I meane of more termes then two. And if your Fraction be a Fraction of Fractions, or otherwise compound you must reduce it to one simple fraction. And then marke well the Denomination of that other Fraction, into which you would turne this: For by that Denominator you must multiply the Numerator of your first Fraction, and the totall product thereof shall you diuide by the Denominator of your first Fraction, and that quotient shall be the Numerator of the Denominator proponed: as for example, I haue this Fraction $\frac{3}{5}$, *which I would turne into ten parts: therefore I multiply this* 10 *by* 3, *that is the Numerator of my Fraction, and there ariseth* 30, *which I diuide by* 5, *and the quotient is* 6, *which must be the Numerator to* 10, *and so* $\frac{3}{5}$ *will be* $\frac{6}{10}$.

☞ To reduce fractions to a denomination appointed.

Scholar. This is easie enough to doe.

Master. Then shall you see another example of the same fraction that is not so easie: as if I would turne $\frac{3}{5}$ into 8 parts, proue you that worke.

Scholar. I must multiply 8 by 3, and there amount

amounteth 24, which I diuide by 5, and the quotient is 4, then is the new fraction $\frac{4}{x}$.

Master. And see you nothing doubtfull in this worke?

Scholar. I see that when 24 was diuided by 5, there remained 4, which I did not passe of, because ye spake nothing of any remainer, but onely of the quotient.

Master. By likelyhood you remember what I said to you in Diuision of whole numbers, that you should not passe of the remainer there, but only note it as a summe that could not be diuided without knowledge of fractions. Wherefore now marke this, that in all diuisions of whole numbers, when there is any remainer, you shall set it ouer a line as a Numerator, and set the Diuisor for the Denominator, and that fraction doth make the Diuision compleat, and is part of of the quotient: as if I would diuide 48 by 5 the quotient will be $9\frac{3}{5}$: so in your former worke when 24 was diuided by 5, the quotient should be $4\frac{4}{5}$, and so the new fraction should be thus, $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$, that is, $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may proue by example of some coyne.

Scholar. Then I take a crowne, whose $\frac{3}{5}$ is 3s. Now if I would proue whether the 3s be $\frac{4}{8}$ and $\frac{4}{6}$ of $\frac{1}{8}$, I shall haue a cumbrous worke to doe.

Master. Indeed for whole pence, your example is a little troublesome: yet turning the

crowne

crowne into halfe pence, it is eaſie enough.

Schol. That will I try.

¶ Firſt I ſée that $\frac{3}{5}$ of a Crowne is 3 chillings, which is 36 pence, or 72 halfe pence. Now if I can finde that this fraction $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equall vnto 3 chillings, then am I fully anſwered.

Becauſe I cannot take $\frac{4}{8}$ of a crowne, I turne the crowne into halfe pence, as you willed me, which makes 120. which I diuide by 8, my quotient is 15, which taken foure times make 60 ob. Now reſteth me to haue $\frac{4}{5}$ of the $\frac{1}{8}$ part of a crowne, whereof $\frac{1}{8}$ part is 15 ob, the 15 being parted in 5 parts, the quotient is 3, which taken foure times maketh 12 ob, which with my 60 before amounteth to 72, which are then equall to $\frac{3}{5}$, my deſire.

Maſter I commend you for your diligence, you might haue wrought it thus: either $\frac{4}{8}$ being abreuiated as before I taught, is $\frac{1}{2}$. Now halfe a crowne is 2 chillings 6 pence. Then $\frac{4}{5}$ of $\frac{1}{8}$ is a fraction of fractions, which if you doe reduce into one entire fraction, as before you haue learned, in ſaying, fiue times 8 is 40, for a new Denominator, and once 4 is 4, for a new Numerator: it maketh $\frac{4}{40}$, and abreuiated alſo make $\frac{1}{10}$. Now the tenth part of a crowne is 6 pence, which put to 2 chillings fiue pence, make alſo 3 chillings, your deſire.

But now one example more for this rule, and then we ſhall end it. If I haue $\frac{7}{15}$ of a Soueraigne (accounting the Soueraigne 20 chillings)

lings) how many ſhillings is that $\frac{7}{15}$?

Scholar. I muſt multiply 7 by 20, and that maketh 140, which I ſhall diuide by 15, and the quotient will be $9\frac{5}{15}$, or elſe in leſſer termes $9\frac{1}{3}$.

Maſter. That is 9 ſhillings, and one third part of a ſhilling, that is 4 pence, as by the ſame Rule you may proue. And this for this time ſhall ſuffice for Reduction. And now I will proceed to Addition.

Addition.

Whenſoeuer you haue any Fractions to be added, you muſt conſider whether they bee of one denomination or not, and if they be of one Denomination, then adde the Numerators together, and ſet that that amounteth for the Numerator ouer the common Denominator, and ſo haue you done: The reaſon is becauſe that ſuch differ little in Addition *or* Subtraction *from the worke of vulgar denominations, where the denominators bee of odde number, as* 3 pence, *and* 5 pence, *make* 8 pence, *where the* Denomination *is not altered. But if the fractions be not of one* Denomination, *or any of them bee mixt of whole numbers and fractions, then muſt you firſt reduce them to one* Denomination, *and after adde them. And if they be many, then adde firſt two of them, and ſo the ſumme that doth amount of the Addition, and the third, and then the fourth, &c. if you haue ſo many.*

Addition of fractions of one denomination.

Scholar.

Scholar. This ſeemeth eaſie enough, now that I haue already learned to reduce, without which I could neuer haue wrought this. And therefore now I ſee good reaſon, why you did place Reduction before Addition.

Maſter. It is well conſidered, but yet refuſe not to expreſſe your vnderſtanding of it by an example.

Scholar. Then would I adde firſt $\frac{7}{18}$ with $\frac{5}{18}$, and becauſe the Denominators are like, (and ſo needeth no reduction) I adde 7 to 5, which maketh 12, and then is my ſumme $\frac{12}{18}$, that is in ſmaller numbers, being abbreuiated $\frac{1}{3}$.

To adde fractions of diuers denominations.

And if I haue many numbers to be added as heere $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$, firſt I muſt reduce them (becauſe they haue diuers denominators (into one Denomination, and then they will be thus: $\frac{150}{400}$, $\frac{320}{400}$, $\frac{360}{400}$, or in leſſer termes, $\frac{15}{40}$, $\frac{32}{40}$, $\frac{36}{40}$, which by Addition doe make $\frac{83}{40}$, that is, $2\frac{3}{40}$.

Maſter. Now may we goe to Subtraction.

Sub-

Subtraction.

Vbtraction hath the same precepts that Addition had: for if the Denominators be like, then must you subtract the one Numerator from the other, and the rest is to bee set ouer the common Denominator, and so your Subtraction is ended: but and if you haue many fractions to be subtracted out of many, then must you reduce them to one Denomination, and into two seuerall fractions, that is, all that must be subtracted into one fraction, and the residue into another fraction, and then worke as I said before.

Subtraction of fractions.

Scholar. For the first example, I take $\frac{15}{12}$ to be subtracted out of $\frac{17}{12}$ and the rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For another example, I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus: $\frac{24}{32}$ and $\frac{28}{32}$.

Then doe I subtract 24 out of 28, and there resteth 4, which I set ouer the common Denominator for a Remainer, thus: $\frac{4}{32}$, that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{1}{6}$ to be subtracted from $\frac{7}{8}$ and $\frac{9}{10}$: and because their Denominators bee diuers, I doe reduce them into one denomination thus: $\frac{1440}{1920}$ $\frac{1600}{1920}$, $\frac{1680}{1920}$, $\frac{1728}{1920}$.

Then doe I adde the two first, & they make $\frac{3040}{1920}$. Also I adde the two last, and they yeeld $\frac{3408}{1920}$. Then doe I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$, that is in smaller termes $\frac{203}{120}$. And thus haue I done with Subtraction, except you haue any more to teach me.

Master. Proue one example or more of two fractions of diuers denominations.

Scholar. I take these two fractions, $\frac{7}{8}$ to be subtracted from $\frac{9}{24}$, which being reduced, will stand thus: $\frac{168}{192}$ and $\frac{72}{192}$. Now would I subtract 168 out of 72, but I cannot.

168	72
$\frac{7}{8}$	$\frac{9}{24}$
192	

Master. Then may you perceiue that you mistake the fractions: for you can neuer subtract the greater out of the lesser, although you may adde, multiply, or diuide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his termes lesser then $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser fraction: for generally if you multiply the Numerators and the Denominators of two fractions crossewaies, that Fraction is the greatest of whose Numerator commeth the greatest summe, as in this example: 7 multiplied by 24, maketh 168; and 9 being multiplied by 8, yeeldeth but 72. therefore is the first fraction 7, the greatest of these two, so can you not subtract it out of a lesser fraction.

The greatest of two fractions.

But if you should subtract a fraction out of a whole number, what should you doe?

Scholar.

Scholar. Marry I would reduce the whole number into a fraction of the same Denonination that my fraction is, and then worke by Subtraction.

Master. So may you doe, but it is easier much, if your fraction be a proper fraction, that is to say, lesse than an vnite, to take an vnite from the whole number, and then turne it into an improper fraction, and so worke your Subtraction. As if I would subtract $3\frac{2}{5}$ from 4, I may take 1 from 4 and turne it into $\frac{5}{5}$, from which I abate $3\frac{2}{5}$, there will remaine $\frac{3}{5}$. And if the first fraction be an improper fraction, then may I take so many vnites from the whole number, that they may make an improper fraction greater then that first, and then worke by Subtraction. As if there bée proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more then 3, and not so much as 4, I must take 4 from 6, & turne them into thirds thus $\frac{12}{3}$, then abate $\frac{10}{3}$, and from $\frac{12}{3}$ there resteth $\frac{2}{3}$: so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$, which is $\frac{10}{3}$, from 6 whole: then set 1 vnder 6, as thus: $\frac{6}{1}$: And then to reduce those two fractions into one Denomination, as héere appeareth $\frac{10}{3}$ from $\frac{6}{1}$. Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$, which maketh $2\frac{2}{3}$ your desire. And thus will I make an end of the worke of subtraction of fractions, and proceéd to multiplication.

$$\begin{array}{cc} & 8 \\ \frac{20}{3} & \times \\ & 3 \end{array} \quad \frac{18}{3}$$

D

Multiplication.

Multiplication of fractions.

Herefore when any two fractions be proponed to be multiplied together, the Numerator of the one muſt bee multiplied by the Numerator of the other: and the ſumme that amounteth ther. of muſt bee ſet for a new Numerator: likewiſe the Denominator of the one muſt be multiplied by the Denominator of the other, and that that amounteth, ſhall bee ſet for the Denominator, and this new third fraction expreſſeth the product of the multiplication of the two firſt fractions proponed, whereof take this example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$

15

$\frac{3}{5}$ $\frac{5}{12}$

60

Scholar. I perceive then that 3, being the Numerator of the firſt fraction, is multiplied by 5, being the numerator of the ſecond fraction, whereof amounteth 15, the numerator of the third fraction. And ſo likewiſe, 5, being the denominator of the firſt fraction, is multiplied by 12 the denominator of the ſecond fraction, whereof amounteth 60 the new denominator, ſo that I perceive how the worke is done, but I doe not perceive how $\frac{15}{60}$ is greater then $\frac{3}{5}$. for if I ſhall vſe my former manner of examination by the parts of ſome coyne, I ſée that $\frac{3}{5}$ of a Crowne is 36 pence, and $\frac{5}{12}$ of a crowne is 25 pence, whereof the one multiplied by the other, doth make 900 pence,

900 pence, which is 15 Crownes: but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 pence, and that is much lesse then any other of both the first fractions.

Master. That difference is betwéen multiplication in whole numbers, and multiplication in broken numbers, that in whole numbers, the summe that amounteth is greater then both the other whereof it came: but in fractions it is contrariwise: for the summe that amounteth is lesser then any of the other two fractions whereof it is produced.

Scholar. I desire much to understand the reason thereof.

Master. Although I purposed to reserue the reasons of workes Arithmeticall for the perfect Booke of Arithmeticke, yet I will shew you this, because of the strangenesse of the worke.

You see in whole numbers, that of two numbers being multiplied together, is made the third number, which third number, doth beare the same proportió to the number multiplied, that the multiplier doth beare to an vnite. And so in fractiõs, the third nũber which amounteth of multiplication, beareth the same proportion to each of the two first fractions, that the other of those two fractions doth beare to an Vnite.

Scholar. Sir, I understand your words thus: when 40 is multiplied by 12, there doth amount 480, which 480 doth containe 40 so many times in it, as 12 doth containe vnites, that is to say, twelue times. And so it

appeareth that 480 doth containe twelue so many times also as 40 doth contain vnites, that is, 40 times. But now I see not how the third number in this example of Fractions can containe any of the two former (as it happened in whole numbers) seeing it is lesser then either of them.

Master. No marvell if you cannot see that thing which is not possible to be seene of any man, how the third number in Multiplication of Fractions should be greater then any of the two former fractions: but yet this may you see (which I said) that the third number in fractions so multiplied, doth beare the same proportion to any of the two former fractions, that the other of those two fractions doth beare to an vnite, as in your example, $\frac{3}{5}$ being multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$. Now say I, that $\frac{15}{60}$ doth beare the same proportion to $\frac{3}{5}$ that $\frac{5}{12}$ doth beare to an vnite, as you may in your owne forme of examination by Coyne try it: for in an old Angell, (which in times past was currant for 7 shillings fiue pence, are 180 halfe pence) which I set for the intire vnite, whose parts (according to the fractions aforesaid) are these, for $\frac{15}{60}$ set 45 halfe pence, for $\frac{3}{5}$ take 108 halfe pence, and for $\frac{5}{21}$ put 75 halfe pence. Now doth 45 beare the same proportion to 108, that 75 doth beare to 180, for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserued till another time, when the knowledge of pro-

proportions in due order shall be taught: yet in the meane season I will shew you how it commeth to passe, that in fractions the third summe must néeds be lesser then any of the other two.

Consider this, that when a fraction is postponed, as in the former example $\frac{3}{5}$ if it be multiplyed by more then 1, it will make more then one entire number. As if I multiply $\frac{3}{5}$ by 5: that is to say, if I take it 5 times, it will make three entire vnites: Example in a Crown, $\frac{3}{5}$ of it maketh 3 shillings, which if I take fiue times, it will amount to 15 shillings, that is, three entire Crownes: so if I take the same $\frac{3}{5}$ but twice, it will yéeld 6 shillings, that is, one entire Crowne, and $\frac{1}{5}$. Now if I take it but once, it cannot be more then it was before, that is 5 shillings. And if I take it lesse then once, it cannot be so much as it was before. Then séeing that a Fraction is lesse then one, if I multiply a fraction by another fraction; it followeth that I doe take the first fraction lesse then once, and therefore the summe that amounteth, must néeds be lesse then the first Fraction. Note. ☞

Scholar. Sir, I thanke you much for this reason. And I trust I doe perceiue the thing, as by example of this same fraction $\frac{3}{5}$ I will expresse. If I take $\frac{3}{5}$ of a Crowne once, that is to say, if I multiply $\frac{3}{5}$ by 1, it will be as it was before, but 3 shillings: so if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then

will it be but halfe ſo much: likewiſe if I multiply it by $\frac{1}{3}$, that is, if I take but the third part of one, it will yéeld but 12 pence, that is, the third part of the firſt Fraction.

And ſo to make an end: if I take it but the twelfth part of once, that is, If I doe multiply it by $\frac{1}{12}$, it will yéeld but the twelfth part of the firſt fraction, which is but 3 pence. And it followeth, that if $\frac{1}{12}$ make three pence, then $\frac{5}{12}$ muſt néeds make fiue times ſo much, that is 15 pence, which was the ſumme that hath giuen the occaſion of all this doubt.

Maſter. Then I perceiue you haue ſufficient vnderſtanding in this ſort of multiplication for this time, wherefore I will proceéd to the reſt.

In Multiplication it happeneth ſometime, that there be whole numbers to be multiplied with Fractions; and may bee in two ſorts: for either the whole number is ſeuerall from the Fraction, and is the Multiplier, or elſe the whole number is ioyned with one, or both of the Fractions, and ſo maketh a mixt number thereof. If it bee in the firſt ſort, then needeth there no reduction, but onely multiply the Numerator of the Fraction by that whole number, and the totall thereof ſet for the new Numerator.

To multiply a whole number into a fraction.

Scholar. I vnderſtand you thus. If I haue $\frac{6}{23}$ to bee multiplied by 16, then muſt I multiply that 16 with 6, which is the Numerator, whereof commeth 96, and that muſt I ſet for the new Numerator, kéeping ſtill 23

for

for the Denominator, and so the fraction will be $\frac{96}{23}$, that is $4\frac{4}{23}$.

Master. And in this sort of worke you may abridge the labour, thus. If it happen the Denominator to be such a number, as may evenly be diuided by the said whole number proposed, then diuide it thereby, and set the Quotient of that diuision for the former Denominator: but reserue still the Numerator, and so is the multiplication ended.

Scholar. Then I saine this example $\frac{7}{20}$ to be multiplyed by 5: and because 5 will iustly diuide 20, therefore I take the Quotient of that diuision, which is 4, and set in stead of 20, and so the fraction will bee $\frac{7}{4}$, that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$ that would haue followed of the other sort of worke.

Scholar. I perceiue it very well.

How to Multiply mixt numbers.

Master. *Now then for the other sort, where the number is mixt, take this way: first to reduce the said whole number and fraction into one improper fraction, (as I shewed you in* Reduction) *and then multiply them together, as if they were proper Fractions.*

Scholar. $13\frac{3}{5}$ being set to bee multiplied by $\frac{5}{8}$, first I must reduce the mixt number, as in this example appeareth by multiplying 13 by 5, and that maketh 65, whereto I must adde the Numerator 3,

$$\frac{13\frac{3}{5}}{68} \text{ by } \frac{5}{8} \qquad \frac{340}{5\ 40}$$

and

and so the fraction will be $\frac{68}{5}$, which two Fractions now I shall multiply after the accustomed forme, and it will be $\frac{340}{40}$, or $\frac{34}{4}$.

Master. You haue done well: and so may you see, that although most part of the formes of Multiplication may be wrought without Reduction, yet some cannot, as namely mixed numbers.

Duplation.

And yet one note more will I tell you of Multiplication, before we leaue it: That is, whensoeuer you would multiply any Fraction by 2, *which commonly is called* Duplation, *you may doe it not onely by doubling the Numerator, but also by parting the Denominator into halfe, if it be euen.*

Scholar. Then if I would double $\frac{5}{12}$, I may chuse whether I will make it $\frac{10}{12}$, or else $\frac{5}{6}$. And indeed I see that is all one, but that the diuiding of the Denominator seemeth the better way to make smaller termes of the Fraction, and so they shall néede the lesse Reduction.

Master. It is so: and now I shall not néed to tell you that multiplication is proued by Diuision, and Diuision likewise by multiplication: but the like workes that I shewed you in multiplication, will I shew you in Diuision also.

Diuisi-

Diuiſion of Fractions.

Diuiſion of Fractions.

WHenſoeuer two Fractions be proponed, that one ſhould bee diuided by the other, I muſt ſet downe firſt the fraction that ſhall be diuided, (which is called the Diuidend) and then after it the other which is the Diuiſor. Then ſhall I multiply the Numerator of the Diuidend by the Denominator of the Diuiſor, and that which amounteth, I muſt put for a new Numerator. Againe, I ſhall multiply the Denominator of the Diuidend by the Numerator of the Diuiſor, and the number that amounteth thereof, I muſt put for the new Denominator. And this third Fraction is the Quotient of the ſaid Diuiſion.

Scholar. This ſeemeth eaſie in forme, as by example thus: If I would diuide $\frac{5}{8}$ by $\frac{2}{6}$, firſt I multiply 5, (being the Numerator of the Diuidend) by 6, which is the Denominator of the Diuiſor, and thereof riſeth 30: then I multiply 8 (being the Denominator of the Diuidend) by 2, being the Numerator in the Diuiſor: and ſo riſeth 16, the which I muſt make in a third Fraction, thus $\frac{30}{16}$.

	30	
$\frac{5}{8}$		$\frac{2}{6}$
	16	

Maſter. Me ſeemeth you are quicker in vnderſtanding now, then you were when I taught

taught you the Art of whole numbers: but that is no maruell: for the more knowledge that any man getteth the readier shall he finde his wit, and quicker in vnderstanding: but yet of two things I will admonish you, which you might haue obserued here for the ease of work, and lightnesse of vnderstanding the nature of the Quotient.

Whensoeuer you diuide one fraction by another, either they be both equall together, or elle the one is greater then the other: if they be equall, their Quotient shall be such, that the Numerator and the Denominator of it shall be equall also. And if the two first fractions bee equall, their Quotient shall declare the same by the vnequality of the Numerator and Denominator, as in these examples following shall appeare.

First, if equall fractions $\frac{4}{9}$ and $\frac{12}{27}$ be equall together, and if the one be diuided by the other, the Quotient will be $\frac{108}{108}$, as you may perceiue by that Rule aforesaid.

Now in the vnequall fractions, as $\frac{4}{9}$ and $\frac{3}{10}$, the Quotient will be $\frac{40}{27}$, where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Note how to know the proportion betweene two numbers.

Maister. The reason is this: When any Fraction is diuided by another, the quotient declareth what proportion the Diuidend beareth to the Diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, which must be founded, not 2, but twice,

twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the Quotient repreſenteth the Diuidend, and the Denominator repreſenteth the Diuiſor. And this is alwaies true, whether the greater fraction be diuided by the leſſer, or the leſſer by the greater. But this proportion will not be exactly knowne, till you haue learned the Art of proportions: notwithſtanding ſomewhat of it I haue declared in the Rule of Reduction. But now for the eaſie remembrance of the Quotient in Diuiſion: as ſoone as you haue ſet downe your two fractions the one againſt the other, then make a ſtraight line for the Quotient: and as ſoone as you haue multiplied the Numerator of the Diuidend, by the Denominator of the Diuiſor, ſet the number that amounteth ouer the ſaid line, and then multiply the other two numbers, and ſet their totall vnder the ſame line.

Scholar. I perceiue you would not haue me truſt to memory till I were better expert, leſt oftentimes I happen by miſſe-remembrance to be abuſed. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$, I muſt ſet the numbers one againſt the other, (as here doth appeare) and then make another line for the quotient in ſome good diſtance,

$$\frac{2}{3} \quad \frac{3}{4}$$

distance, where I may set the numbers of the quotient, as sone as any of them is multiplied. So then as sone as I haue multiplied 2 by 4, which maketh 8, I shall set that 8 ouer that line, thus: And then multiply 3 by 3, which yeeldeth 9: and that 9 must be set vnder the same line, and then will the whole quotient appeare thus $\frac{8}{9}$: wherby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but how may I perceiue that?

$$\frac{8}{9}$$

Master. Although you might better perceiue it by the Rule of Reduction, yet this example may be declared in common coines as in a common shilling of 12 pence, of which $\frac{2}{3}$ maketh 8 pence, and $\frac{3}{4}$ doth make 9 pence, and so you may easily see that their proportions doe agree. And if you had taken this example before when you toke the example of $\frac{5}{8}$ and $\frac{3}{6}$ your Quotient would appeare (as this doth) more easie to vnderstand; whereas that Quotient being $\frac{30}{16}$, is not an easie proportion for you to perceiue, being yet little acquainted with proportions.

Scholar. If there be whole numbers to be diuided by a fraction, how shall I performe it?

To diuide a whole number by a fraction.

Master. *When any whole number shall bee diuided by a fraction, you must multiply the said whole number with the Denominator of the fraction, and set the totall thereof for the new* Numerator, *and for the* Denominator

set

ſet the Numerator *of the fra-*
ction.

Scholar. Then 20 diuided by $\frac{5}{4}$ will make $\frac{80}{3}$, as here appeareth $\frac{80}{3}$.

80 by $\frac{5}{4}$
3

Maſter. *Euen ſo: but if you would diuide the fraction by the whole number, then multiply the Denominator by the ſame whole number, and ſet the totall for the Denominator, without changing the Numerator.*

To diuide the fraction by the whole number.

Maſter. Then to diuide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth $\frac{20}{23}$, by $\frac{4}{1}$ in this example $\frac{20}{92}$.

$\frac{20}{23}$ by $\frac{4}{1}$
92

Maſter. You ſay well. And by the ſame example you giue me occaſion to remember another briefe way to doe the ſame: for if you had diuided the ſaid numerator, by 4, and ſet the quotient for the numerator, keeping ſtill the old denominator, it would haue béene not onely as well done, but alſo in a fraction of leſſer termes.

Another briefe way.

Scholar. I gueſſe it to be euen ſo, by a like worke that you taught me in multiplication: And for proofe thereof $\frac{20}{23}$ being the dividend, and 4 the diuiſor, I diuide the Numerator 20 by 4, and the quotient is 5, which I ſet for 20 ouer 23, thus, $\frac{5}{23}$: And I ſée that it is all one with $\frac{20}{92}$, as by diuiding or abreuiating both theſe termes by 4, and ſo reducing them

to

to their leaſt denomination, I may eaſily proue: as appeareth by this example, $\frac{20}{92}$ $\frac{5}{23}$.

Maſter. You conceiue it well. And if there be mixt numbers (either one or both) you muſt firſt reduce that mixt number into an improper fraction, & then worke as you haue learned.

Scholar. That was ſufficiently taught in Multiplication. Therefore I pray you goe forward to ſome other thing.

Maſter. Then take this note yet for Diuiſion: if the denominators be like, then diuide the Numerators as if were in whole numbers, and the quotient whether it be fraction, whole number, or mixt, is a good quotient for that diuiſion. And generally, if one of the Numerators may iuſtly diuide the other, by that quotient multiply the Denominator of the leſſer Numerator, and ſet it that doth amount in the rowme of the ſame denominator, and then for a numerator to it, ſet the Denominator of the other fraction.

Scholar. Then if I would diuide $\frac{3}{4}$ by $\frac{12}{17}$, I ſée that 3 will diuide 12, and the Quotient will be 4, by which I muſt multiply the other 4, that is the Denominator vnder 3, and then it is 16, which I ſet for the denominator 4, and ouer it inſtead of 3, I muſt ſet 17 the other Denominator, and ſo it is thus, $\frac{17}{16}$.

Maſter. And ſo is $\frac{17}{16}$ inſtead of $\frac{51}{48}$, which would haue riſen by the common worke, as here appeareth.

$$\frac{3}{4} \text{ by } \frac{12}{17} \qquad 51 \qquad 48$$

And

And now for mediation (which is to diuide by 2) marke this, if the Numerator be an euen number, set the halfe of it in his place without the Diuisor, and so haue you done: and if the Numerator be not euen, then double the Denominator.

Scholar. That is, if I would mediate $\frac{6}{11}$, I may make the quotient $\frac{3}{11}$, and if I would mediate $\frac{7}{11}$ I must make it $\frac{7}{22}$.

Maister. And thus will I make an end of the workes of common fractions for this time, not doubting but you can apply them both to the Rule of Progression, and also to the Golden Rule, without any other teaching then you haue learned before, which might seeme tedious to repeat, in regard you haue sufficient knowledge in Reduction, Addition, Subtraction, Multiplication, and Diuision: And therefore will I goe in hand with the Rule of Proportion, or Golden Rule, which now will appeare easie enough.

The Golden rule direct, in Fractions.

Maister.

Herefore as touching the Golden rule, for the placing of the 3 numbers proponed in the question whereby to find the fourth, and for the forme of their worke, with other like notes, I

The rule of proportion, in fractions.

refer you to that which you haue already learned.

But this easie forme of working by fractions shall you note, that if your three numbers be fractions, for an apt worke and certaine, multiply the numerator of the first number in the question, by the denominator of the second: And all that againe multiply by the Denominator of the third number, and the totall thereof shall you keepe for to be the Diuisor. Then multiply the Denominator of the first number by the numerator of the second, and the whole thereof by the Numerator of the third, and the totall thereof shall be your Diuidend.

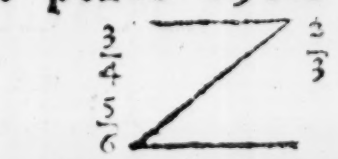
Note this for a generall rule.

Now diuide this Diuidend by the diuisor which you found out before, and that number shall be the fourth number of the question which you seeke for, as in this example.

A question of veluet.

If $\frac{3}{4}$ of a yard of veluet cost $\frac{2}{3}$ of a Soueraigne, esteemed at 20 shillings, *what shall $\frac{5}{6}$ cost?*

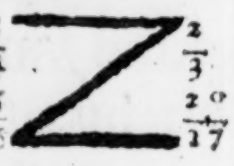
Scholar. If it please you to let mee make the answer, I would first place these three numbers as I learned in whole numbers, thus.



And then according to your new rule, I must multiply 3, being Numerator, in the first number, by 3 the Denominator of the second, and thereof commeth 9, which I multiply againe by 6, the denominator of the third number, and so haue I 54, which I keepe for the diuisor. Then multiply I 4 the denominator of the first, by 2 the numerator of the second,

and

and there ariſeth 8 which againe I multiply by 5, the Numerator of the third, and it maketh 40. Then muſt I diuide 40 by 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$, in leſſer termes, and then the Figure will ſtand thus.



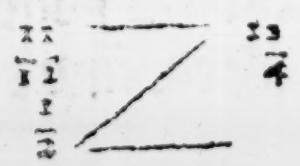
But what that is in money I cannot tell, except I ſhall worke it by Reduction, as you taught mee.

Maſter. It forceth not now, you may reduce it when you liſt, but it were diſorderly done here to mingle diuers works together, where we doe not ſeeke the value of the thing in common money, but in an apt number, which yee haue well done. And therefore will I yet ſhew you another like way of eaſineſſe in worke, how you may change your three fractions into three whole numbers, by which you ſhall worke as if the queſtion were propoſed in whole numbers. The firſt number you ſhal find as I taught you: now to find the diuiſor of the ſecond number, take the Numerator for the ſecond fraction: and for the third number take that that ariſeth of the multiplication of the denominator of the firſt, by the numerator of the third, & then worke your queſtion.

A queſtion of Siluer.

Scholar. *For example hereof, I put this queſtion. If* $\frac{11}{12}$ *of* 1 pound *waight of ſiluer be worth* $\frac{12}{4}$ *of a Souereigne, what is* $\frac{1}{2}$ *of* 1 pound *weight worth?*

For the anſwere, firſt I place the fractions in order thus:



Then to turne these fractions into whole numbers, I multiply 11, which is the numerator of the first by 4 (the denominator of the second) and there commeth 44, which I multiply by 2 the denominator of the third, and so amounteth 88, which I set for the diuisor in the first place. Then in the second place I set 12 which is the numerator of the second fraction, & in the third place I set the sum that amounteth of 12, being the denominator of the first number, multiplied by one, being numerator in the third number, and so the figure will stand as here you see.

88 | 12
12 |

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I diuide by 88, & the quotient will be $1\frac{56}{88}$, or in lesser termes, $1\frac{7}{11}$, and then the figures will stand thus:

$\frac{11}{12}$ | $\frac{12}{4}$
$\frac{1}{2}$ | $1\frac{7}{11}$

Master. These two formes now you vnderstand well enough, and as for any other at this time I will not repeat, onely this shall you marke for the proofe of this Rule whether your worke be well wrought or no. Multiply the first number by the fourth, and note what amounteth: then multiply the second by the third, and marke what amounteth also. Now if those two numbers so amounting be equall, then is your worke well done, else you haue erred. And this shal suffice for the former rule.

The proof of the golden Rule.

The

The Backer Rule, or Reuerſe Rule in Fractions.

BVt in the Backer Rule, this ſhall you note for eaſe of worke, that you multiply the numerator of the firſt by the numerator of the ſecond, and the whole thereof by the denominator of the third, and that amounteth thereof, ſhall be the diuidend. Then multiply the denominator of the firſt, by the denominator of the ſecond, & that whole by the numerator of the third, and that that ariſeth thereof ſhall be the diuiſor. Example of this.

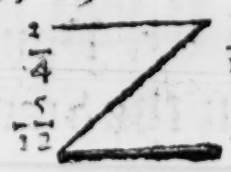
The backer Rule in Fractions.

Note this alſo for a generall Rule.

I did lend my friend $\frac{3}{4}$ *of a Porteguiſe ſeauen moneths, vpon promiſe that he ſhould doe as much for me againe, and when I ſhould borrow of him, he could lend me but* $\frac{5}{12}$ *of a Porteguiſe: now I demand how long time muſt I keepe his mony in iuſt recompence of my loane, accounting* 13 *moneths in the yeere?*

A queſtion of Loane.

Scholar. The firſt number muſt be the firſt mony borrowed, that is $\frac{3}{4}$ of the Porteguiſe: the ſecond number the 7 moneths, that is $\frac{7}{13}$ of a yeere: and the third number the money that was lent in recompence, that is $\frac{5}{12}$ of a Porteguſe: then I ſet the numbers thus:



Then (as you taught me) I multiply 3 (being numerator in the firſt number by 7 the numerator of the ſecond number, and it maketh 21, which I multiply by 12 the dominator of the third, and ſo haue I 252 for the diuidend: then I multiply 4 the denominator of the firſt, by 13 the denominator of the ſecond, & it yeldeth 52, which I multiply againe by 5 the numerator of the third, and it will make 260, that is the diuiſor. Then muſt I diuide 252 by 260, ſo it will be in the ſmall fraction $\frac{63}{65}$ of a yeere.

Maſter. And thus doe you ſée ſome eaſe in working, better then to multiply and diuide tediouſly ſo many Fractions.

Statute of Aſſiſe of Bread and Ale.

Another queſtion yet will I propoſe, to the intent you may ſee thereby the reaſon of the Statute of aſſiſe of bread and ale, which in all ſtatute Bookes, in Latine, French & Engliſh is much corrupted for want of knowledge in this Art, for the right vnderſtanding whereof I propone this queſtion.

Queſtion of Bread.

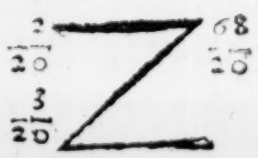
When the price of a quarter of wheat is 2 ſhillings, *the farthing white loafe ſhall weigh* 68 ſhillings, *then I demand what ſhall ſuch a loafe weigh when a quarter of wheat is ſold for* 3 ſhillings?

Scholar. This queſtion muſt be wrought as it is propoſed in whole numbers, and not in Fractions.

Maſter. You ſéeme to ſay reaſonably, howbe it in the Statute of aſſiſe, the rate is made by

by the proportion of parts in a pound waight Troy, else could it not be a Statute of any long continuance, seeing the shillings doe change often, as all other monies doe: but this Statute being well understood, is a continuall rule for euer, as I will anon declare by a New Table of Assise, conuerting the shillings into ounces, and parts of ounces.

Wherefore here by a shilling you must understand $\frac{1}{20}$ of a pound waight, and so by a penny $\frac{1}{20}$ of an ounce: wherefore although you might worke this question proponed by whole numbers well enough, for that time when the Statute was made, yet to apply it to your time, & to make it serue for all times generally, it is best to worke it by fractions, setting for 2 shillings $\frac{2}{20}$, and for 68 shillings, $\frac{68}{20}$, & so for three shillings $\frac{3}{20}$, and then will the Figure of the question stand thus:



In which question because all the denominators bee like, you shall worke onely with the numerators.

Scholar. Then shall I multiply 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient will be $45\frac{1}{3}$: but how shall I make a fraction of that, to stand with the other?

Master. Haue you so soone forgotten what was taught you so lately? this is his forme.

$$\frac{45\frac{1}{3}}{20}$$

Scholar. I remember it now, and then it ſignifieth 45 twenty parts, and the third deale of one twenty part.

Note what a ſhilling is.

Maſter. So is it that maketh in ſhillings 45 ſhillings, 4 pence, whereby you may note one great error in the Statute Bookes, which haue conſtantly 48 ſhillings in that Aſſile. And by this Rule if you examine the Statute, you ſhall find many ſummes falſe. Wherefore for the true vnderſtanding of that Statute, and ſuch like as I haue made mention of it, and ſomewhat recogniſed it, ſo do I wiſh that all Gentlemen & other Students of the Lawes would not neglect this Art of Arithmeticke, as vnneedfull to their ſtudies. Wherefore to encourage them thereto, and to gratifie both them and all other in generall, I will exhibite a Table of that part of the Statute in two columnes, and in a third columne. I will adde the correction of thoſe errors which haue crept into it.

Here followeth the Table.

The

The price of a quarter of Wheate		The waight of a farthing white loafe by the Statute Bookes.			The correction by iuſt Aſſiſe.		
s	d	l	s	d	l	s	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	2	8	0	3	8	0
2	6	2	14	$4\frac{1}{5}$	2	14	$4\frac{4}{5}$
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	$10\frac{2}{7}$
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	$2\frac{2}{3}$
5	0	1	8	$2\frac{1}{2}$	1	7	$2\frac{2}{5}$
5	6	1	4	$8\frac{1}{4}$	1	4	$8\frac{3}{22}$
6	0	0	2	8	1	2	8
6	6	0	19	11	1	0	$11\frac{1}{13}$
7	0	0	19	1	0	19	$5\frac{1}{7}$
7	6	0	18	$1\frac{1}{2}$	0	18	$1\frac{2}{5}$
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	$0\frac{1}{4}$	0	15	$1\frac{1}{3}$
9	6	0	14	$0\frac{3}{4}$	0	14	$3\frac{15}{19}$
10	0	0	13	$7\frac{1}{2}$	0	13	$7\frac{1}{5}$
10	6	0	12	$11\frac{1}{4}$	0	12	$11\frac{3}{4}$
11	0	0	12	$4\frac{1}{4}$	0	12	$4\frac{4}{11}$
11	6	0	11	10	0	11	$9\frac{22}{23}$
12	0	0	11	4	0	11	4

In the common Bookes there is no further rate of Aſſiſe made, then into 12 s the quarter of Wheat, but in an ancient Copy of 200 yeeres old (which I haue) there is added the rate of Aſſiſe vnto 20 s the quarter, but yet was that Aſſiſe alſo either wrong caſt at the firſt penning, or elſe corrupt ſince that time, for lacke of iuſt knowledge in the Rule of Proportion, which I will adde here alſo to gratify ſuch as be deſirous to vnderſtand truth exactly.

The price of a quarter of Wheat.		The waight of a farthing white loafe by the Statute Bookes.			The correction by iuſt Aſſiſe.		
s	d	l	s	d	l	s	d
12	6	0	11	0	0	10	$10\frac{14}{25}$
13	0	0	11	$0\frac{1}{2}$	0	10	$5\frac{7}{13}$
13	6	0	10	$1\frac{1}{2}$	0	10	$0\frac{8}{9}$
14	0	0	9	7	0	9	$8\frac{4}{7}$
14	6	0	9	$2\frac{1}{3}$	0	9	$4\frac{16}{29}$
15	0	0	9	$1\frac{1}{2}$	0	9	$0\frac{4}{5}$
15	6	0	9	$1\frac{1}{4}$	0	8	$9\frac{9}{31}$
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	$2\frac{10}{11}$
17	0	0	8	3	0	8	0
17	6	0	7	10	0	7	$10\frac{9}{35}$
18	0	0	7	6	0	7	$6\frac{2}{3}$
18	6	0	7	3	0	7	$4\frac{8}{37}$
19	0	0	7	2	0	7	$1\frac{17}{19}$
19	6	0	5	10	0	6	$11\frac{9}{13}$
20	0	0	5	6	0	6	$9\frac{3}{5}$

These two Tables I haue set seuerall, because no man should thinke that I would either adde or take away from any Law those parts which might of right seeme either superfluous, either diminute: but yet I may not bee so curious as to neglect manifest errors, which is not onely my part, but euery good Subiects duty with sobriety to correct. And for auoiding of offence, I haue rather done it in this Priuate Booke, then in any Booke of the Statutes it selfe, trusting that all Men will take it in good part.

Scholar. I would wish so, but I dare not so hope, sith neuer good man that would reforme errour, could escape the venimous tongues of enuious detracters, which because they either cannot, or list not to doe any good themselues, doe delight to barke at the doings of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, whereunto I am not vnprouided of fit answers, but I will not seeme so hasty to make the answers, before I heare their obiections, but as I trust that men are of a better nature, and more gratefull now then some haue beene in times past, as I haue done in the Statute of Assise for bread in rate of shillings, so will I set forth the like Table in pounds and ounces, and the parts thereof, that it may be easily applied to all times: but I meane not by this to alter any word of the Statute,

Concerning the following Tables.

Statute, being ſo good an ordinance, and of ſo great continuance) but onely to make it as a kinde of expoſition and declaration of the ſaid Statute, truſting that thereby the Statute may be better vnderſtood, and conſequently better put in execution. And here you ſhall note, that I haue accounted the ſhillings after the rate of 60 ſhillings to the pound weight, becauſe I eſteeme it the moſt apt for our time. Wherefore in the firſt Columne you find the price of Wheat directly againſt it; in the ſecond Columne, you may find the weight of a Farthing white Loafe in this our time: and if you double the number (as I haue done in the third Columne) then haue you the weight of the Halfe-penny White-Loafe; and ſo in the fourth Columne is ſet the weight of a penny white-Loafe. It needeth not to tell that the ſight doth teſtifie, how that euery Columne is parted into three ſmaller pillars, whereof the firſt Columne hath theſe three titles, pounds, ounces, and penny weights. And as in the firſt columne 12 pence make a ſhilling, and 20 ſhillings make a pound, ſo in the other three Columnes 20 pence weight maketh an ounce and 12 ounces doe make a Pound.

A pound weight.

Gentle

GEntle *Reader*, touching the vnderſtanding of the *Table* following, wherein according to our time, *Maſter Record* alloweth 60 *pence* to the *ounce*, and 3 *pound*, or 60 *ſhillings* to the *pound*, and thereupon after the rate o' 60 *ſhillings* to the *pound Troy*, doth he frame or produce this his *Table*, beginning at 3 *ſhillings the quarter*, till he come to 40 *ſhillings* 6 *pence the quarter*. And this his *proportion* (for that he hath not ſet downe any one *Example* to continue the worke) hath beene hard for many to conceiue or comprehend, and therefore the only chiefe cauſe why I haue written this *digreſſion* for the better vnderſtanding of him therein.

The firſt thing therefore that is ſought for in this *Table*, as in the other aforeſaid, is a *Maxime*, grounded vpon the *Statute*, which is this. When the *quarter of Wheat* is ſold for *two ſhillings*, then the *farthing white-loafe* ſhall wey 68 *ſhillings*, whereby *a ſhilling* is meant $\frac{1}{20}$ of a *pound*, and by a *penny* $\frac{1}{20}$ of an *ounce*. Now therefore for a *generall Rule*, to finde what weight the *Farthing-white-loafe* ſhall wey at 3 *ſhillings the quarter*, til you come to 40 *ſhillings* 6 *pence the Quarter*, is thus to be wrought. Comming to the *firſt ground*, and working by the *Backer Rule*, ſay; if *two ſhillings the quarter*, giue, or allow the *Farthing white-loafe* to wey 68 *ſhillings*, what weight ought the *Farthing-white-loafe* to wey at 3 *ſhillings the quarter*? Worke, and you ſhal finde 45 *ſhillings* 4 *pence*,

as

es before in the correction of the *firſt Table* is noted. Then for the *ſecond worke*, ſay by the *Rule of 3 direct*; if 20 *pence* giue one *ounce*, what giueth 45 *ſhillings*, 4 *pence*? Multiply and diuide, and you ſhall finde 544 *ounces*, which 544 *ounces* being multiplyed by 3, for 3 *pounds*, or 60 *ſhillings*, yeeldeth 1632 *ounces*, which diuided by 20, produceth 81 *ounces*, and $\frac{12}{20}$, or rather $\frac{3}{5}$ of an *ounce*, equall vnto 12 *penny weight*, which is *halfe an ounce*, and 2 *penny weight*, and ſo maketh in all 6 *pounds*, 9 $\frac{1}{2}$ *ounces*, and 2 *penny weight*. Now the next way to continue this *Table*, to know the weight of the *Halfe-penny White-loafe*, is thus. Multiply 1632 *ounces* by 2, and it bringeth forth 3264 *ounces*, and diuided by 20, it yeeldeth 163 *ounces* and $\frac{1}{5}$, which is equall to 13 *pounds*, 7 *ounces*, and 4 *penny weight*, as *M. Record his Table noteth*.

Thirdly, for the weight of the Penny White-loafe, multiply 1632 *ounces* by 4, and diuide by 20, and after by 12, as before, and you ſhall find 27 *pounds*, 2 *ounces*, and 8 *penny weight*, &c. This *Method*, or elſe by doubling the *Farthing white-loafe*, for the *weight of the Halfe-penny white loafe*, and ſo doubling the *halfe-penny White-loafe*, for the *weight* of the *Penny White-loafe*, is the order to continue the *Table* to the end thereof.

The price of a quarter of Wheat.

po.	shil.	pc.
0	3	0
0	4	6
0	6	0
0	7	6
0	9	0
0	10	6
0	12	0
0	13	6
0	15	0
0	16	6
0	18	0
0	19	6
1	1	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	1	6

The waight of a Farthing white loafe.

po.	oũ.	penny waight.
6	9	12
6	8	16
6	8	$12\frac{4}{5}$
6	8	$8\frac{2}{7}$
6	8	$8\frac{2}{3}$
6	8	$6\frac{2}{5}$
6	6	$16\frac{8}{11}$
6	6	12
6	6	$11\frac{1}{13}$
6	6	$10\frac{3}{5}$
6	6	$4\frac{4}{5}$
6	6	$1\frac{1}{2}$
6	6	12
6	6	4
6	5	$15\frac{3}{7}$
6	5	$8\frac{4}{11}$
6	5	$3\frac{1}{5}$
6	5	$11\frac{13}{16}$
6	5	$10\frac{14}{23}$
6	5	$5\frac{7}{13}$
6	5	$0\frac{2}{43}$

The price of a Quarter of Wheate.

£	s	d
0	1	0
0	2	0
0	3	0
0	4	0
0	5	0
0	6	0
0	7	0
0	8	0
0	9	0
0	10	0
0	11	0
0	12	0
0	13	0
0	15	0
0	16	0
0	18	0
0	19	0
1	1	0
1	2	0
1	4	0
1	5	0
1	6	0
1	7	0
1	8	0
1	9	0
1	10	0
1	11	0
1	13	0
1	14	0
1	16	0
1	17	0
1	19	0
2	0	0

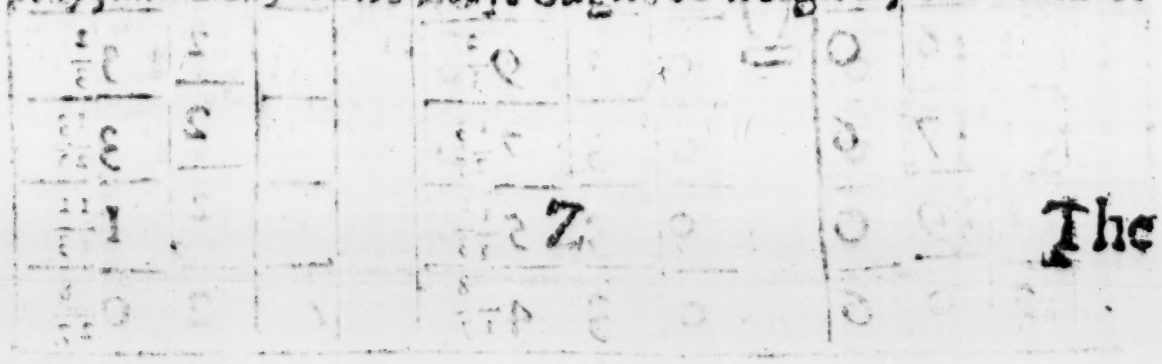
The weight of a Halfe-penny white loafe.

Pe. oũ ces.	Penny waight
13	7 4
9	0 16
6	9 12
5	5 $\frac{3}{5}$
4	6 8
3	10 $12\frac{4}{9}$
3	4 16
2	8 $12\frac{4}{5}$
2	5 $13\frac{5}{11}$
2	3 4
2	2 $\frac{2}{13}$
2	1 $2\frac{2}{13}$
2	0 $9\frac{3}{5}$
1	9 $15\frac{1}{5}$
1	8 8
1	7 4
1	6 $2\frac{2}{3}$
1	5 $3\frac{1}{19}$
1	4 $6\frac{2}{5}$
1	3 $10\frac{6}{7}$
1	2 $16\frac{8}{11}$
1	1 $3\frac{19}{23}$
1	0 $11\frac{1}{13}$
1	1 $1\frac{3}{25}$
1	1 12
1	2 $3\frac{19}{23}$
1	3 $16\frac{8}{11}$
1	4 $6\frac{2}{5}$
1	5 $3\frac{1}{19}$
1	6 $2\frac{2}{3}$
1	7 4
1	8 8
1	9 $15\frac{1}{5}$
2	0 $9\frac{3}{5}$
2	1 $2\frac{2}{13}$
2	2 3 4
2	3 $13\frac{5}{11}$
2	4 $12\frac{4}{5}$
2	5 8
3	0 $5\frac{1}{3}$
3	4 16
3	10 $12\frac{4}{9}$
4	6 8
5	5 $5\frac{3}{5}$
6	9 12
9	0 16
13	7 4

The waight of a Penny white loafe.

Pe.	oũ- ces.	Penny waight.
27	2	8
18	1	12
13	7	4
10	11	$1\frac{1}{5}$
9	0	16
7	$5\frac{1}{2}$	
6	12	$10\frac{2}{5}$
6	9	$5\frac{3}{5}$
4	11	$6\frac{10}{11}$
4	8	
4	6	
4	2	$4\frac{4}{13}$
3	7	$19\frac{1}{5}$
3	4	$10\frac{2}{5}$
3	2	
2	10	$7\frac{3}{19}$
2	8	$12\frac{4}{5}$
2	7	$1\frac{5}{7}$
2	5	$13\frac{5}{11}$
2	4	$7\frac{5}{23}$
2	3	4
2	2	$2\frac{6}{25}$
2	1	$2\frac{2}{13}$
2	0	$0\frac{8}{42}$

HAuing ſpoken before for the vnderſtanding of the *Table* placed by *Al Record*, a man indued with rare knowledge in *Arithmeticall and Geometricall proportions*, touching the *Statute* of *Coynage*, *and the Standard thereof*, as appeareth in his Epiſtle of this *Booke*, *dedicated to King Edward the ſixt*. inſinuating vnto his Highneſſe that the *Standard of Coyne* is much altered from the 14 yeere of *King Edward the third*, (when this *Statute and Aſſiſe* was confirmed) to the *Standard of this our time*. For it appeareth that in *King Edward the thirds* time, when the *Aſſiſe of Bread and Drinke* was eſtabliſhed, that a *Sterling penny*, *round without clipping*, did then weigh 32 *cornes of wheat drie*, and taken out of the middle of the *eare*, and 20 of theſe *pence* made an *ounce*, & 12 *ounces* made a *pound Troy*. And ſo from the *weight of a penny* to 20. *ſhillings ſterling*, which then weighed 12 *ounces*, tooke *Bread his weight* and proportion: And now finding 60 *pence* is an *ounce*. That onely cauſe (I perceiue, for the zeale of a Commonwealth) mooued him to ſet downe the ſame *Table in this priuate Booke*; meaning not thereby to alter any word of the *Statute* being ſo good an ordinance and of ſo long continuance, but as a kinde of expoſition by the way, that thereby the *Statute* may be better vnderſtood, and ſo conſequently better put in execution. Which *Aſſiſe of bis*, is three times greater then the *Statute* now alloweth. Therefore alſo, (to gratifie ſuch as are deſirous of knowledge, according to theſe *prices of a Quarter of wheate*) I haue added to this *Author theſe three other new Tables following*, and reduced their *prices* into their iuſt proportions of *Sterling money*, and alſo reduced the *Money* into knowne *weight Troy*, according to the *Statute*. And thereafter according to proportion in my other three *Tables*, haue I noted the *iuſt weight* that a *Farthing*, *Halfe penny*, *and Penny white-Loafe* ought to weigh by the *Statute*.



The price of a Quarter of Wheate.

l.	s.	d.
0	0	0
0	0	6
0	0	9
0	1	0
0	1	3
0	1	6
0	1	9
0	2	0
0	2	6
0	3	0
0	3	6
0	4	0
0	4	6
0	5	0
0	5	6
0	6	0
0	6	6
0	7	0
0	7	6
0	8	0
0	8	6
0	9	0
0	9	6
0	10	0
0	10	6
0	11	0
0	11	6
1	0	0
1	0	6
1	1	0
1	1	6
1	2	0
1	2	6
1	3	0
1	3	6
1	4	0
1	4	6
1	5	0
1	5	6
1	6	0
1	6	6
1	7	0
1	7	6
1	8	0
1	8	6
1	9	0
1	9	6
2	0	0

The weight of a Farthing white loafe in Sterling money by Assise.

l.	s.	d.	Penny waight.	grains.
0	0	2	4	5
0	0	3	2	18
0	0	4	1	15 3/7
0	0	5	1	1 1/3
0	0	6	0	18 3/5
0	0	7	0	16
0	0	8	0	14 2/5
0	0	9	0	12 4/9
0	0	10	0	11 1/11
0	0	11	0	10 2/3
0	1	0	0	9 3/5
0	1	3	0	8 4/5
0	1	6	0	8 0/9
0	1	9	0	7 3/7
0	2	0	0	7 1/5
0	2	6	0	6 6/11
0	3	0	0	6 2/3
0	3	6	0	6 2/13
0	4	0	0	5 3/5
0	4	6	0	5 1/3
0	5	0	0	5 1/15
0	5	6	0	4 12/17
0	6	0	0	4 4/5
0	6	6	0	4 4/9
0	7	0	0	4 2/21
0	7	6	0	3 18/19
0	8	0	0	3 17/20
0	8	6	0	3 16/21
0	9	0	0	3 15/22
0	9	6	0	3 14/23
0	10	0	0	3 13/24
0	10	6	0	3 12/25
0	11	0	0	3 11/26
0	11	6	0	3 10/27
1	0	0	0	3 9/28

The waight of a Farthing white loafe in Troy weight by Assise.

l.	s.	d.	Penny waight.	grains.
0	0	2	4	5
0	0	3	2	18
0	0	4	1	15 3/7
0	0	5	1	1 1/3
0	0	6	0	18 3/5
0	0	7	0	16
0	0	8	0	14 2/5
0	0	9	0	12 4/9
0	0	10	0	11 1/11
0	0	11	0	10 2/3
0	1	0	0	9 3/5
0	1	3	0	8 4/5
0	1	6	0	8 0/9
0	1	9	0	7 3/7
0	2	0	0	7 1/5
0	2	6	0	6 6/11
0	3	0	0	6 2/3
0	3	6	0	6 2/13
0	4	0	0	5 3/5
0	4	6	0	5 1/3
0	5	0	0	5 1/15
0	5	6	0	4 12/17
0	6	0	0	4 4/5
0	6	6	0	4 4/9
0	7	0	0	4 2/21
0	7	6	0	3 18/19
0	8	0	0	3 17/20
0	8	6	0	3 16/21
0	9	0	0	3 15/22
0	9	6	0	3 14/23
0	10	0	0	3 13/24
0	10	6	0	3 12/25
0	11	0	0	3 11/26
0	11	6	0	3 10/27
1	0	0	0	3 9/28

The price of a quarter of Wheat.

£	s	d
1	19	0
1	17	6
1	16	0
1	14	6
1	13	0
1	11	6
1	10	0
1	8	0
1	7	0
1	5	6
1	4	0
1	2	6
1	1	0
0	19	6
0	18	0
0	16	6
0	15	0
0	13	6
0	12	0
0	10	6
0	9	0
0	7	6
0	6	0
0	4	6
0	3	0

The waight of the Halfe-penny white loafe in Troy whaigt by Afsile.

po. oũ.	penny waight
0	4
0	4
0	4
0	4
0	4
0	4
0	5
0	5
0	5
0	6
0	6
0	6
0	7
0	8
0	8
0	9
1	0
1	1
1	3
1	6
1	9
2	3
3	0
4	0
6	0
8	0

The waight of a penny white loafe in Troy waight by Afsile.

po. ofi.	penny ces.	waighr.
0	8	0
0	9	0
0	10	0
0	11	0
0	12	0
0	13	0
0	14	0
0	15	0
0	16	0
0	17	6
0	18	0
0	19	0
1	0	6
1	1	0
1	2	0
1	3	0
1	4	0
1	5	0
1	6	0
1	7	0
1	8	0
1	9	0
1	10	0
1	11	0
1	12	0
1	13	0
1	14	0
1	15	0
1	16	0
2	0	0
2	3	0
2	7	0
3	0	0
3	7	0
4	6	0
6	0	0
7	0	0

Scholar

Scholar. Sir, I doe thanke you moſt heartily for this, not onely in mine owne name, and in the name of all Students, but alſo in the name of the whole Commons, to whom the reſtitution of this Aſſiſe (I truſt) ſhall bring reſtitution of the weight in Bread, which long time hath béene abuſed. And if you know any like things more, wherein you would vouchſafe to declare the errours, and ſet forth the truth, you cannot but obtaine great thankes of all good hearted men that loue the Common-wealth.

Maſter. I haue ſundry things to declare, but I haue reſerued them for a priuate Booke by it ſelfe, yet notwithſtanding becauſe the Statute of the rate of meaſuring of ground is ſo common, that it toucheth all men, and yet no more common then needfull, but ſo much corrupt, that is, too farre out of all good rate, not onely in the Engliſh Bookes of Statutes, commonly Printed, but alſo in the Latine Books, and in the French alſo, (for I haue read of each ſort, and conferred them diligently) I will giue you a Table for the reſtitution of thoſe errours, as may ſuffice for this preſent time. And firſt I will propoſe one queſtion to you touching the vſe of that Statute, whereby you may perceiue the order how to examine the whole Statute, and euery parcell thereof, and the queſtion is this.

A queſtion of meaſuring of ground.

When the Acre of ground doth containe foure Perches in breadth, then muſt it containe 40 Perches

ches in length. Then doe I demand of you, how much shall the length of an Acre be, when there is in the breadth of it 13 Perches. But before you shall answer to this question, I will declare vnto you another Statute, which is the ground of the former Statute. And that Statute is this.

It is ordained, that three Barly-cornes dry and round, shall make vp the measure of an inch: 12 inches shall make a foot, and 3 foot shall make a yard, (the common English bookes haue an Elne) fiue yards and a halfe shall make a Perch, and 40 Perches in length, and 4 in breadth, shall make an Acre. This is that Statute, whereby you may perceiue, that the intent of the Statute is, that one Acre should containe 160 square Perches. Now let me heare you answer to the question. Statute measure. An Acre.

Scholar. **As I perceiue by the words of the Statute, a Perch to be the $\frac{1}{160}$ part of an Acre, so could I make these numbers all in fractions, and so worke the question: but seeing I may doe it also in whole numbers, I take that forme for the most ease; therefore thus I set the question in forme. Then doe I multiply 40 by 4, and it maketh 160, which I diuide by 13, and the quotient is $12\frac{4}{13}$.**

4 — 40
13 — $12\frac{4}{13}$

Master. **Now turne that $\frac{4}{13}$ into the common parts of a Perch, as they be named in the former Statute: howbeit it shall be best to take one of the least parts in Denomination**

for auoiding of much labour, as Feete, whereof the Perch containeth $16\frac{1}{2}$.

Scholar. Then to returne $\frac{4}{13}$ into Feete, I multiply $16\frac{1}{2}$ by 4, and it maketh 66, which I muſt diuide by 13, and the quotient is $5\frac{1}{13}$.

Maſter. So I find, that if the Acre hold in bredth 13 Perches, it ſhall containe in length 12 Perches, 5 Foot, and $\frac{1}{13}$ of a Foot, which is not fully an Inch, for the Inch is $\frac{1}{12}$ of a Foote. But heere all the Statute Bookes in Latine and Engliſh (that I haue ſéene) doe note it to be 13 Perches, 5 Foote, and one Inch, which maketh aboue 13 Perches too many in the Acre: ſo that I would haue thought the errour to haue crept into the Printed Bookes, by the great negligence that Printers in our time doe vſe, ſaue that in written Copies of great antiquity, I doe finde the ſame: yet haue I one French copy which hath 12 Perches $\frac{1}{4}$ and one Foot, and that miſſeth very little of the truth.

Note this errour.

Scholar. Then I ſée it is true that I haue often heard ſay, that the trueſt Copies of the Statutes, be the French Copies.

Maſter. That is often true, but not generally, as I haue by conference tried diuerſly: but in this Statute the French Booke is moſt corrupt: in all other places lightly.

But now to performe my promiſe, I will ſet forth the Table for meaſuring of an Acre of ground, onely by ſuch parts as the Statute doth mention, becauſe at this time I doe of purpoſe

purpose write it for the better vnderstanding of that Statute, and hereafter with other things intend to set forth this same more at large.

In this Table following, I haue not done as in the other Statute before compared by restitution with the faults crept into the Statute, but onely haue written that true measure, which the equity of the Statute doth pretend. For it were vile to iudge of so noble Princes and worthy Counsellors, as haue authorised and set forth this Statute, that they would make one Acre in any forme greater then another, but euery one to be iust and equall with each other, which is the ground also of my worke: and hereby may all men perceiue how needfull Arithmeticke is to the Students of Law. But now I thinke best to make an end of these matters for this present time, sith the Table hath in it none obscurity that I should need to declare.

The breadth of the Acre	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an Inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{9}{17}$
18	8	14	8	0
19	8	6	11	$\frac{7}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{9}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	7	$\frac{6}{13}$
27	5	15	3	$\frac{1}{5}$

The breadth of the Acre	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an Inch.
28	5	11	9	$0\frac{3}{7}$
29	5	8	6	$0\frac{12}{29}$
30	5	5	6	0
31	5	2	7	$0\frac{29}{31}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$0\frac{13}{17}$
35	4	9	5	$0\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$0\frac{8}{37}$
38	4	3	5	$0\frac{13}{19}$
39	4	1	8	$0\frac{4}{13}$
40	4	0	0	0
41	3	14	10	$0\frac{28}{41}$
42	3	13	4	$0\frac{2}{7}$
43	3	11	10	$0\frac{32}{43}$
44	3	10	6	0
45	3	9	2	0

Scholar.

Scholar. Indeed Sir, I vnderstand the Table (as I thinke) by those other which you set forth before. For in the first columne is set the Perches of the breadth of an Acre, and then in the two columnes following appeareth how many Perches, and how many foote that same Acre must haue for his length.

Master. You take it well: howbeit to speake exactly of breadth and length, and the first columne doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limit his length, and the shorter side doth betoken the breadth, yet it is no great abuse in such Tables, where a man cannot well change the title, to let the name remaine, although the proportions of the numbers doe change: for still by the first columne is expressed the measure of the one side, and by the two other Pillars in one columne, is set forth the measure of the other side. And this shall be sufficient now for the vse of the Golden Rule.

The Rule of Fellowship.

NOw somewhat will I touch certaine other Rules, which for their seuerall names may seeme diuers Rules, and distinct from this, but indeed they are but branches of it: yet because they haue seueral workings in appearance, but also

alſo pleaſant in vſe, I will giue you a taſte of each of them. As for the Rule of Fellowſhip, both ſingle and double, with time and without time, I ſhall need to ſay little more then I haue already ſaid in teaching the workes of whole numbers: yet an example or two will wee haue to refreſh the remembrance of the ſame, and to declare certaine proper vſes and applications of it, as this for one.

The Rule of Fellowſhip without time.

A queſtion of inequall ſociety.

Foure men got a booty or prize in time of warre, the prize is in value of money 8190 pound, and becauſe the men be not of like degree, therfore their ſhares may not bee equall; but the chiefeſt perſon will haue of the booty the third part, and the tenth part ouer: the ſecond will haue a quarter, and the tenth part ouer; the third will haue the ſixt part: and ſo there is left for the fourth man a very ſmall portion, but ſuch is his lot (whether he be pleaſed or wroth) he muſt be content with one 20 part of the prey: Now I demand of you what ſhall euery man haue to his ſhare?

Scholar. You muſt bée faine to anſwer to your owne queſtion, elſe it is not like to be anſwered at this time.

Maſter. The ſoyme to vnderſtand the ſolution of this queſtion, and all ſuch like, is this: Reduce all the Denominators into one number by Multiplication, except that any of them be parts of ſome other of them, for all ſuch parts you may ouerpaſſe, and take for them all thoſe numbers, whoſe parts they be: As in this example the ſhares be theſe $\frac{1}{3}\ \frac{1}{10}; \frac{1}{4}\ \frac{1}{10}; \frac{1}{6}\ \frac{1}{10}$, if

I multiply all the denominators together, beginning with 3, and so goe on vnto 20, it will make 144000: but considering that 3 is a part of 6, I will omit that 3, and likewise 10, which is a part of 20, I may ouerpasse also, and then is there but 3 denominators to multiply, that is, 4, 6, & 20 which make 480, which summe I take for my worke, because all the denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{1}{3}$ and $\frac{1}{10}$, the $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48, which I put in one summe for the first mans share, and it maketh 208. Then for the second mans share, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must haue $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole summe: so that if the whole prey had béene but 480 pound, then were the question answered: but because the summe was of greater value, by this meanes now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number of that I found by multiplying the Denominators, and in the second place the summe of the booty, And looke what proportion is betweene the first number and the second, the same proportion shall be betweene the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the third

The reason of this rule.

third place, one of the parts or ſhares, and then worke by the former Rule of proportion or Golden Rule. And becauſe I haue foure ſeuerall parts of the firſt number, by which I would finde out foure like parts of the ſecond number, therefore muſt I make foure ſeuerall figures.

Scholar. Now I truſt I can anſwer to your queſtion, as by your fauour I will proue:

And to trie it, I ſet the foure figures thus, marked with A, B, C, D, to ſhew their order,

A

480 — 8190
208

B

480 — 8190
160

C

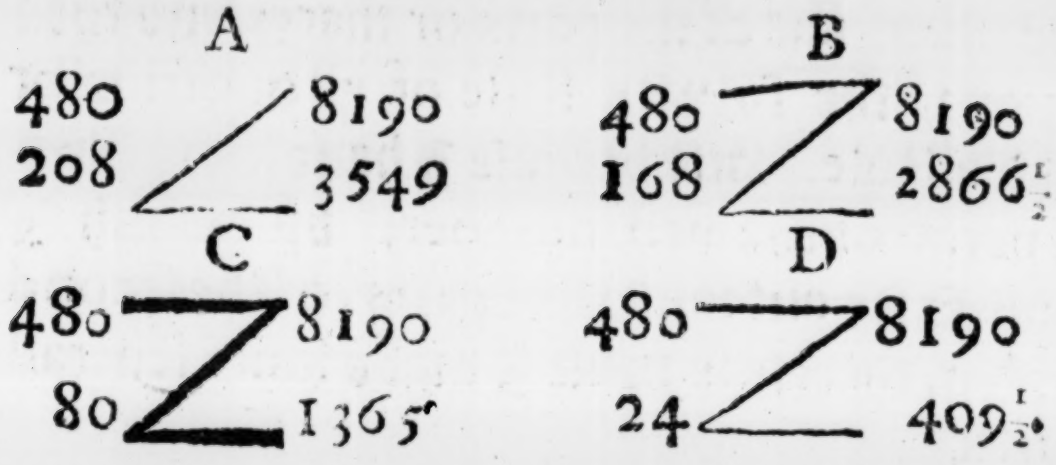
480 — 8190
80

D

480 — 8190
24

And then in each of them I multiply the ſecond number by the third, and diuide their totall by the firſt, and ſo amounteth the fourth ſumme which I ſeeke for: For if I doe multiply 8190 by 208, it maketh 1703520, which being diuided by 480, maketh in the quotient 3549 for the firſt mans portion.

And ſo working with the other three figures, I finde for the ſecond man $2866\frac{1}{2}$, and for the third man 1365, and then for the fourth man $409\frac{1}{2}$, and ſo euery mans ſhare is ſet forth in the figure here annexed.



The proof by Addition.

And thus I thinke I haue done well.

Maſter. If you miſdoubt your working, and liſt to proue it, adde all the ſhares togeſher; and if they make the totall, then ſeemeth it well done.

Scholar. I may ſet them thus: and then by Addition the laſt ſumme doth amount, that is, 8190, and therefore (as you ſay) it ſæmeth to be well wrought.

	3549
	2866 1/2
	1365
	409 1/2
	8190

But I beſæch you, is there any doubt in this triall, that you vſe that word, Sæmeth?

Maſter. You may eaſily conjecture, that if you did aſſigne the firſt mans ſhare to the laſt, and ſo change all the reſt, and one had anothers ſhare, yet would the Addition appeare all one, and therefore is not the proofe exact.

But if you will make a iuſt proofe for the firſt mans part, take 1/3 and 1/10 of the whole ſumme, and if it agrée with the number in the figure, then it is well done. And ſo doe for the ſecond, third, and fourth ſummes, and this proofe faileth not. Now will I propound certaine other queſtions, which haue bæne ſet forth

The iuſt proofe.

forth by certaine learned men, albeit not without some ouersight, which questions I protest heartily, I doe not repeat to deprаue those good men, whose labours and studies I much praise, and greatly delight in. But only according to my profession, to seeke out truth in all things, and to remoue all occasions of errour as much as in me lieth: and for that cause I will onely name the questions without hurting the Authours name.

The first question is this.

A question of building.

Foure men did build an house, which cost them 3000 crownes, their shares were such that one man should pay $\frac{1}{2}$ of the summe, and sixe crownes ouer: the second should pay $\frac{1}{3}$ and 12 crownes ouer: the the third man must lay out $\frac{2}{3}$ abating 8 crownes: and the fourth man should pay $\frac{1}{4}$ and 20 crownes more. Can you answere to this question?

Scholar. No, I cannot sir, and that you know best of any man, for I know no more then you haue taught me.

An impossible question.

Master. Then I dare say you cannot doe it, neither yet the best learned man that euer did propose it: for the question is impossible. For declaration whereof, I will be bold to vse first the representation of the numbers in their aptest forme (although I haue not yet taught that manner of worke) because it may appeare plainely that the question is not possible. For here I haue set the parts, and added them, and they make the whole summe, and $\frac{3}{4}$ and 30 more. Now, how is it possible

to

to diuide truely either gaines, either charges, so that the particulars $1\frac{1}{4}$ shall be more then the totall?

$\left\{\begin{array}{rcl} \frac{1}{2} & + & 6 \\ \frac{1}{3} & + & 12 \\ \frac{2}{3} & - & 8 \end{array}\right\}$ $\left\{\begin{array}{rcl} \frac{1}{4} & + & 20 \end{array}\right\}$

$1\frac{3}{4} + 30$

Scholar. It is against the forme of proofe by addition of parts.

Master. You say truth. And (because you shall perceiue it the better) I will trie it after the vulgar forme, as in this figure you see, where the $\frac{1}{2}$ with 6 ouer, is 1506, for the totall as you heard before, is 3000, the $\frac{1}{3}$ and the 12 more is 1012: the $\frac{2}{3}$ would be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all being added in one summe, doe make 5280, where the totall summe should bee but 3000, which summe of 3000 if you diuide by $\frac{3}{4}$, you shall haue $\frac{3}{4}$ of it, that is 2250, and thereto adde 30 more, then will those 3 summes make 5280: whereby you may see how this forme (as well as the other) doth declare that the particulars in that question would make more then the whole summe by $\frac{3}{4}$, and 30 more, and therefore can that question not be accepted as a possible thing, but yet doe certaine learned men propound such questions, and answere to them:

	1506
	1012
	1992
	770
	5280

	2000
	2250
	5280

There-

Therefore somewhat to say to their excuse (rather of their good meaning, then for their doing) I will anone declare what may be said for their defence: but in the meane season, I will propound the question as it may be wrought by good possibilitie.

As if foure men build a house together, and it cost them 3000 crownes, and then for the partition they agree thus: that as often as the first man doth pay 6 crownes so often the second man shall pay 4, the third man 8, and the fourth man 3. Or else thus: that the first man shall pay double so much as the fourth, and the second man shall pay $\frac{1}{3}$ of the first mans charge: the third man shall pay double so much as the second: (and these two waies are to one end:) but further for their agreement it is appointed also, that the first shall giue 6 crownes ouerplus, and the second 12, and the fourth shall giue 20: but the third man shall giue no ouerplus, but shall haue 8 crownes abated of his charge.

The forquestion of building now possible.

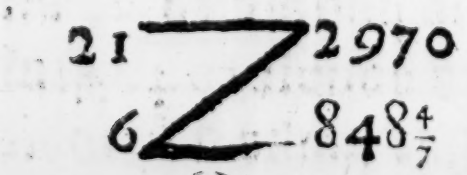
Now is the question possible to be assoiled, and this the way to doe it. Marke the proportion of the seuerall charges, and set out small numbers in that rate, by which you may reduce the worke to the Golden Rule, as here in the first forme, the numbers are already named, 6,4,8,3: and in the second forme (although they be but plainely named) yet they may be the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Now adde these together, and they make 21, which 21 must be set for the first number in the Gol-

den Rule: for if it, with the ouerplus of each mans charge would make the totall summe of the charges, then were those seuerall summes the charges of each man, besides his ouerplus: but now it is not so.

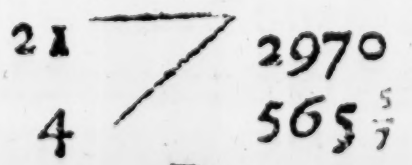
The Rule.

But yet this is true, (so excellent are conclusions Arithmetical,) that loke what proportion each of their seuerall summes doth beare to 21, the same proportion doth the iust charges of euery man (besides his ouerplus) beare to the totall of the charges, the ouerplus being deducted: wherefore this may you note, that before you doe apply the totall of his charges to the Golden rule, you must deduct the ouerplus, which is 6, 12 and 20, that is in the whole 38: but then 8 must be restored for the abatement of the third man, and then remaineth to be deducted 30: take 30 therefore out of 3000, and there will rest 2970, which I must set in the Golden Rule for the second summe; & for the third summe I must put each of the small numbers before mentioned, which although they bee not seuerall charges, yet they represent them in proportiõ. And so making fore euery mans charge a seuerall question, the figures will be 4 which I marke with foure letters, A, B, C, D, thus.

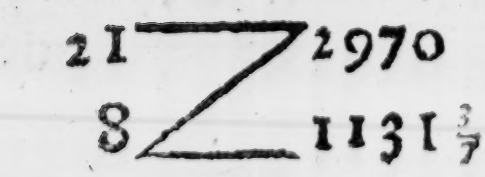
A



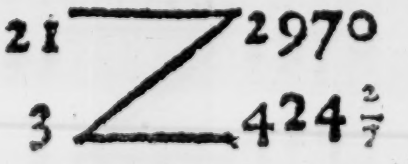
B



C



D



Where I haue ſet for briefeneſſe the ſumme of euery mans charge in the fourth place, preſuppoſing that you can tell how to trie out that fourth ſumme by ſo many examples as ye haue had.

Scholar. As I truſt that I vnderſtand this forme, ſo I deſire much to know what may be ſaid for them that miſtake this queſtion.

Maſter. You ſeeme ſo deſirous to know this errour, that haue forgotten to examine, whether this worke be without fault.

Scholar. Me ſeemeth this worke to be well done, becauſe the Addition of the foure ſeuerall numbers doth make the totall ſumme of 2970, which was to be diuided into ſuch foure parts.

Maſter. But then haue you forgotten that the firſt man muſt pay ſixe Crownes more beſides his ſhare, and the ſecond man 12 Crownes more, the third man 8 Crownes leſſe, and the fourth man 20 crownes more: for without theſe, your firſt totall of 3000 Crownes will not be made.

Scholar. Then muſt I adde to the firſt mans ſumme 6 more, and it will be $854\frac{4}{7}$, and to the ſeconds ſumme, I muſt adde 12, and it will be $577\frac{5}{7}$: from the thirds ſumme I muſt abate 8, and then will the ſumme be $1123\frac{3}{7}$: then adding vnto the fourths ſumme

20, it will be 444 $\frac{2}{7}$, and these foure summes will make 3000, which is the whole charge, as in this example it may appeare, where first I gather the $\frac{14}{7}$, that maketh 2, and so proceed I in the Addition to the end

854	$\frac{4}{7}$
577	$\frac{5}{7}$
1123	$\frac{3}{7}$
444	$\frac{2}{7}$
3000	

Master. Now haue you well done, and this worke in the same summes is brought of other learned men for the true solution of the question, as it was first proponed, which (as I said) was impossible: and now examine if both agrée with the summes in the question proponed.

The first man must pay $\frac{1}{2}$ and 6 ouer of the totall summe: how thinke you, is 854 $\frac{4}{7}$ the halfe, and 6 more of 3000?

Scholar. No that it is not, for it should be 1505: and for the second man 1012: and for the third man 1992, and for the fourth man 770: whereof not one summe agréeth to this worke. But I marueil, that so wise men could be so much ouer-séene.

Maister. It is commonly séene, that when men will receiue things from elder Writers, and will not examine the thing, they séeme rather willing to erre with their Ancients for company, then to be bold to examine their workes or writings. Which scrupulositty hath ingendred infinite errors in all kinds of knowledge, and in all ciuill administration, and in euery kinde of Art. But these

these Learned men did not meane any o-ther thing by this question, then to finde such numbers as should beare the same proportion together, as those numbers in the question proponed did beare one to another: which thing you shall perceiue more plainely by another question of theirs, that is this.

A question of a Testament.

A man lying vpon his death-bed bequeathed his goods, (which were worth 3600 Crownes) in this sort. Because his wife was great with child, and hee yet vncertaine whether the childe were male or female, he made his bequest conditionally, that if the Wife bare a Daughter, then should the Wife haue baife his goods, and the Daughter $\frac{1}{3}$, *but if shee were deliuered of a Sonne, then that Sonne should haue* $\frac{1}{2}$ *of the goods, and his Wife but* $\frac{1}{3}$. *Now it chanced her to bring forth both a Sonne and a Daughter; the question is: How shall they part the goods agreeable to the Testator his Will?*

Scholar. If some cunning Lawyers had this matter in framing, they would determine this Testament to be quite voide, and so the Man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it: and also, it might haue chanced, that shee should haue brought forth neither Sonne nor Daughter, as often hath beene seene; so is the Will insufficient in that point also.

Maſter. Such Scanners ſhould ſéeme too cunning, and yet not ſo cunning as cruell: for the minde of the Teſtator is to be taken fauorably for the aide of the Lygatories, when there ariſeth ſuch doubt. But let vs try this worke, not by force of Law, but by proportion Geometricall, ſéeing the Teſtator did minde to prouide for each ſort of them.

Scholar. If the Sonne ſhall haue $\frac{1}{2}$ by force of the Teſtament, ſo muſt the mother haue $\frac{1}{3}$. Againe, becauſe ſhée hath a Daughter alſo, therefore ought ſhée to haue $\frac{1}{2}$, and the daughter $\frac{1}{3}$, that is both wayes $\frac{1}{3}$ $\frac{1}{3}$, and $\frac{1}{3}$ $\frac{1}{24}$, which commeth to the whole goods, and $\frac{2}{3}$ more.

Wherefore it ſéemeth alſo impoſſible.

Maſter. In this matter, the mind of the Teſtator is ſo to be vnderſtood, that ſuch proportion ſhould be betwéene the portion of the wife and the ſonne as is betwéene $\frac{1}{3}$ and $\frac{1}{2}$, that is, the ſonne muſt haue $\frac{1}{6}$ for $\frac{2}{6}$ to his Mother, ſo ſhall he haue 3 to 2, that is, as much as his Mother, and halfe as much more; and the mother muſt haue the like rate in compariſon to her daughter. Then muſt I find out three numbers in ſuch proportion, that the firſt may as much as the ſecond, and halfe as much more, (that is) in proportion ſeſquialtera; and the ſecond to the third, in that ſame proportion: ſuch numbers be 9, 6, 4.

Scholar. I pray you Sir, how ſhall I find out thoſe numbers?

Maſter. That will I gladly ſhew you.

Whatſoeuer

To finde three numbers in any proportion.

Whatsoeuer the proportion be of any three numbers, multiply the Terms of that proportion together, and the number that amounteth, shall be the middle number of the 3 : then multiply that middle number by the lesser terme, and diuide that totall by the greater, and the least number of the three will amount. So if you multiply that middle number by the greater extreame, and diuide the totall by the lesser extreame, then will the greatest number of that Progression amount.

To finde the proportion betweene two numbers.

Scholar. Then in this example to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must diuide (as you taught me in diuision) $\frac{1}{2}$ by $\frac{1}{3}$, and the quotient will be $\frac{3}{2}$, that is, $1\frac{1}{2}$. Whereby I perceiue that the proportion in this question, is as 3 to 2. Therefore as you taught mee euen now, I multiply 3 by 2, and the summe is, 6, which must be the middle number: then I multiply the middle number 6 by 2, which is the least terme, and the summe is 12, that I doe diuide by 3, being the greater terme, and the quotient is 4: so is 4 the least number of the three. Then I multiply 6 by 3, whereof commeth 18, and that I diuide by 2, and so haue I 9, which is the greatest number of the three.

Master. Another way yet may you finde the third number in any Progression, if you haue two of them: for if the middle number be one of them which you haue, then mul-

ply it by it ſelfe (as in this example, 6 by 6, maketh 36) and that totall diuide by the other number which you haue, and the third number will be the quotient.

Scholar. When I diuide 36 (which commeth of 6 multiplied by it ſelfe) by 4, the quotient will be 9: and if I diuide 36 by 9, the quotient will be 4. But what if I know the firſt number and the third, and would haue the middle number?

Maſter. Multiply the two numbers together, and in their totall you muſt ſéeke the root of that number, and it ſhall be the middle number: but becauſe as yet you haue not learned how to extract Roots, therefore vſe the firſt forme which I haue taught you, til I teach you to extract Roots. And now goe forwards with the anſwer to the ſame queſtion.

Note.

Scholar. I perceiue then that the Sonne muſt not haue $\frac{1}{2}$ of the goods, neither the Mother $\frac{1}{3}$, nor yet the Daughter $\frac{1}{3}$, but yet muſt the goods be diuided into ſuch proportion, that the Sonne ſhall haue 9 Crownes for 6 to his mother, and the mother ſhall haue 6 Crownes for euery 4 to her daughter. Then I apply it to the Golden Rule in three examples, as followeth.

Where the firſt number is the Addition of thoſe three numbers 9,6,4: and the third is one of them ſeuerally: the ſecond is the total of the goods in that Teſtament: and then by

by the worke of the Golden Rule, I finde out the fourth number in euery worke: that is, for the Sonne $1705\frac{5}{19}$, for the mother $1136\frac{16}{19}$, and for the daughter $757\frac{17}{19}$: the which summmes added together, doe make the summe of the whole goods as may be seene by this Example.

19 | 3600
9

19 | 3600
6

19 | 3600
4

$1705\frac{5}{19}$
$1136\frac{16}{19}$
$757\frac{17}{19}$
3600

And this (me thinketh) I doe perceiue, because in this case there is a necessary remedy deuised against an vrgent inconuenience: therefore those Learned men thought they might vse the like liberty in that other question.

Master. Your grosse is good, but they had so good reason for them in the one, as they haue in the other: As in another example of theirs, it may better appeare, as in this:

A man left vnto his three Sonnes 7851 crownes to be parted in such sort, that the first Sonne should haue $\frac{1}{2}$, the second sonne $\frac{1}{3}$, and the third sonne $\frac{1}{4}$, which is not possible: for $\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{1}{4}$ doe make $\frac{26}{24}$, or $\frac{13}{12}$; that is, $1\frac{1}{12}$, so is it more then the whole, but reduce these Fractions into one denomination, the least that they will come to, and they will be

Another question of a Testament.

be $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, *and ſo may you part the goods into ſuch proportion as theſe three numerators beare together, that is, the firſt to haue 6 for euery 4 to the ſecond; and the ſecond to haue 4 as often as the third hath 3; and ſo their portions will be for the firſt,* 3623 $\frac{7}{13}$, *for the ſecond* 2415 $\frac{9}{13}$, *and for the third* 1811 $\frac{10}{13}$; *and theſe three ſhares added together, will make the totall ſumme of the whole goods, as you may eaſily ſee in this example.*

	3623 $\frac{7}{13}$
	2415 $\frac{9}{13}$
	1811 $\frac{10}{13}$
	7851

Another queſtion is there propoſed thus.

Another like queſtion.

There are 450 *crownes to be diuided between three men; ſo that the firſt man muſt haue* $\frac{1}{2}$ *and* $\frac{1}{3}$, *the ſecond man* $\frac{1}{3}$ *and* $\frac{1}{4}$; *the third man ſhall haue* $\frac{1}{4}$ *and* $\frac{1}{5}$.

Scholar. I marueil that any man ſhould be ſo ouer ſæne, to propone that queſtion as a thing poſſible, ſith $\frac{1}{2}$ $\frac{1}{3}$, $\frac{1}{3}$ $\frac{1}{4}$, $\frac{1}{4}$ $\frac{1}{5}$, doe make 1 $\frac{13}{11}$ that is almoſt double the whole ſumme.

☞ But I perceiue it might be thus propoſed: that as often as the firſt man did receiue 50 crownes, ſo often the ſecond man ſhould receiue 35, and the third man 27: for $\frac{1}{2}$ and $\frac{1}{3}$ is equall to $\frac{50}{60}$: and ſo is $\frac{1}{3}$ and $\frac{1}{4}$ equall to $\frac{35}{60}$, and $\frac{1}{4}$ and $\frac{1}{5}$ is $\frac{27}{60}$: and ſo working the queſtion, the three figures will appeare in this forme: whereby the firſt mans portion is found to be 200 $\frac{50}{56}$: the ſecond mans part is

112 — 450
50 — 200 $\frac{50}{56}$

112 — 450
35 — 140 $\frac{35}{56}$

112 — 450
27 — 108 $\frac{17}{56}$

140 $\frac{35}{56}$

140 $\frac{35}{56}$: the third mans share 108 $\frac{27}{56}$: which in the whole both make 450 Crownes to be diuided betweene them.

Maſter. And thus you are (I thinke) ſufficiently inſtructed in the Rule of Fellowſhip.

The Rule, of Alligation.

NOw will I goe in hand with the Rule of Alligation: which hath his name for that by it there are diuers parcels of ſundry prices, and ſundry quantities alligate, bound, or mixed together: wherby alſo it might be well called the Rule of Mixture; and it hath great vſe in compoſition of Medicines, and alſo in mixtures of Metals, and ſome vſe it hath in mixtures of Wines: but I wiſh it were leſſe vſed therein than it is now adayes. The order of this rule is this.

The Rule of Mixture.

When any ſummes are propoſed to be mixed, ſet them in order one ouer another, & the common number (wherevnto you will reduce them) ſet on the left hand; then marke what ſummes be leſſer then that common number, and which be greater, and with a draught of your Penne euermore linke two numbers together, ſo that one be leſſer then the

The reaſon of this Rule.

the common number, and the other greater then hée: (for two greater or two ſmaller can not well be linked together:) and the reaſon is this, that one greater and one ſmaller, may be ſo mixed that they will make the meane or common number very well: but two leſſe can neuer make ſo many as the common number, being taken orderly: no more can two ſummes greater then the meane, euer make the meane in due order, as it ſhall appeare better to you hereafter. And, as it is of neceſſity to linke euery ſmaller, (once at the leaſt) with one greater, and euery greater with one ſmaller, ſo it is at liberty to linke them oftner then once: and ſo may there be to one queſtion, many ſolutions. When you haue ſo linked them, then marke how much each of the leſſer numbers is ſmaller then the meane or common number, and that difference ſet againſt the greater numbers which bée linked with thoſe ſmaller, each with his match ſtill on the right hand, and likewiſe the exceſſe of the greater numbers aboue the meane, you ſhall ſet before the leſſer numbers, which bée combined with them. Then ſhall you (by Addition) bring all theſe differences into one ſumme, which ſhall be the firſt number in the Golden rule, and the ſecond number ſhall bée the whole maſſe that you will haue of all thoſe particulars: the third ſumme ſhall bée each difference by it ſelfe, and then by them ſhall be found the fourth number, declaring the iuſt por-

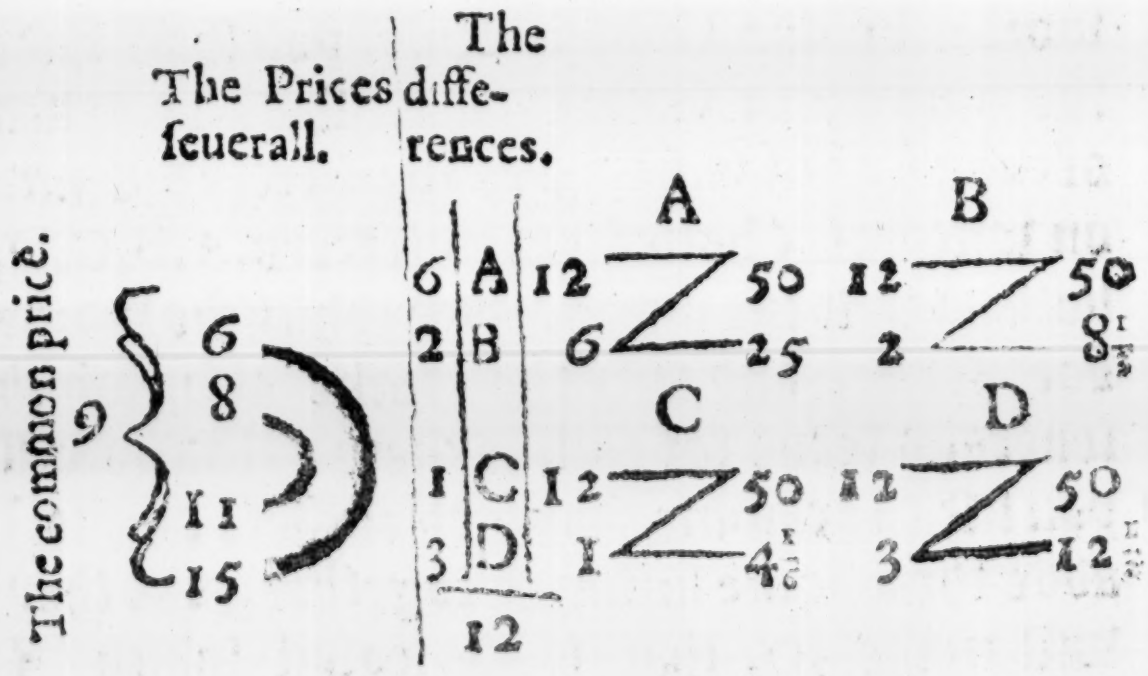
portion of euery particular in that mixture: As now by these examples I will make it plaine,

A question of mixing of wines.

There are foure sorts of Wine, of seuerall prices. one of 6 pence, *a gallon, another of* 8 pence, *the third of* 11 pence, *and the fourth of* 15 pence, *the gallon. Of all these wines would I haue a mixture made, to the summe of fiftie gallons, and so the price of each gallon may be* 9 pence. *Now demaund I, how much must bee taken of euery sort of Wine?*

Scholar. If it shall please you to worke the first example, that I may marke the applying of it to the rule: then I trust I shall bee able, not onely to doe the like but also to see the reason in the order of the worke.

Master. Marke then this forme, and the placing euery kinds of number in it.



Here (you see) I haue set downe the seuerall

rall prices which be 6, 8, 11, 15, and haue linked together 6 with 15, and 8 with 11. The common price 9, I haue set on the left side, and the difference betweene it, and euery particular price, I haue set on the right hand not against the summe (whose difference it is) but against the summe that it is linked withall, so the difference of 15 aboue 9, is 6, which I haue set, not against 15 but 6, that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue set against 15. So likewise the difference betweene 8 & 9, is but 1, that I haue set against 11, and the difference of 11 aboue 9 (which is 2) I haue set against 8. Then adde I all those foure differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of Gallons that I should haue, and the third summe is euery particular difference. Now if you worke by the Golden Rule, you shall finde the number of Gallons that shall be taken of each sort of wine: For the better distinction whereof, I haue set these letters, A, B, C, D, both against the numbers for which the works doe serue, and ouer the worke also, which seuerally serue for each of them. And now (if you list to examine the truth of these works,) adde these foure summes together, and they will make 50, that is the totall which I

The proof of this Rule.

would

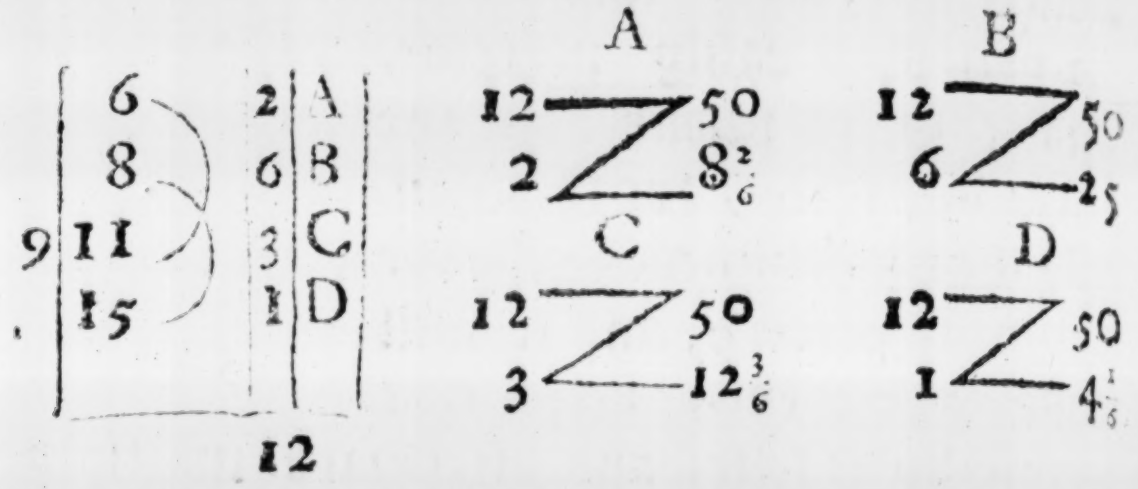
would haue, as by this example you may easily perceiue. And (for to proue how the prices doe agree) doe this: multiply the totall summe 50, by the common price 9, and it will make 450: then keepe that summe by it selfe, and afterward multiply euery seuerall summe of gallons, by the price belonging to the same gallons; & if that summe doe agree with this which you haue kept first, then is your worke well done. As here 25 is the number of gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you shall set downe, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of gallons, and it will make $66\frac{4}{6}$: so againe $4\frac{1}{6}$ multiplied by 11: doth make $45\frac{5}{6}$. And last of all, $12\frac{1}{3}$ multiplied by 15, maketh $187\frac{3}{6}$: and these added together do make 450, as in the example annexed you may see, wherefore seeing it doth agree with the former summe of 50, multiplied by 9, I may iustly affirme this worke to b. good, and well done.

25	
8	$\frac{1}{3}$
4	$\frac{1}{6}$
12	$\frac{1}{3}$
50	
150	
66	$\frac{4}{6}$
45	$\frac{5}{6}$
187	$\frac{3}{6}$
450	

And now to proue how you can doe the like I propound the same question, onely willing you to vse some other forme of combining or linking the summes.

The variation of this question.

Scholar. That shall I proue with your fauour, & therefore I combine 8 with 15 and 6 with 11, and then the forme will be as followeth.



Whereby amounteth the same summe in totall of the differences as did before: and yet now the differences be altered as the combination is changed, whereof I understand the reason by your former worke. And therefore here appeareth no strange thing, but that now I have $8\frac{2}{6}$ gallons of 6 pence, and 25 gallons of 8 pence, & 12 gallons and $\frac{3}{6}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{6}$ of 15 pence: so that multiplying $8\frac{1}{6}$ by 6, it maketh 50, and then 25 multiplyed by 8. maketh 200: likewise $12\frac{3}{6}$ multiplied by 11 yeld $137\frac{3}{6}$, and $4\frac{1}{6}$ multiplied by 15, maketh $62\frac{3}{6}$, which 4 summes added into one, will yeld in the total 450, which agreeth with the multiplication of 50, (being the total summe of gallons) by 9 the common or meane price.

	50
	200
	$137\frac{3}{6}$
	$62\frac{1}{2}$
	450.

Master, Seeing you conceive this worke so well, I will propound another example vnto you of more varietie, in the Alligations or combinings, as thus:

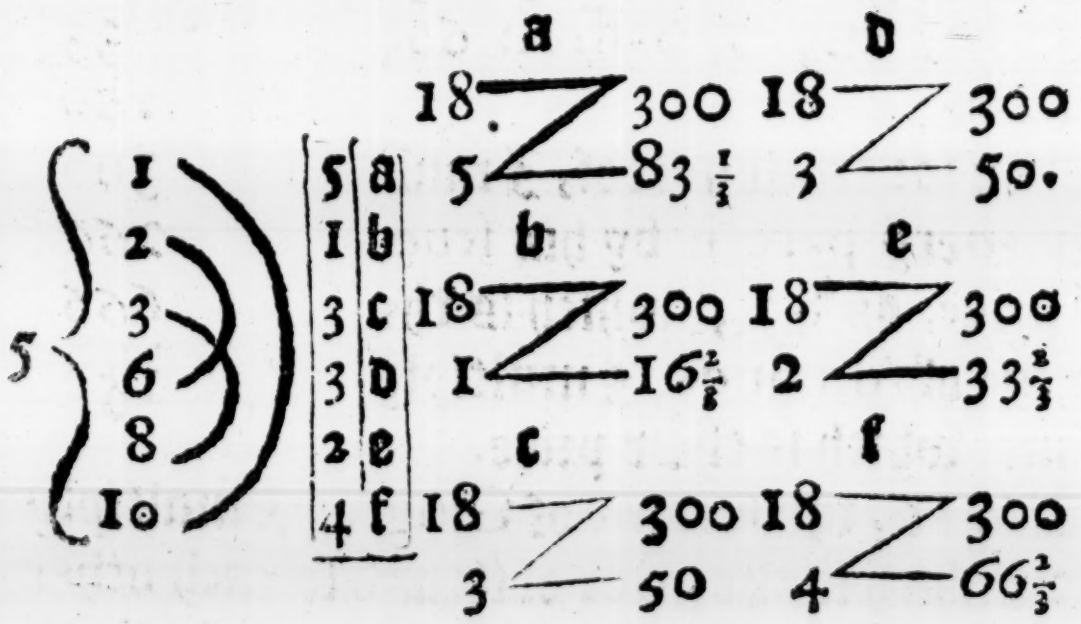
A

A Merchant being minded to make a bargaine for Spices, in a mixt maſſe (that is to ſay,) of Cloues, Nutmegs, Saffron, Pepper, Ginger, and Almonds: the Cloues being at 6 ſhillings, Saffron at 10 ſhillings, Pepper at 3 ſhillings, Ginger at 2 ſhillings, and Almonds at 1 ſhilling. A queſtion of Spices.

Now would he haue of each ſort ſome, to the value of 300 pound in the whole, and each pound one with another, to beare in price 5 ſhillings: How much ſhall he haue of each ſort?

Scholar. That will I try thus.

Firſt, I ſet downe thoſe ſixe ſeuerall prices, and at the left hand I ſet the common price, 5 ſhillings. Then I linke them thus, 1 with 10, 2 with 6, and 3 with 8: as in the example following.



Maſter. I had minded to haue combined them in more variety: but I am content to ſée your owne worke firſt, and then more varieties in combination may follow anon.

Scholar. Then to continue as I began, I sæke the difference betwæne 1 and 5, (which is 4) and that I set against 10; then against 1 I set 5, which is the excelle of 10 aboue 5, so I gather the difference betwæne 2 and 5, which is 3: and that I set against 6, because it is combined with 2: and likewise the difference of 6, aboue 5, (which is 1) I set against 2. Then take I the Difference of 3 from 5, which is 2, and that I set against 8: and before that 3, I set the difference of 8 aboue 5, which is 3. Then gather I all these differences by Addition, and they make 18, which I set for my first number in the Golden rule, and so appeareth by those workes, that of Almonds I must take $83\frac{1}{3}$ pound, of Ginger $16\frac{2}{3}$ pound, Pepper 50 pounds, of Cloues, 50 pounds, of Nutmegs $33\frac{1}{3}$ pounds, and of Saffron $66\frac{2}{3}$ pounds.

	$83\frac{1}{3}$
	$33\frac{1}{3}$
	150
	300
	$266\frac{2}{3}$
	$666\frac{2}{3}$
	1500

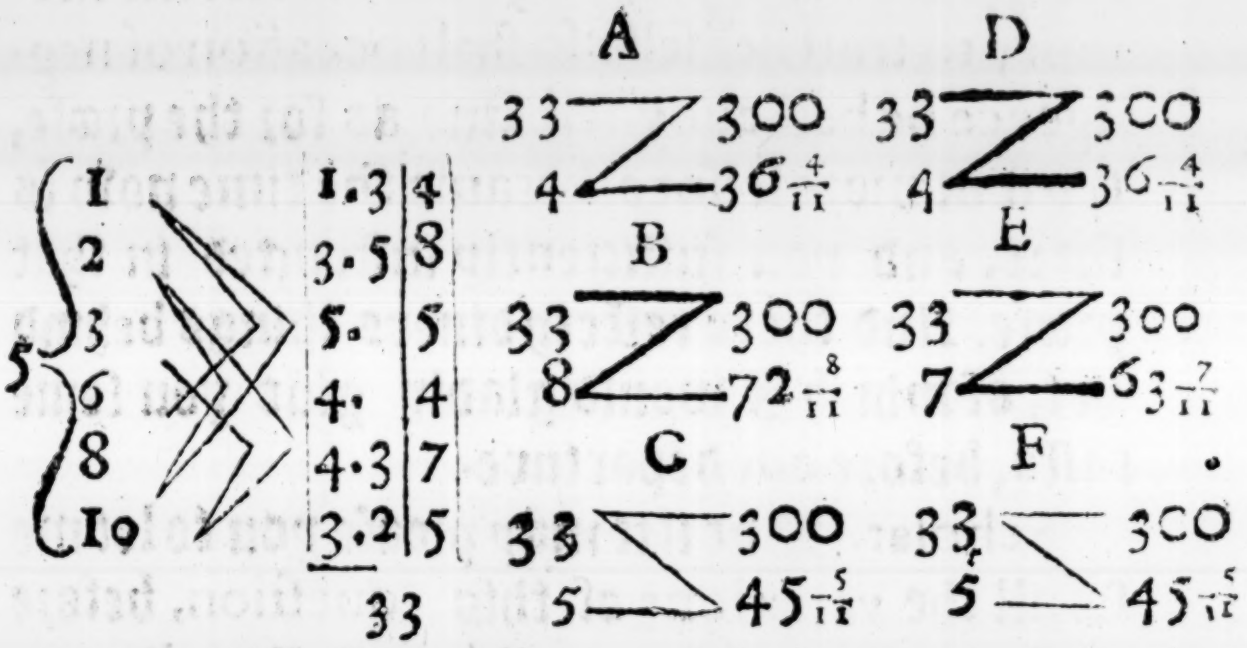
Then for triall hereof, I multiply euery parcell by his seuerall price, as $83\frac{1}{3}$, which is the summe of Almonds, I multiply by one, which is their price.

Also $16\frac{2}{3}$ the summe of Ginger, I multiply by 2, which is the price of it: and so each other in his kinde, as this Table annexed doth represent, and then adding them altogether, I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300, by the common price 5, where-

fore

fore it appeareth well wrought.

Master. Now I will make the alligation to proue your cunning somewhat better: but because you shall not thinke your selfe pressed so much, I will also note the differences, as by this example you may see, where I



haue alligate 1 with 6 & 8, and therefore haue I set against 1 both their differences, that is 1 and 3: Likewise, because 2 is combined with 8 and 10, I set before him their differences, 3 & 5. Against 3 I haue set onely 5, which is the difference of 10, wh' [illegible] whom 3 is combined onely. Likewise 6 [illegible]ly alligate to 1, & therefore is the differe[illegible] from 5, which is 4 onely set against [illegible]inked with 1 and 2, and therefore h[illegible]inst him, both their differences, [illegible] 10 is ioyned with 2 and 3, th[illegible]e their diff[illegible]rences, 3 and 2. [illegible]'efor you, in another col[illegible] differences redu- ☞ ced into [illegible]ry seuerall sort,

and

and haue also added them together, whereby appeareth that they make 33, and so consequently you see the workes of the Golden Rule set forth. For the sixe drugges I haue added the letters, A, B, C, &c. as before.

But I would not wish you to cleaue still to these Elementary aydes, but accustome Memory to trust her selfe: so shall occasion of negligence be best auoyded. And as for the proofe, try it at more leisure, because the time now is short, and you sufficiently instructed in that proofe. And there resteth diuers things behind yet, of which I would gladly giue you some taste, before our departure.

Note.

Scholar. But if it may please you to let me see all the variations of this question, before you goe from it, for me thinketh I could vary it two or three wayes more yet.

Master. I am content to see you make two or three variations, but I would bee loath to stay to see all the variations, for it may be varied aboue 300 wayes, although many of them would not well serue this purpose.

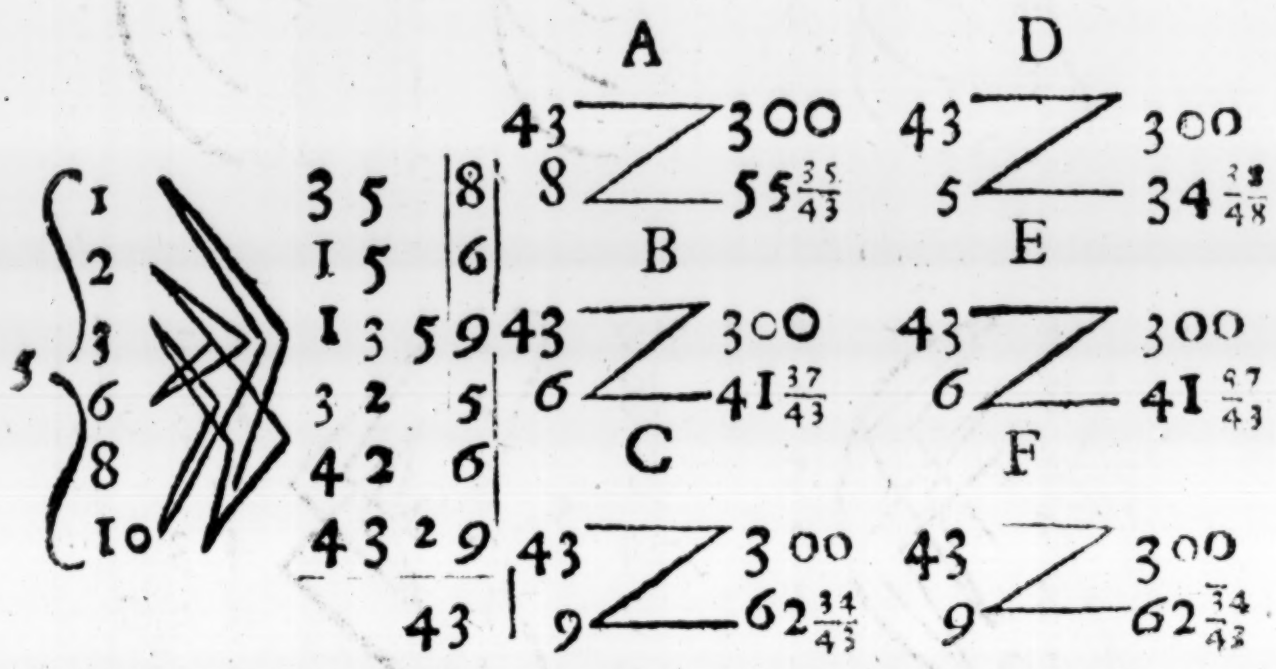
Scholar. I thoug[h] impossible to make so many variations.

Note.

Master. Marueil[l]at, for some questions of this Rule [a]ed aboue 1000 wayes: but I woul[d] get such fantasies, till a time o[f] And now goe forward with [t]his question.

Scholar. For I like the

the first number 1 with 8 and 10. and 2 I combine with 6 and 10: then ioyne I 3 with 6,8,and 10, as in this fozme.



And so doth there appeare the proportion of weight foz euery kinde of Drugge in this mixture. Now foz the tryall.

Master. Nay, stay there: you shall not néed to make tryall in one example so often, oz if you list doe it by your selfe, I am content. But now set forth (for declaration that you conceiue the Rule) two or three examples of seuerall combinations, and then will we passe to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these be the va ties: in which, as the

combinations are ſeuerall, ſo doth it plainely appeare, that the differences by which the proportion of each ſeuerall kinde is taken, are alſo ſeuerall. And yet I ſee in the three firſt of theſe fiue varieties, and in one other before, the totall ſumme of the differences to be one, that is to ſay, 18, whereby I perceiue that the variety of their mixture doth depend on the variety of their differences ſeuerall, and not of the variety of their totall ſum e.

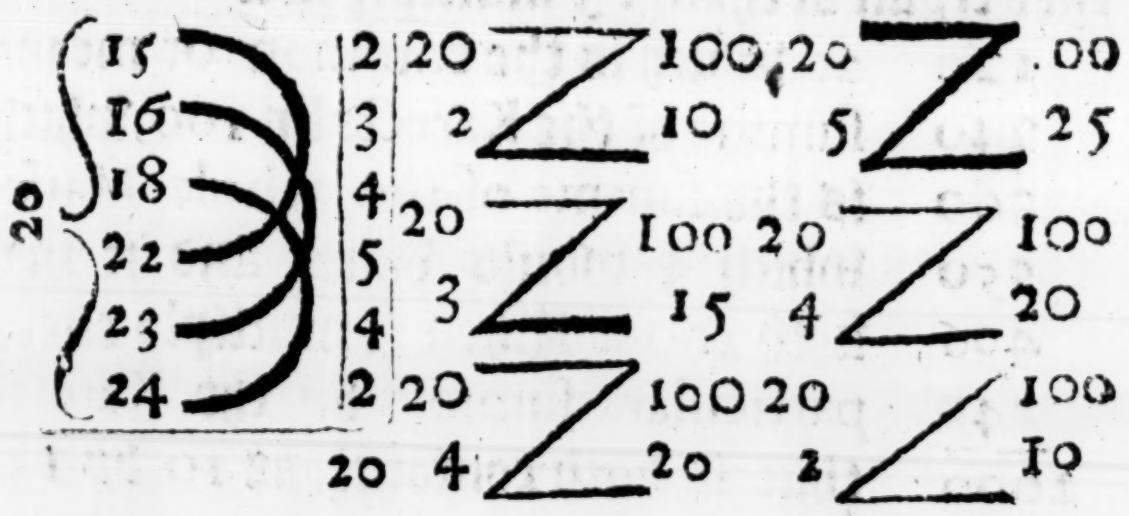
Maſter. So is it. S eing you conceiue it ſo well, I will mak nd of this Rule, onely exhibiting vnto te Queſtion of two of the mixture of A that by it you may deuiſe others like ciſe your ſelfe therein alſo, becauſe th erueth often

in

in busineſſe of charge, not ſo much for Goldſmiths, as of coynage in Mints. First I demand of you this queſtion; *If a Mynt maſter haue Gold of 22 Kareets, and ſome of 23 Kareets, ſome of 24: Againe ſome of 15 ſome 16, and ſome of 18 kareets, and would mixe them, ſo that he might haue 100 ounces of 20 kareets: How much muſt he take of each ſort?*

A queſtion of mixing Gold.

Scholar. To know that, I anſwer in order, thus:



Maſter. You haue wrought the queſtion well: but how chanced you made no doubt of that new name Kareet?

Scholar. Becauſe I thought it out of time to demaund ſuch queſtions now, ſeeing you make ſo much haſte to end: and againe in this caſe the proportion of the number, is ſufficient for my purpoſe in this worke: truſting that another time you will inſtruct mee as well of this, as of ſun ther things, which as I haue heard y of, ſo I haue a great deſire to then

Maſte iſwer is reaſonable, and your re uſt (with Gods helpe)

I intend to satisfie. And now to goe forward with this matter, let me see your examination of this last worke.

Scholar. First for the one part I adde together all the particular summes, as they appeare in the worke, and they make 100, as here by their Addition doth appeare.

And so it seemeth that the summes are well gathered: but for the further tryall of them: I multiply first

10	
15	
20	
25	
20	
10	
100	

120 20 which is the common or meane
240 summe of the Karects by 100, which
360 is the summe of the whole Masse,
550 which I would haue, and it ma-
406 keth 2000. Then I multiply euery
240 particular summe by the Karects
2000 that it doth containe, as 10 by 15, and that maketh 150.

Likewise I multiply 15 by 16, and it yeeldeth 240: so 20 by 18, maketh 360. And 25 by 22, yeeldeth 550: likewise 20 by 23, bringeth forth 460; and last of all 10 multiplied by 24, yeeldeth 240: which summes all ioyned together make 2000, that doth agree with the like summe before, wherefore I may well say, that the worke is good. And now if (it please you) I would set forth for varieties of this question to proue my wit

Master. Goe to let me

Scholar. Here be four

And

3	4	7
3		3
2		2
2		2
5	4	9
5		5

28

2	3	5
3	4	7
4		4
5		5
5	4	9
4	2	6

36

2	3 4	9
4		4
3		3
5		5
5	2	7
5	4	9

37

4		4
4		4
2	3 4	9
2		2
2		2
5	4 2	11

32

And more yet could I make, but not like to the number that you speake of in the variation of the other question.

Master. That will I teach you at more leisure, seeing it is a thing rather of pleasure then of any necessity.

A question of mixing of siluer.

But now for your exercise in this Rule, one other question I will propose. A Mint-master hath sixe Ingots of siluer of sundry fineneſſe, some of foure ounces fine, and some of fiue ounces, some of sixe, and other of eight, some of 11, *and other of* 12, *and his desire is to mixe* 500 *pounds weight, so that in the whole Masse euery pound weight should beare nine ounces of fine siluer: How much shall he take (say you) of euery sort of siluer?*

Scholar.

Scholar. To finde out that, I set the numbers thus in order.

And gathering the differences, it will appeare, that, of the first sort there must be $43\frac{11}{23}$: of the second like much: of the third sort $65\frac{5}{23}$: and of the fourth sort as much: of the fift sort $195\frac{17}{23}$ and of the sixt sort $86\frac{22}{23}$, which in the whole will make 500 pound weight, and in ounces after 9 ounces fine 4500, that is of the first sort $173\frac{21}{23}$: and of the second sort $217\frac{9}{23}$: of the third sort $391\frac{3}{23}$: of the fourth sort $521\frac{17}{23}$, of the fift sort $2152\frac{4}{23}$, and of the sixt sort $1043\frac{11}{23}$, which altogether doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

Master, This is well done of you, therefore now make three or foure varieties, and so an end of this Rule.

Scholar. These 4 varieties I set for example.

Master.

Maſter. And by theſe it appeareth, that you can finde out more, with which I will not now meddle, ſaue onely (for to ſhew you an eaſie helpe in drawing the lines of Combination) I will ſet forth two varieties here.

And this ſhall ſuffice now for the Rule of Alligation or mixture: for by theſe examples may you eaſily coniecture ſuch other as dos appertaine to it, as well for the due working, as for variety of drawing the lines of combination.

Scholar. Sir, albeit it pleaſed you erewhile to put me from my muſing at the many varieties that may fall in theſe combinations and termed them fantaſies; yet my fantaſy

fantaſy giueth me, that the conſideration of this ſhould in many other examples and caſes of importance be very néedfull, and the knowledge of it moſt profitable. Therefore ye may well thinke, that at another time conuenient I will requeſt you to aide me herein.

Maſter. Truth it is that this conſideration may fall in practiſe as well Politicke as Philoſophicall, and ſundry wayes in them be applied: Therefore when time ſhall fall fit for the diſcuſſing of this conſideration, you ſhall not want my helping hand.

The Rule of Falſhood.

The occaſion of the name.

NOw will I briefly alſo teach you ſomewhat of the Rule of Falſehood, which beareth his name, not for that it teacheth any fraud or Falſhood, but for that by falſe numbers taken at all aduentures, it teacheth how to finde thoſe true numbers that you ſeeke for.

Scholar. So might any other rule be called the Rule of Falſhood, for they worke by wrong numbers, and by them finde out the right numbers: ſo doth the Rule of Alligation, the Rule of Fellowſhip, and the Golden Rule partly.

Maſter.

Maſter. In the Golden Rule, the Rule of Fellowſhip, and the Rule of Alligation, although the numbers that you worke by, bée not the true numbers that you ſéeke for, yet are they numbers in iuſt proportion, and are found by orderly worke: whereas in this Rule the numbers are not taken in any proportion, nor found by orderly worke, but taken at all aduentures.

And therefore I ſometimes being merry with my friends, and talking of ſuch queſtions, haue cauſed them that propoſed ſuch queſtions, to call vnto them ſuch children or Idiots as happened to be in the place, and to take their anſwere, declaring that I would make them ſolue thoſe queſtions, that ſéemed ſo doubtfull.

And indéed, I did anſwer to the queſtion, and worke the triall therof alſo by thoſe anſweres which they happened at all aduentures to make: which numbers ſéeing they be taken as manifeſt falſe, therefore is this Rule called the Rule of falſe Poſitions, and for briefeneſſe, the Rule of Falſhood: which Rule for readineſſe of remembrance, I haue compriſed in theſe few verſes following, informe of an obſcure Riddle.

Geſſe at this worke as hap doth leed,
By chance to truth you may proceed,
And firſt worke by the queſtion,
Although no truth therein be don.

Such

Such falſhood is ſo good a ground,
That truth by it will ſoone be found.
From many bate too many mo,
From too few take too few alſo;
With too much ioyne too few againe:
To too few adde too many plaine.
In croſſe-wiſe multiply contrary kind,
All truth by falſhood for to find.

The ſenſe of theſe verſes, and the ſumme of this Rule is this:

The expoſition of the Rule.

When any *queſtion is proponed* appertaining to this *rule*, firſt imagine any *number* that you liſt, which you ſhall name the *firſt poſition*, and put it in ſtead of the *true number*, and then worke with it as the *queſtion* importeth: and if you haue miſſed, then is the *laſt number* of that worke either too *great* or too *little*: that ſhall you note as hereafter ſhall be taught you, and you ſhall call it the *firſt errour*.

Then beginne againe, and take another *number*, which ſhall be called the *ſecond poſition*, and worke by the *queſtion*: if you haue miſſed againe, note the exceſſe or default as it is, and call that the *ſecond errour*. Then multiply croſſe-wiſe the *firſt poſition* by the *ſecond errour*, and againe the *ſecond poſition* by *the firſt errour*, and note their *totals* ſeuerally by the names of *totals*. Then marke whether the two *errours* were *both alike*, that is to ſay, both *too much*, or both *too little*: or whether they bee *vnlike*, that is, the one too *much*, and the other too little:

little: for, if they be like, then ſhall you *ſubtract* the one *totall* from the other (I meane the *leſſer* from the *greater*) and the *remainer* ſhall be your *Diuidend*: ſo muſt you abate the *leſſer errour* out of the *greater*, and the reſidue ſhall be the *Diuiſor*. Now diuide the *Diuidend* by that *Diuiſor*, and the *quotient* will ſhew you the *true number that you ſeeke for*. But, and if the *errours be vnlike*, then muſt you adde both thoſe *totals* (which you noted) together, and take that *whole number* for the *Diuidend*, ſo ſhall you adde both *errours* together, and that *whole number* ſhall bee the *Diuiſor*, and the *Quotient* of that Diuiſion ſhall giue you the *true number* that the *queſtion* ſeeketh for, and this is the whole rule.

Scholar. This rule ſéemeth ſo vnlike any other, that without ſome example I ſhall not eaſily vnderſtand it.

Maſter. With a good will: propoſe halfe a ſcore ſundry queſtions & examples of variety, for the better vnderſtanding of the worke hereof: and for the firſt take this example. *A Maſon was bound to build a wall in* 40 *daies, and it was couenanted ſo with him, that euery day that hee wrought, he ſhould haue for his wages* 2 ſhillings 1 penny, *and euery day that he wrought not, hee ſhould be amerced* 2 ſhillings *ſixe* pence, *ſo that when the wall was made, and the reckoning taken of the daies that he wrought, and of the other that hee wrought not, the Maſon had cleerely but fiue* ſhillings *fiue* pence *for the worke. Now doe*

A queſtion of Maſonrie. The firſt example.

doe I demand how many daies did he worke of those 40, *and how many did he not worke?*

Scholar. I pray you expresse the order of the worke, that I may partly by imitation, and partly by comparing it with the Rule, be able againe to doe the like.

Maister. This order shall you kéepe in the worke of this rule: first take some number(as you list) at aduenture, as for example, I say hée plaied 12 daies, and wrought 28 daies. Now cast you the wages of euery day, and sée whether it will agrée with the summe of 5 shillings 5 pence.

Scholar. The 28 daies that he wrought after 25 pence the day, yéeldeth 700 pence. Then 12 daies that hee wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Maister. Hée had but 65 pence, and by this supposition he should haue had 340: therefore is this summe too much by 275, which summe I must set downe after this sort as you sée héere, where first I haue made a crosse, (commonly called S. Andrewes crosse) and at the ouer corner, on the left hand

12
X
275 +

I haue set the first position 12: and at the other corner vnder it I haue set 275, which is the first errour, with this figure +, which betokeneth too much, as this line, — plaine without a crosse line betokeneth too little.

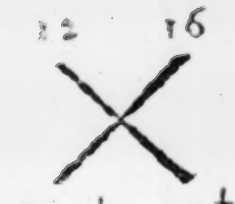
On

On the right hand of the croſſe I haue left two like roomes for the ſecond poſition, and his error. Therefore to proſecute the worke, I ſuppoſe he plaied 16 daies, and wrought 24.

Scholar. I was a while in doubt why you named the daies of his working, ſeeing they be not ſet in the figure: and I doubted how you knew them, or elſe whether that you did ſuppoſe them at all aduentures, as you did the daies that hee plaied: but now I gather, that ſeeing 40 daies is the whole time limited, then the daies that hee plaied being ſuppoſed, the reſt of 40 muſt needs be the daies that hee wrought, and therefore 28 followed 12 of neceſſity, and 24 followeth 16 alſo of neceſſity: but yet I ſcarſe perceiue why you ſet not in the figures as well 28 as 12.

Maſter. It forceth not which of them I take, ſo that in the ſecond poſition I take the numbers of the ſame nature that is here both of working daies, or both of idle. but now examine you this ſecond poſition.

Scholar. If he plaied 16 daies, then abating 16 times 30 pence, the ſumme will be 480 pence, and for 24 dayes that he wrought, euery day yeelding 25 pence, the totall is 600 pence: ſo that abating 480 out of 600, there reſteth 120, and as you ſay, it ſhould be but 65: therefore it is too much by 55: that muſt be ſet on the right hand of the figure at the neather part, and ouer it on the ſame ſide 16, which is the ſecond poſition, thus:



And as I gather by your wordes, it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

Master. So were it. But this shall you marke, that, of what nature soeuer the two positions be, of the same nature is the quotient. Therfore when the positions in this question are 12 & 16, which both being numbers of the playing dayes, the quotient shall declare the true number of playing dayes: whereas if the positions had bene 28 and 24, which are supposed to be the working dayes, then would the Quotient declare the true number of the working dayes, and not of playing dayes, as it will doe now. And therefore to continue the workes of this question, and to finde the true number of playing dayes, I must multiply crosse-wise the first position by 55, that is the second errour, and the totall will bee 660. Then I multiply 275 and 16, and it yeeldeth 4400. Now because the errors are like, that is to say, both too much, I must subtract 660 out of 4400, and so remaineth 3740, which is the dividend. Againe, I must subtract the lesser errour 55 out of 275, that is the greater errour, and there will remaine 220, which will bee the diuisor: then diuiding 3740 by 220, the quotient will be 17. Wherefore I say now constantly, that 17 is the true number of dayes that the Mason plaied: and then it followeth that he wrought 23 dayes, and so is the question answered.

Now

Now for the order of triall of this worke, there needeth none other triall but onely this, to worke with this number according to the queſtion, and if it agree, then appeareth the number to bee it that you would haue. The proof of this rule.

And here now ſeeing he wrought 23 daies, and muſt haue for euery day 25 pence, the whole ſumme commeth to 575. Then againe ſeeing he plaied 17 daies, and muſt abate 30 pence for euery day, the whole ſumme of the abatement will be 510: Therefore I ſubtract 510 out of 575, and there will remaine 65, which maketh 5 ſhillings, 5 pence, the cleere wages of the Maſon for his worke, according to the queſtion.

Scholar. Now I truſt I vnderſtand the worke and the Rule ſo well (and the better by this proofe) that I can be able to doe the like. And for a proofe, I take the ſame queſtion all ſaue the laſt number, where I will ſuppoſe that he had 10 ſhillings, for his wages cleere: And now to geſſe at the number of the dayes that hee wrought, I ſuppoſe firſt that hee wrought 20 daies: then ſay I, if he wrought 20 daies, his wages muſt be 500 d: then did he play other 20 daies, for which muſt be abated 600 d, and then he loſeth 100 d. And ſo am I at a ſtay, for it is not like to your former worke.

Maiſter. You ſhould haue required of mée ſome queſtion, & not haue taken a queſtion of your own phantaſying, vntill you were more expert in this art: for ſo might you as well

happen to an impossible question, as on a possible: but now to goe forward, consider that this number is too little by 220, seeing hee should gaine by your supposition 120 pence, and in this position he loseth 100, those both make 220, which you shall set downe for the first errour with this signe —, betokening too little, as here in this forme following doth appeare.

20

And now for the rest, goe forward your selfe once againe.

220—

Scholar. As my errour hath vttered my folly, so it hath procured mee better vnderstanding.

Now therefore considering this position not to solue the question, I take another, supposing that he wrought 30 dayes: Then for his wages he must be allowed 750 pence, and for the 10 dayes which hee wrought not, hee must abate 300 pence, and so remaineth clere 450 pence, but it should be onely 120 pence, therefore is it too much by 330, which I set downe in the figure with the former position and his errour; and the figure appeareth thus:

Now must I multiply in crosse wises 20 by 330, and it will be 6600.

20 30

Then againe I multiply 30 by 220, and it will be also 6600. Wherefore if I shall subtract the one

220 — 330 +

out

out of the other, there will remaine nothing to be the Diuidend.

Maſter. In this you forget your ſelfe againe: for in as much as the ſignes in the errours be vnlike, therefore muſt you worke by Addition, adding together thoſe two totals to make the Diuidend, and alſo adding the two errors to make the Diuiſor. And becauſe you ſhall no more forget this part of that Rule, take this briefe remembrance.

Vnlike require Addition,
And like deſire Subtraction.

Scholar. You meane, that if the errors haue like ſignes, then muſt the Diuidend and the Diuiſor be made by Subtraction, as is taught before: And if thoſe ſignes bee vnlike (as in this laſt example they bee) then muſt I by Addition gather the Diuidend and the Diuiſor. Therefore muſt I adde 6600 to 6600, and it will be 13200, which will bee the Diuidend. Then againe I adde 220 to 330, and it will be 550, which muſt be the Diuiſor: wherefore diuiding 13200, by 550, the quotient will be 24, whereby I know that the Maſon wrought 24 dayes, and then it followeth that he plaied 16 daies.

Maſter. Examine your worke, whether it be agreeable to the queſtion or no.

Scholar. For 24 dayes worke the wages muſt be 600 pence, and for 16 dayes which the Maſon wrought not, there muſt be abated 480 pence, and then remaineth clere to the

Maſon 120, as the queſtion importeth: wherfore it is euident that 24 is the true number of dayes that he wrought.

Maſter. Although you ſéeme now to vnderſtand this worke, yet to acquaint your minde the better with the new Trade of this Rule, I thinke it good to propone to you 5 or 6 examples more before I make an end of it.

Scholar. Sir, I thanke you that you doe ſo conſider my commodity and profit in knowledge, for vndoubtedly it is practiſe and exerciſe that maketh men prompt and expert in euery kinde of knowledge.

Maſter. You ſay well, ſo that they follow ſome certaine precepts to gouerne and rule their practiſe by, elſe may practiſe procure cuſtome of error, and a repugnance to exactneſſe of knowledge: namely, as long as the error is not plainely knowne to the vulgar ſort. But to returne to your worke.

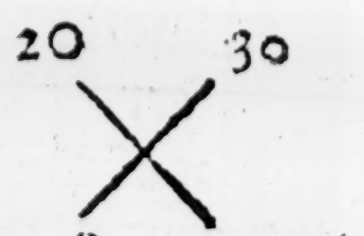
A queſtion of wares, the ſecond example.

There is a ſeruant that hath bought of veluet and damaske for his Maſter 40 *yards, the veluet at* 20 ſhillings *a yard, and the damaske at* 12 ſhillings, *and when he commeth home, his Maſter demandeth of him how much hee hath bought of each ſort: I cannot tell (ſaith he) exactly: but this I know, that I paid for damaske* 48 ſhillings, *more then I paid for veluet: now muſt you geſſe how many yards there is of each ſort.*

Scholar. Although the geſſe ſéemeth difficult, yet I will proue what I can doe: for I remember your ſaying, that it forceth not

not how fond or false the geſſe be, ſo it be ſomewhat to the queſtion, and not an anſwer of a contrary matter.

Therfore firſt I imagine that he bought 20 yards of Damaske, for which he ſhould pay after the former price 240 ſhillings: then muſt he néds haue of veluet other 20 yards (to make vp the 40 yards) and that would coſt 400 ſhillings. So that the totall of the price of the Damaske is leſſe then the ſumme paid for Veluet 160 ſhillings, and ſhould be more by 48. Therefore the firſt errour is 208 too little. Then begin I againe, and ſuppoſe he bought of Damaske 30 yards, that coſt 360 ſhillings, then had he but 10 yards of Veluet, which coſt 200 ſhillings: and now the price of the Damaske is greater then the price of the veluet by 160 ſhillings, and ſhould bée but 48, therefore is the ſecond errour 112 too much, which I ſet in forme of figure as here doth appeare. Then doe I multiply in croſſe wayes 208 by 30, and the ſumme will be 6240 Alſo I multiply 112 by 20, and there will amount 2240. And in as much as the ſignes of the errours be vnlike, I know I muſt worke by Addition, therefore adde I thoſe two torals together, & they make 8480, which is the Diuidend: then adde I alſo the two errours together, 208 and 112, and they make 320, which is the Diuiſor: wherefore



diuiding 8480 by 320, the quotient will be $26\frac{1}{2}$, which is the true ſumme of yards of Damaske that he bought, and in veluet 13 yards $\frac{1}{2}$, and that appeareth by examination, thus: $26\frac{1}{2}$ yards of Damaske at 12 ſhillings the yard, maketh 318 ſhillings: then in Veluet hee had but 13 yards, and $\frac{1}{2}$, and coſt 270 ſhillings, at 20 ſhillings the yard. Now ſubtract 270 out of 318, and there will remaine 48, which is the number of ſhillings that the Damaske did coſt more then the veluet.

Maſter. Now ſhall you haue a queſtion of another kinde.

A queſtion of debt, the third example.

There are three men that doe owe money to mee, and I haue forgotten what the totall ſumme is, and what the particulars be.

Scholar. Why, then it is impoſſible to know the debt.

Maſter. Peace: you are too haſtie, there is more helpe in it then yet you ſée, I haue three ſeuerall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the firſt and the ſecond to amount to 47 pound, the debt of the firſt man and the third man did make 71 pound, and the ſecond man his debt with the third, did riſe to 88 pound. Now can you tell what euery man did owe, and what was the whole ſumme?

Scholar. Nay in good faith: but as I perceiue that it muſt bee found by coniecture, ſo will I gueſſe at it, ſuppoſing that the firſt man did owe 20 pound, and the ſecond man 30, and the third.

Maſter.

Maſter. Nay ſtay there, you are too farre gone already: you may not ſuppoſe a ſeuerall ſumme for euery man, for it is enough to ſuppoſe one ſumme for the firſt man, and let the other riſe as the queſtion importeth. Therefore ſeeing you ſet the firſt man his debt to be 20 pound, the ſecond man cannot owe 30 pound, for the declaration is, that their debts added together did make 47 pound, ſo muſt the ſecond man his debt bee but 27 pound. Now the ſecond debt with the third, muſt make 88: therefore ſubtract 27 out of 88, and there will remaine 61, as the third man his debt. Then ſaith the declaration, that the firſt and third mans debts doe make 71: but by this ſuppoſition they make 81, that is 10 too much, which I muſt ſet for the firſt errour. Now worke you the ſecond poſition.

Scholar. I ſuppoſe the firſt mans debt to be 24 pound: then muſt the ſecond mans debt (by your declaration) be but 23 pound, ſeeing both they make but 47 pound. And the ſecond man his debt with the third, doe make 88 pound, and the ſecond man oweth but 23: therefore the third man muſt owe 65 pound. Now the third mans debt with the firſt, ſhould make by the declaration 71 pound, and they doe make 89 pound, that is 18 pound, too much, and that is the ſecond errour, which I ſet downe with the firſt, and their poſitions in this forme, and then I doe multiply in croſſe wates 20 by 18, & it is 360.

And

And 10 by 24 maketh 240. Also because the signes of the errours bée like, I must worke by subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the diuidend: then doe I subtract 10 out of 18 by the same reason, and so is the Diuisor 8, which is found 15 times in 120: therefore I say that the first man did owe 15 l. and then the second man must owe 32 l: for those 2 doe make 47 l, and the third mans debt is 56: for so much remaineth if I abate 15 out of 71, or if I take 32 out of 88.

10 24
10+ 18+

The fourth example.

Master. *For the fourth example, take this easie question for the variety in worke. Two men hauing seuerall summes, which I know not, doe thus talke together: the first saith to the second, if you giue me* 2 shillings *of your money, then shall I haue three times so much money as you. The second man answereth; It were more reason that our summes were made equall, and so will it bee if you giue mee* 3 shillings *of your money. Now gesse what each of them had.*

Scholar: I imagine that the first had 9 s.

Note.

Master. Consider euermore in your imagination that you take a likely summe, as in this question, take such a summe, that hauing 2 added vnto it, may bée diuided into 3 parts euen.

Scholar. Why? I remember you said before, it forceth not how fondly soeuer I guessed.

Master.

Maſter. As for the poſſibility of the ſolution, it is truth: but for eaſineſſe in worke, the apteſt numbers are moſt conuenient.

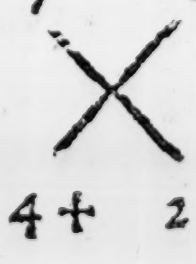
Scholar. I thought no leſſe, and therefore I toke 9 as an apt number to be parted into three: but I perceiue I ſhould haue conſidered the aptneſſe of that partition after the addition of two vnto it, and then 7 had beene more meet.

Maſter. That is truth, and then ſhould the ſecond man his ſumme be 5: for although hee haue now but the third part of 9, that is 3, yet you muſt remember that he lent the firſt man 2, and ſo had he 5.

Scholar. Then to goe forward: if the ſecond man had three of the firſt man, then ſhould hee haue 8, and the firſt man but 4: ſo hath he double to the firſt man: yet he ſaid in the queſtion they ſhould haue equall: wherefore it appeareth that he hath 4 too much.

Therefore I note that error with his ſuppoſition, & geſſe againe that he hath 10 ſhillings: whereunto I adde 2 ſhillings borrow of the ſecond man, and then he hath 12 ſhillings: ſo the ſecond man hath remaining but foure, whereunto if I adde the 2 that hee lent to the firſt man, ſo had he but 6 ſhillings at the beginning.

Then take 3 s from the firſt man, and glue to the ſecond, then hath the firſt man but 7, and the ſecond hath 9, which are not equall, but there are 2 too



many,

many, wherefore I ſet downe both the poſitions with their errors as before you ſée, & multiply a croſſe, ſo commeth there 40 and 14: and becauſe the ſignes be like, I take 14 out of 40, and ſo reſteth 26 to bée diuided, then likewiſe I take 2 out of 4, and there reſteth 2, by which I diuide 26, and the quotient will be 13, which is the ſumme that the firſt man had. And ſo appeareth that 2 being added thereto, the ſumme will bee 15, ſo hath the ſecond man but 5, and before he had 7: then take 3 from the firſt, and put to his 7, and ſo haue each of them 10, and that is equall as the queſtion would.

The fift example: A queſtion of Lambs.

Maſter. *For the fifth example, take this queſtion. One man ſaid to another, I thinke you had this yeere two thouſand Lambes: So had I, ſaid the other; but what with paying the tithe of them, and then three ſeuerall loſſes, they are much abated: for at one time I loſt halfe as many as I haue now left, and at another time the third part of ſo many, and the third time $\frac{1}{4}$ ſo many. Now gueſſe you how many are left.*

Scholar. Becauſe here is mention made of certaine parts, I muſt take a number that may haue all theſe parts, that is to ſay: $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{4}$ which will be 24, howbeit 12 hath the ſame parts. Therefore I take firſt 12 to bee the number that doth remaine, ſo hath he loſt 6, 4, and 3, that is 13, and the whole 25, but it ſhould be 2000.

Maſter. Ye are deceiued yet ſtill, you haue for-

forgotten the 10 part, which muſt bée defalked, that is 200, ſo there remaineth but 1800, and now goe on againe.

Scholar. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 tw few, which I ſet for the errour. Then for the ſecond poſition I take 24, whoſe halfe is 12, the third part 8, and the quarter 6, whereby riſeth 50, which is too little by 1750, therefore I ſet downe both the poſitions with their errours thus:

And multiply in croſſewaies 1775 by 24, where of commeth 42600. Alſo I multiply 1750 by 12, and there ariſeth 21000. And becauſe the ſignes are like, I doe ſubtract the one from the other, and ſo remaineth the Diuidend 21600. Then doe I ſubtract 1750 out of 1775, and there reſteth 25, by which I diuide 21600, and the quotient is 864, whereof the halfe is 432, and the third part is 288, the quarter is 216, which all being added together, will make 1800. And if you adde thereto the tenth which was abated before, then will the whole ſumme be 2000.

12	24
1775–	1750–

864
432
288
216
1800

And now doth there come a queſtion to my memory which was demaunded of me, but I was not able to anſwer to it: and now me thinketh I could ſolue it.

Maſter. Propone your queſtion.

Scholar.

A queſtion of ſheep and tillage, the ſixth example.

Scholar. *There is ſuppoſed a Law made, that (for furthering of tillage) euery man that doth keepe ſheepe, ſhall for euery tenne ſheepe eare and ſow one Acre of ground: and for his allowance in ſheepe paſture, there is appointed for euery foure ſheepe one Acre of paſture. Now is there a rich ſheepe-Maſter which hath 7000 Acres of ground, and would gladly keepe as many ſheepe as hee might, by that Statute I demand how many ſheepe ſhall hee keepe?*

Maſter. Anſwer to the queſtion your ſelfe.

Scholar. Firſt I ſuppoſe hée may kéepe 500 ſheepe, and for them hée ſhall haue in paſture after the rate of foure ſheepe to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this errour is too little by 6825. Therefore I gueſſe againe that hée may kéepe 1000 ſheepe, that is in paſture 250 Acres, and in tillage, 100 Acres, which maketh 350, that is too little by 6650. Both theſe errours with their poſitions I ſet downe as you ſée, and multiply them croſſe 6825 by 1000, and it maketh 6825000, alſo I multiply 6650 by 500, and there commeth 3325000, which ſumme I ſubtract out of the former, and there remaineth 3500000 for the diuidend: likewiſe I ſubtract the leſſer errour out of the greater, and there reſteth 175, by which I diuide 3500000 (the diuidend aforeſaid)

500 1000
6825 - 6650 -

and

and the quotient will be 20000, ſo that by this rate, he that hath 7000 Acres of ground may keepe 20000 ſheepe.

Maſter. You haue done well, notwithſtanding both this laſt queſtion, and the next before might be wrought without the ſecond poſition by the Rule of proportion, as this: When in this queſtion ye found in the firſt error that for 500 ſheepe, there muſt be 157 Acres, then might you reduce it to the Golden Rule thus: Another way of working.

175 | 500
7000 | 2000

If 175 *Acres will admit in allowance* 500 *ſheepe, then* 7000 *will haue* 2000 *And ſo by one poſition, with the helpe of the Golden Rule may you anſwere that queſtion.*

Likewiſe for the queſtion of Lambes, when you had found that 12 came of 25, you might haue ſet the figure as followeth, and haue ſaid:

25 | 12
1800 | 864

If 25 doe leaue but 12 what ſhall 1800 leaue? and it would appeare to be 864.

Scholar. Sir, I thanke you for this aid, for it doth much ſhorten the worke of this Rule.

Maſter. Yet againe, I will ſhew you an other way to anſwere to this laſt queſtion without the Rule of falſe poſition, and that by the Rule of Fellowſhip, for it appeareth in the propoſing of the queſtion, that 10 ſheepe Another way yet.

must

muſt haue in paſture two Acres and $\frac{1}{2}$, and for them muſt there be eared but one Acre: ſo it followeth, that for 2 Acres eared, there muſt be 5 ſet to paſture: and if you put them both into one ſumme, they will make 7. Therefore loke what proportion 7 being this totall, doth beare to 5 and to 2, ſuch proportion ſhall any totall in this queſtion beare to the paſture ground, and the eared ground.

Scholar. This ſeemeth wondrous aptly. Therefore to proue it, I demaund this by the former ſuppoſition: If a man haue 300 Acres, how much ſhall he leaue in paſture, and how much ſhall he turne to tillage? You ſay that as 7 is to 5, ſo ſhall 300 be to the Acres of paſture: and as 7 is to 2, ſo is 300 to the Acres of tillage, whereof for both I haue ſet examples here following, whereby appeareth that of Paſture, there ſhall be 214 $\frac{2}{7}$ Acres, and of Tillage 85 $\frac{5}{7}$, which both ſummes added together, doe make 300.

7 ⟌ 5
300 ⟌ 214 $\frac{2}{7}$

7 ⟌ 2
300 ⟌ 85 $\frac{5}{7}$

Another queſtion, the ſeuenth example.

Maſter. *Now take another example: A man hath three ſiluer cuppes with one couer, the couer wayeth 18 ounces, the ſecond cuppe wayeth euen halfe the weight of the firſt and the third. Now if the couer be put to the firſt Cuppe, they wey iuſt as much as all the three Cuppes doe wey: and if the couer be ioyned with the ſecond Cuppe, they wey as much as the ſecond twice, and the third: and if the*

Couer

Couer be put to the Cuppe, they will make twice as much as the first and the second Cup. Now try you what was the iust weight of euery Cup.

Scholar. I doe set the weight of the first Cuppe to be 9 ounces, then in as much as these two (that is to say, the couer and the first Cuppe) doe wey the weight of the three Cuppes, I see that the three Cuppes must wey 27 ounces, for so much is 18 and 9. Also because the first and the third doe wey double so much as the second, therefore is it the third part of that weight, that is 9, and then would it follow, that the third Cuppe also should wey 9 ounces; but then the question saith, that the Couer being ioyned to the second Cuppe, they wey as much as the second twice, and the third once, that should bee 27, and so it doth: that being ioyned with the third Cuppe, they should wey twice as much as the first and the second, that should be 36, and they wey but 27, so is that errour 9 too little. Then beginne I againe, and say, that the first Cuppe doth wey 12 ounces, which I ioyne with the Couer, and they make 30 ounces: then seeing the second is of that weight, it must needes wey 10 ounces, and the third must wey 8 ounces, seeing the first and the third must wey 20 ounces. Now put I the Couer to the second Cuppe, and they wey 28 ounces, which should bee euen so: then ioyne I the couer with the third Cup, & so should it wey twice the first & the second,

that is 44 ounces, and they wey but 26, that is 18 too litle: those errors with their positions I set down, and multiply in crosse

waies 9 by 12, whereof commeth 108: Also 9 by 18, and that yéldeth 162: and in as much as the signes be like, I abate the lesser out of the greater, and there doth remaine 54. Then doe I also abate the lesser errour from the greater, and so remaineth 9, by which I diuide 54, and the quotient is 6; which I take for the true weight of the first Cup, which being torned with the Couer, must wey as much as the three Cups, so doe they wey but 24 ounces. Then séeing the second Cup is the third part of that weight, for the other two Cups (you say) must wey double his weight, the weight of the second Cup is 8 ounces, and so the weight of the third must be 10 ounces. Now put the Couer to the second Cup, and it will make 26 ounces: that must be the weight of the second twice, and the third once, that is, twice 8, and once 10, and so is it. Againe, put the couer to the third Cup of 10 ounces, and they must wey twice as much as the first and the second, that is 28, and so is all agréable.

Master. Then answer to this question.

A question of water: the eight example.

There is a Cesterne with foure Cockes, containing 72 barrels of water: and if the greatest Cocke be opened, the water will auoid cleane in sixe houres; at the second Cocke it will aske eight houres; at the third

third Cocke it will auoide in no leſſe then nine houres: and at the ſmalleſt it will require twelue houres. Now I demand in what ſpace will it auoid, all the Cockes being ſet open?

Scholar. First, I imagine it will auoid in two houres.

Maſter. Then muſt there auoid by the firſt Cocke $\frac{1}{3}$ of the water, that is 24 barrels, and by the ſecond Cocke $\frac{1}{4}$, that is 18, and by the third Cocke $\frac{1}{5}$, that is 16 Barrels, and by the ſmalleſt Cocke $\frac{1}{6}$, that is 12 Barrels, all which ſummes put together, doe make 70, as by their Addition it doth appeare, but it ſhould be 72; therefore the errour is 2 tw few.

Scholar. Then will I beginne againe by your fauour, becauſe I thinke I vnderſtand the worke, and put three houres for the due time: ſo ſhall there runne out at the greateſt Cocke $\frac{1}{2}$, that is, 36 Barrels, and at the ſecond hole $\frac{3}{8}$, that is 27, and at the third Cocke $\frac{1}{3}$, that is 24, and at the ſmalleſt hole $\frac{1}{4}$, that is 18 Barrels, which altogether doe make 105, and ſhould be but 72, ſo is it too much by 33; therefore doe I ſet the errors in order of the figure with their poſitions, and worke by Multiplication, in croſſe, ſaying, two times 3 is 6, and two times 33 maketh 66; and becauſe the ſignes are vnlike,

24
18
16
12
70

2 3

2 — 33 +

I must adde those two totals together, which make 72: also I adde the two errors, and they make 35, by which I diuide 72, and the Quotient ariseth $2\frac{2}{35}$, whereby I see that all the Cockes being set open, the water will auoid in two houres, and $\frac{2}{35}$ of an houre.

Master. This exercise maketh you to grow expert in the Rule. Therefore I will in-ure you somewhat more with a question or two.

A question of partners. The ninth example.

There were two men that had been partners, and had in account between them 300 *Duckets; whereof the one should haue for his part* 180, *and the other* 120: *but in the parting of them, they fell at variance; so that each of them catched as many as he could: yet afterward being reconciled, they agreed that he which had gotten most part of them, should lay downe* $\frac{3}{4}$ *of them againe, & he that had gotten least, should lay downe* $\frac{1}{3}$ *of those which hee had taken, and then parting them into two equall parts, each man to haue halfe thereof, and so had they their iust portions, as they ought: now I demand of you what each of them had gotten by the scambling?*

Scholar. I suppose he that had least, got 108 duckets, then the other had 192: wherefore in laying downe againe of the 192, there was put downe $\frac{3}{4}$, that is 144, and so had hee left but 48. Also of the 108, there was laid downe 36, that is $\frac{1}{3}$, and so hee had left 72. Then I put together 144 and 36, and it maketh 180, which I part into two parts euen, and so commeth 90 to bee giuen to each of them:

them: which ſumme put to 72, maketh 162, and ioyned to 148, it maketh 238: and now I doubt how I ſhall goe forward.

Maſter. You néede not to take but one of them, which you liſt, the greater or the ſmaller, for all commeth to one purpoſe: and ſo may you compare it that you take to any of the other ſummes, remembring that you make comparíſon to the ſame in the ſecond worke: as for example of the firſt part. If you compare 138 with the leſſer ſumme due, that is, 120, ſo is it 18 too much; and if you compare it with the greater ſumme, then is it 42 too little. Againe, if you compare 162 to the greater ſumme, the errour will be 18, as it was in the other: but it will haue a contrary ſigne: and if you compare it with the leſſer ſumme, it will be 42 too much: ſo that the error both waies is either 18, or 42: and as for the ſignes it little forceth, for in them is nothing conſidered here, but likeneſſe & vnlikeneſſe, which in this caſe doth neither further or hinder: But now goe on with the worke. Note.

Scholar. If it be ſo, then am I out of my greateſt doubt. Then I ioyne that 90 (which I found as the halfe of the latter partition) vnto 48, which is left with the one man, and ſo hath he 138, which (I may ſay) is 18 for many, for the leaſt ſhould be but 120: that error doe I note, and then make a new poſition, ſuppoſing the one man to haue 204, and the other to haue 96; wherefore of the 204, there

must bée laid downe 153, and so remaineth with him 51. Also of the 96, there must be laid downe $\frac{1}{3}$, that is 32, and so resteth with that man 64: Now of the 153 and 32, I make one summe, as 185, which I must diuide into two equall parts, and so each man shall haue $92\frac{1}{2}$, whereunto if I adde their former portions reserued, then the one shall haue $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherefore take the lesser summe now againe, as I did before, that is $143\frac{1}{2}$, and finde that he hath too many by $23\frac{1}{2}$; for he should haue but 120, so haue I for my two positions two errors, which I set down as here may be seene, each error vnder his position, & then by the Rule I doe multiply in crosse-wayes 108 by $23\frac{1}{2}$, and there riseth 2538, which I note, then againe I multiply 96 by 18, and thereof amounteth 1728.

108	96
18 +	$23\frac{1}{2}$ +

Now because the signes are both like, that is, both too many, I must worke by Subtraction, and so abating 1728 out of 2538, there will rest for the Diuidend 810: then for the Diuisor I subtract 18 out of $23\frac{1}{2}$, and there remaineth $5\frac{1}{2}$, by which I diuide 810, and the quotient will be $147\frac{3}{11}$, which is the iust portion of him that had the least summe. And if I doe subtract it out of 300, being the totall summe, then will there remaine $152\frac{8}{11}$, as the portion that the other did get.

Master.

Maſter. For the proofe of this worke, you may chooſe whether you will examine thoſe numbers according to the forme of the queſtions, or elſe worke by other two poſitions for to finde the ſecond number: and if thoſe poſitions bring the ſame numbers that did amount by the firſt two poſitions, then doth each work confirme other. Note

Scholar. By your patience, I will proue both wates, not onely to ſeeke their agreement, but alſo to accuſtome my minde to thoſe workes, for I perceiue it is exerciſe that muſt be the chiefe engrauer of theſe Rules in my memory.

Maſter. You conſider it well: then go to.

Scholar. Firſt I will by two other poſitions try to finde the portion of him which had moſt.

Maſter. Although you may doe it with any poſitions, yet to ſee the agreement of your worke the better, take the ſame poſitions that you did before, comparing them now to the greater, as you did before vnto the leſſer.

Scholar. Then I ſuppoſe that hee that had moſt, had 192, ſo had the other 108. Now if I take $\frac{3}{4}$ out of 192, that will be 144, and there will reſt to that man but 48. And from the ſecond which had 108 if I take $\frac{1}{3}$, that is 36, there will remaine to him 72: then ioyning 144 with 36, it will make 180, the halfe whereof being 90, If I adde to each of thoſe

two mens portions remaining with them, the one shall haue 138, and the other 162, of which two I take the greater, (that is 162) and see it to bee 18 too few, for it should be 180, that errour I note vnder this position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take $\frac{1}{4}$ of 204, and it will bee 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is 32, and there remaineth to him 64; now put I that 32 to 153, and it valueth 185: which being parted in equall values, maketh $92\frac{1}{2}$ to be added to each mans remainder, and to the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greatest summe, and it is $23\frac{1}{2}$ too little, that doe I note also and set both these errors vnder their positions, as in this example following doth appeare.

And then multiplying 102 by $23\frac{1}{2}$, there doth arise 4512.

Againe, I multiply 204 by 18, and it maketh 3672, which I doe subtract out of 4512, because the signes be like, and there resteth 840 for the diuidend, then subtracting 18 out of $23\frac{1}{2}$ there will remaine $5\frac{1}{2}$, which I must take for the Diuisor. And so diuiding 840 by $5\frac{1}{2}$, the quotient will be $152\frac{8}{11}$, whereby I haue found an agreeable summe to that which I found by the former Positions, for him that had most

most, which I dee subtract out of 300, that is the totall, th re will rest $147\frac{3}{11}$, which was the portion of him that had the least part.

Master. So by diuers politions, you see that one doth confirme the worke of the other. Now examine those two numbers by the forme of the question, & so shall you proue your worke good also.

Scholar. If that he which gate most had $152\frac{8}{11}$, then must dee lay downe $\frac{1}{4}$ of this summe. That is $114\frac{6}{11}$, and so shall remaine with him but only $38\frac{2}{11}$. The other which had least, that is $147\frac{3}{11}$, must put downe of his summe $\frac{1}{3}$, that is $49\frac{1}{11}$, and so doth there remaine with him yet $98\frac{2}{11}$. Then doe I adde together $114\frac{6}{11}$, and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, which I must part into two equall parts, and that will be $81\frac{9}{11}$ to be giuen to each of them: putting $81\frac{9}{11}$ vnto $38\frac{2}{11}$, there doth amount 120 iust, which is the true portion of him that should haue the lesser summe: and adding $81\frac{9}{11}$ $98\frac{2}{11}$, the totall will bee 180, the true portion of the other. And so is the worke by this proofe also tryed to be good. And this I marke by the way, that in their scambling he got most (as it chanceth often) that ought to haue had least by iust partition.

Master. Let your study bee to learne truth and iust art of proportion, & to distribute and part according thereunto, as often as occasion shall be ministred. And here would I make an end of this rule, saue that I remember

one

one pleasant question which I cannot ouer-passe, which I will declare somewhat largely, because you shall as well vnderstand some reason in the pleasant inuention, as apt proceeding in the witty working thereof.

The tenth example of mixture of gold and siluer.

Hiero king of the Syracusans in Sicilia, had caused to be made a Crowne of gold of a wonderfull weight, to be offered for his good successe in wars: in making whereof, the Goldsmith fraudulently tooke out a certaine portion of gold, and put in siluer for it, so that there was nothing abated of the full weight, although there was much of the value diminished.

Which thing at length being vttered, (as no euill can alwaies lie hid) the King was sore mooued: and being desirous to trie the truth without breaking of the crowne, proponed the doubt to Archimedes, vnto whose wit nothing seemed vnpossible, which although presently he could not answer vnto, yet he had good hope to deuise some policy for that invention, and so musing thereon, as he chanced to enter into a baine full of water to wash him, hée obserued, that as his body entred into the baine, the water did runne ouer the Tub, whereby his ready wit of such small effects conjecturing greater workes, conceiued by and by a reason of solution to the Kings question, and therefore reioycing exceedingly, more then if hée had gotten the Crowne it selfe, forgate that hée was naked, and so ranne home, crying as hée ranne, *Ἑυρηκα Ἑυρηκα*, I haue found, I haue found. And thereupon caused two massy pieces,

pieces, one of gold, and another of ſiluer, to be prepared of the ſame weight that the ſaid Crowne was of: and conſidering that gold is heauier of nature then ſiluer, and therefore gold of like weight with Siluer, muſt nœds occupy leſſe rome by reaſon it is more compact and ſound in ſubſtance, he was aſſured that putting the maſſe of gold into a veſſell brim full of water, there would not ſo much water run ouer, as when hee ſhould put in the ſiluer maſſe of the like weight. Wherefore hee tried both, and noted not onely the quantities of the water at each time, but alſo the difference or exceſſe of the one aboue the other, whereby hæ learned what proportion in quantity is betwœne Gold and Siluer of equall weight. And then putting the crowne it ſelfe into the veſſell of water brim full (as before) marked how much water did runne out then, and comparing it with the water that ranne out when the Gold was put in noted how much it did exœd that: and likewiſe comparing it to the water that ranne out of the ſiluer, marking how much it was leſſe then that, and by thoſe proportions found out the iuſt quantity of gold that was taken out of the Crowne, and how much ſiluer was put in ſtead of it: But ſeeing Vitruuius which writeth this Hiſtory doth not declare the particular worke of this triall, it ſhall be no inconuenience to ſuppoſe an example for declarations ſake, wherein although the true and iuſt

pro-

proportion be not expressed, yet the forme of triall shall be truely set forth. And for an example, I suppose the weight of the crowne to be 8 pound, and so of each the other two Masses. And when the Masse of Gold was put into the water, I imagine that there ran out two pound of water: and when the masse of siluer was put in, I suppose there ranne out three pound $\frac{1}{2}$. Againe, when the Crowne was put in, there ranne out two pound $\frac{1}{4}$: Now to know what quantity of siluer was in the Crowne, works by the Rule of false position, and imagine that there was two pound of siluer, then must there be fiue pound of gold, then say thus by the Rule of Proportion: If eight pound of gold doe expell two pound of water, what shall fiue pound expell? and it will be 1 pound $\frac{1}{2}$. Againe for the siluer: if eight pound of siluer expell three pound $\frac{1}{2}$ of water, what shall 2 pound of siluer put out? it will bee $\frac{7}{8}$, now adde those two weights of water together, & they will make two pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{2}{4}$ so is it too much by $\frac{1}{8}$.

Scholar. Now doe I understand the worke as I thinke, therefore I pray you let me worke the rest of the question. And because this first supposition did erre, I note that position and his errour, and take a new position, esteeming the Siluer to be but one pound, so must there be in gold seuen pound. Then say I: if eight pound of gold doe yéeld two

pound

pound of water, what ſhall ſeuen pound yéld? and it will be 1 pound $\frac{3}{4}$. Againe, if 8 pound of ſiluer expell 3 pound $\frac{1}{2}$ of water, what ſhall 1 pound expell? and it will be $\frac{1}{16}$. Now muſt I adde thoſe two ſummes together, and they make 2 pound $\frac{5}{16}$. and they ſhould make 2 pound $\frac{1}{4}$, ſo is it too little, by $\frac{1}{16}$. Therefore I ſet the poſitions, with their errors, in order as here followeth: And then I multiply in croſſe wayes 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: Likewiſe 1 multiplied by $\frac{1}{8}$, maketh $\frac{1}{8}$. And becauſe the ſignes be vnlike, I muſt adde theſe two ſummes, which make $\frac{1}{4}$: and that is the dividend.

2 1

$\frac{1}{8}$ + $\frac{1}{16}$ −

Againe, I muſt adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be $\frac{3}{16}$, that is the Diuiſor. Now I ſhall diuide $\frac{1}{4}$, by $\frac{3}{16}$, and the quotient will be $\frac{16}{12}$, that is, 1 $\frac{1}{3}$: whereby I know that there was put 1 pound and $\frac{1}{3}$ of ſiluer into the Crowne, and ſo much Gold taken out for it.

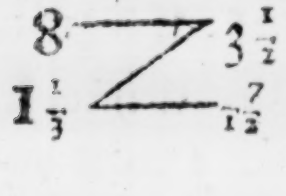
Maſter. Proue it now by examination, according to the queſtion.

Scholar. If there were 1 pound $\frac{1}{3}$ of ſiluer, then was there of gold 6 pound $\frac{2}{3}$. Now ſay I by the Rule of Proportion:

If 8 pound of gold expell two pound of water, what ſhall 6 pound $\frac{2}{3}$ expell?

8 — 2

$6\frac{2}{3}$ — $1\frac{1}{3}$



It will be 1 pound $\frac{2}{3}$. Againe If 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall $1\frac{1}{3}$ expell? It will be $\frac{7}{12}$. Now must I adde together 1 pound $\frac{2}{3}$, and $\frac{7}{12}$, & they will make 2 pound, $\frac{9}{36}$, that is 2 pound $\frac{1}{4}$, according to the suppositiõ of the question: whereby I perceiue the worke to be well done. And I cannot but much reioyce of this excellent inuention, so my desire is kindled vehemently to be perfectly instructed in euery part thereof, and namely in this point, whether the proportion betweene water and gold be such, that for 8 pound of gold put into a vessell full of water, there shall runne out two pound of water, & for as much siluer, whether 3 pound $\frac{2}{3}$ of water would auoyd.

Master. I perceiue your meaning, and coniecture your imagination to be thus; that if you knew the exact proportion betweene gold and siluer, and water, both in their weight and quantities, then could you easily finde out the mixtures of them, which thing I haue reserued for another worke that intreateth of such matters especially. And at this time you must consider that you learne Arithmeticke, which intreateth of the manner to solue doubtfull questions touching number, without regard what matter is signified by that number: else were it necessary in Arithmeticke to teach all Arts, seeing in it may be moued questions of all Arts.

Note.

But ſeeing you are ſo deſirous to know theſe things, I will tell it you in ſuch a ſort, that you ſhall practiſe your Art in finding it, and propound it in forme of a queſtion. Gold beareth greater proportion to water, then ſiluer doth, and their two proportions be in proportion together, as forty eight to twenty fiue. But to helpe you ſomewhat in this riddle, you ſhall note that the proportion of quick ſiluer vnto water, is the iuſt middle number proportionall in Progreſſion Geometricall betweene the proportion of gold and ſiluer vnto water.

A queſtion of the proportion of gold, ſiluer, and quick-ſiluer, vnto water.

And this proportion **is** $\frac{290}{21}$. **Now if you will know the iuſt** numbers **of theſe** three proportions, **then muſt you finde out three** numbers **in** Progreſſion Geometricall, **whereof the** middlemoſt **muſt be** $\frac{290}{21}$, **and the** firſt **muſt be vnto the** laſt, **as** 25 **to** 48. **And thus I will leaue you to finde thoſe** numbers **when you be at leiſure.**

Scholar. **Yet Sir, I thanke you heartily for thus much, for now I ſee the poſſibility to finde them out. Howbeit, becauſe this** queſtion **ſeemeth ſtrange, if it might pleaſe you to inſtruct me ſomewhat in the order of working for it, I ſhould the more eaſily finde the true working.**

Maſter. **You deſire too much if you will ſtudy for nothing: Therefore to occaſion you to ſtudy the better, I will leaue this doubt wholly to your owne ſearch: But as touching the generality of the Rule, Archimedes** needed not to take two Maſſes of Gold

and

and Siluer equall in weight with the Crowne, for the proportion might as well be found in any other weight, yea, although the Masse of Gold were of one weight, and the Masse of Siluer of another. As for example: If the Crowne were of eight pound weight as I did suppose, and I haue not so much other fine Gold, but onely one pound, and trying that by Water, and finding that it doth expell but ¾ of an ounce of water, yet then by it I may inferre, that 8 Pound of Gold would expell 6 ounces of water. And likewise of Siluer, whereof if I had but two pound, and find that it doth expell three ounces of water, then might I affirme that 8 pound would expell 12 ounces, that is, one pound weight: and so is it as good as if the three Masses were all of one weight. And thus for this time I will make an end of this other part of Arithmeticke.

Sch. Although I cannot sufficiently thanke you for this, yet your promise made mee to looke for the Art of extraction of Rootes, whereof hitherto I haue learned nothing.

Master. I will not breake my promise, but intend (God willing) to performe it within this three or foure moneths, if I perceiue this my paines to bee well taken in the meane season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the meane time, you shall take this Addition, not for the second Part of Arithmeticke which I promised, but for an ang-

augmentation of the firſt part, vnto which I would haue annexed the extraction of Rootes ſquare and cubike, namely for examples of the Statute of Aſſiſe of wood, but that in the ſecond part I muſt write of diuers other Roots, and thought it beſt to reſerue thoſe Rules alſo with their examples vnto the ſame ſecond part.

Scholar. Sir, although I cannot recompence your goodneſſe, yet I ſhall alwayes doe mine endeuour to occaſion you not to repent your benefit on me thus imployed.

Maſter. That recompence is ſufficient for your part.

FINIS.

The third part

or Addition to this Booke,
entreateth of breife Rules, called
Rules of Practise, of Rare, Plea-
sant, and commodious effect,
abridged into a briefer me-
thod then hitherto hath
beene published.

With diuers other necessa-
rie Rules, Tables, and Questi-
ons, not only profitable for Mer-
chants, but also for Gentlemen,
and all other occupiers
whatsoeuer, as by the
Contents of this
Booke may
appeare.

Set forth by IOHN MELLIS
Schoole-master.

* *
*

The first Chapter of this Addition, entreateth of briefe Rules, called Rules of *Practise*, with diuers necessarie questions, profitable *not only for Merchants, but also for all other occupiers whatsoeuer.*

He working of Multiplicati- 1 Rule.
on in Practise, is no other thing then a certaine manner of multiplying of one kinde by another: whereupon is brought forth the product of the proponed number, which is accomplished by the meanes of Diuision in taking the *halfe*, the *third*, the *fourth*, the *fifth*, or such other parts of the summe, which is to be multiplied.

And for the better vnderstanding of such conuersions, you shall vnderstand that in the manner and vse of these rules of practise, you ought first to know the euen or aliquot parts of a shilling, which in this table following doth appeare.

Item { 6 4 3 2 1 } *pence is the* { 1/2 1/3 1/4 1/6 1/12 } *of a shilling.*

Wherein as you see according to the order

of theſe rules of Practiſe: at 6 pence the yard of any thing, you muſt take $\frac{1}{2}$ of your number which is to be multiplied, and the product that commeth thereof ſhall be ſhillings, if any vnite doe remaine it is 6 pence.

For 4 d, take the $\frac{1}{3}$ of the number that is to be multiplied, and the product alſo produceth ſhillings, if any vnites doth remaine, each one ſhall be worth in value 4 pence. The like is to be vnderſtood of the other 3, &c.

I *Example.*

At 6 d the yard, what 379 yards?

189 ſ——6 d

II

At 4 d the yard, what 104 yards?

34——8 d

III

At 3 d the yard, what 5014. yards?

1253. ſ——6 d

IIII

At 2 d the yard, what 532 yards?

88 ſ——8 d

V.

At 1 d the yard, what 409?

34 ſ——1 d

Here you may ſee in the firſt example, that 379 yards at 6 d the yard, are worth 189 ſ, 6 d in taking the $\frac{1}{2}$ of 379. And in the ſecond example the 104 yards at 4 pence the yard, are worth 34 ſ 8 d, in taking the $\frac{1}{3}$ of 104. Likewiſe in the third example, 5014 yards at

3 d

3 d the yard, bringeth forth 1253 ſ 6 d in taking the $\frac{1}{4}$ of 5014. Alſo in the fourth example at 2 d the yard, maketh 88 ſ 8 d.

And laſtly, in the fifth example: 409 yards at 1 d the yard, amounteth to 34 ſ 1 d, in taking the $\frac{1}{12}$ of 409: and ſo is to be done alſo of all other queſtiõs the like, when the number of the pence is any of the euen or aliquot parts of 12 d

Item, to bring the products of theſe ſhillings, and all other the like into pounds is very eaſie in diuiding of it in your mind by 20, for it is to be vnderſtood that as often as 20 is found in that product, ſo many pounds doth it containe: which with facilitie to performe, alwaies ſtrike off the figure towards your right hand, with a right downe daſh of your pen, for the 0 that appertaineth to the 20 And then begin at the left hand, in taking the halfe of the reſt. And if that at the laſt any vnite doe remaine, the ſame ſhall be ioyned with the figure that is cut off, which ſhall repreſent the odde ſhillings contained in that worke.

As for example, in your third queſtion at 3 d the yard, which amounteth to 1253 ſ 6 d the product whereof maketh 62 li 13 ſ 6 d, as here you may ſee, is eaſily performed by this example.

	1
125	3
62	13 ---- 6

Alſo for the working of one peny the yard, it is ſomething harſh and hard to take the $\frac{1}{12}$ of ſome products: therefore to caſe that hard worke, you ſhall firſt bring your deliuered

ſumme into groats, by taking $\frac{1}{4}$ part of the product, and if any vnites remaine of that $\frac{1}{4}$ part, as ſometimes there may, they are pence: & muſt be ſignified with a line from the *groats* with their title of *Pence*; and becauſe that 60 *groats* maketh a *Pound* or twentie ſhillings, *ſtrike* off the firſt figure toward your right hand, for the o that appertaineth to 60 (as you did euen now for the o that belongeth to 20:) Then in taking the $\frac{1}{6}$ of that product, if there do remain any vnites, the ſame ſhal you ioyne with the figure that you cut off, eſteeming them as *groats*, which keepe in your minde, & by taking the $\frac{1}{3}$ part of them, you ſhall turne them into *ſhillings*, and ſo haue you done: *As for example*, by a queſtion or two hereafter proponed, ſhall more plainely by the worke appeare.

At 1 d, the yard, what 54368 yards?

1359|2 *groats*.

$\frac{1}{6}$ li ——— 226-10 ſ.-8 d.

Here in taking the $\frac{1}{6}$ part of 1359, in comming to the laſt worke, the $\frac{1}{6}$ part of 39 being taken, the *remainder* is 3, which ioyned with the two that was cut off, maketh 32 *groats*, which conuerted into *ſhillings* by taking the $\frac{1}{3}$ part, maketh as appeareth 10 *ſhillings* 8 d. Many other wayes there are, but none more apt for a *young learner* to vnderſtand then this: wherfore this one way well impreſſed in memorie, is better then 20 wayes doubtfully vnderſtood.

As

At 1 peny the yard, what 4533 yards?

1133 | 3 groats-1d

li 188—17-9d

At 1 peny the yard, what 64768 yards?

16192 groats

li 269-17 4d.

NOw followeth alſo to be vnderſtood that if the number of pence be not an aliquot part of 12, you muſt reduce them into ſome aliquot part of 12: and after the aforeſaid manner, you ſhall make of them two or three products, as need ſhall require, and adde them together into one ſumme. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken. 2 Rule.

for pence	take	and	or	&
5	3	2	4 2	1
7	4	3	6	1
8	4	4	6	2
9	6	3	4.4	1
10	6	4	4.4 and	2
11	6 4 &	1	4 4	3

Here in the firſt note of this *Table* at 5 d, you ſhall firſt take for 3 d the $\frac{1}{4}$ of the number that is to be multiplied: and likewiſe for 2 d, the $\frac{1}{6}$ of the ſame number, adding together both the products: But if you will worke by 4 and 1, you muſt for 4 d, firſt take the $\frac{1}{3}$ of the number that is to be multiplied: and for 1 d, take the $\frac{1}{12}$ of the whole ſumme, or rather, which is more better, for 1 peny you may take the $\frac{1}{4}$ of the product which did come of the 4 pence:

pence: becauſe that 1 d, is the $\frac{1}{4}$ of 4 pence. The totall ſummes of theſe two numbers ſhall bee the ſolution to the *queſtion*. And in like manner it to be done of all others, as by theſe examples following ſhall appeare.

I

At 5 d the yard, what	758 yards?
3 d	189——6 d
2 d	126——4 d
ſhillings,	325——10 d

Otherwiſe.

At 5 d the yard, what	758 yards?
4 d	252——8 d
1 d	63——2 d
ſhillings	315——10 d

II

At 7 pence the Ell, what	563 Elles?
4 d	187——8 d
3 d	140——9 d
ſhillings	328——5 d

III.

At 8 d the pound, what	112 pound?
4 d	37——4 d
4 d	37——4 d
ſhillings	74——8 d

Otherwiſe.

At 8 pence the pound, what	112 pounds?
6 d	56——0
2 d	18——8
ſhillings	74——8 d

IIII.

IIII

At 9 pence the Elle, what — 356 Elles?

6 d	178——0
3 d	89——0
shillings	267——0 d

V.

At 10 pence the peece, what — 795 peeces?

6 d	397——6
4 d	265——0
shillings	662——6

VI.

At 11 pence the pound, what — 7576 pounds

6 d	2788——0
4 d	2525——4
1 d	631——4
	6944——8d

Pounds. 347–4 s–8 d

1 Here in this first example, where it is demanded (at 5 d the yard) what will 758 cost? First, for 3 d, I take the $\frac{1}{4}$ of 758: and thereof commeth 189 s. 6 d: Then for 2 d I take the $\frac{1}{6}$ of the same 758, which amounteth to 126 s 4 d, these two summes added together, doe make 315 *shillings* 10 *pence*: and so much are the 758 *yards* worth at 5 d the *yard*.

1 Item also for the same againe: First for 4 d, I take the $\frac{1}{3}$ of 758; and thereof commeth 252 s 8 d: then for 1 peny I take the $\frac{1}{4}$ of the same 758, that is to say, of 252 s, 8 d, and it yeeldeth me 63 s, 2 d: which both added together maketh 315 s—— 10 d, as before.

2 *Item, for* 7 d *there is taken the* $\frac{1}{3}$ *and the* $\frac{1}{4}$ of the whole summe which is to be multiplied,

and adde them together, that is to ſay, firſt, for 4 pence there is taken $\frac{1}{3}$ of 563 : which comes to 187 ſ—8 d, as appeareth by the worke, and for 3 d there is taken the $\frac{1}{4}$ of the whole Summe, which amounteth to 140ſ–9 d. Both which products added together, do make 328 ſ–5 d, and ſo much comes 563 *elles* to at 7 d the *elle*.

3 *Item, for the firſt* 8 d *there is taken for* 4 d, the $\frac{1}{3}$ of the whole ſumme, and another $\frac{1}{3}$ for the other 4 d, which added together, as in the example doth euidently appeare, amounteth to 74 ſ–8 d.

Againe, for the ſecond worke of 112 li. there is taken firſt the $\frac{1}{3}$ of the whole ſumme for 6 d, which comes to 56ſ: then for that 2 d you haue to take $\frac{1}{6}$ of the whole ſumme, or if you will, the $\frac{1}{3}$ of the product that came of 6 d either of which maketh 18 ſ–8 d. Theſe two ſummes being added together, doe make 74 ſ. 8 d. as in the third example appeareth.

4 *Item, for* 9 d *there is taken for* 6 *pence* the $\frac{1}{2}$ of the whole ſumme: and the $\frac{1}{4}$ of the whole ſumme for 3 d, or otherwiſe for the 3 d, you may take the $\frac{1}{2}$ of the product that came of 6 d, becauſe 3 pence is the $\frac{1}{2}$ of 6 d: which added together, as plainely appeareth in the *fourth example*, amounteth to 267 ſ, 0 d.

Item, for 10 d, *firſt there is taken for* 6 d, the $\frac{1}{2}$ of the whole ſumme, which amounteth to 397 ſ–6 d. Then for 4 d there is found 265 ſ, both which added together, make 662

ſhillings

ſhillings, 6 d, as appeareth in the *fift example*: It may alſo be wrought, *as appeareth by the ſecond note in the Table*, by 4 d twice taken, and the $\frac{1}{2}$ of the product of 4 d: or elſe by the $\frac{1}{6}$ of the whole ſumme, &c.

Item, for 11 d, *there is firſt taken the* $\frac{1}{2}$ *for* 6 d: then the $\frac{1}{3}$ of the whole ſumme for 4 d: Laſtly, the $\frac{1}{4}$ of the laſt product for 1 d. All which 3 ſummes added together, maketh in ſhillings 6944 ſ-8 d, and in pounds 347-4 ſ-8 d.

Item, likewiſe by the ſame reaſon, when you will multiply (by ſhillings) any number that is vnder 20 ſ, *you ſhall haue in the product pounds, if you know the euen or aliquot parts of* 20, *which are here in this little Table ſet downe to ſight.* 3 Rule.

Item ſ { 10, 5, 4, 2, 1 } is the { $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$ } of one pound.

So that for 10 ſ *which is the* $\frac{1}{2}$ *of a pound*, you may take the $\frac{1}{2}$ of the number which is to be multiplied, and you ſhall haue in your product pounds: if an vnite doe remaine, it ſhall be worth ten *ſhillings*.

Likewiſe for 5 *ſhillings you muſt take the* $\frac{1}{4}$ of the number which is to be multiplied, and if there doe remaine any *vnites*, they ſhall be fourth parts of a *Pound*, euery vnite being in value fiue *ſhillings*.

For 4 *ſhillings take the* $\frac{1}{5}$ *of the number* which

is

is to be multiplied: and if there doe remaine any *vnites*, they shall be fift parts of a *Pound*, each vnite being in value 4 *shillings*.

For 2 *shillings you must take the* $\frac{1}{10}$ *of the* number to be multiplied, wherefore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same number (which is neerest your rigt hand) from all the other figures with a smal right downe line or dash with a pen, & so haue you done: for al the other figures which doe remaine toward your left hand from the same figure that you doe separate, shall be the said $\frac{1}{10}$ of a *Ponud*: & that figure so separeted towards your right hand, shall be so many peeces of 2 s the peece, the which figure you must double to make thereof the true number of shillings, as by the example shall appeare.

Finally for 1 *shilling needeth small worke*, for it is to many shillings as be proponed in the summe, which to bring into pounds, hath beene already taught in the *first rule*.

Example.

At 10 s the peece, What	li	6543 peeces ?
$\frac{1}{2}$		3271 10 s.
At 5 s the Elle. What	li	4373 Elles ?
$\frac{1}{4}$		1093 . 5 l.
At 4 s the yard, What	li	7839 yards ?
$\frac{1}{5}$		1567 . 16 s.
At 2 s the pound waight, What	li	7527 pound
$\frac{1}{10}$		752 . 14 l.

At 1 ſ the peece, What 757 | 3 peeces?
$\frac{1}{20}$ li 378 13 ſ.

NExt, followeth in order to be vnderſtood, that if the number of ſhillings be not ſome euen or aliquot part of 20, you muſt then conuert the ſame number of ſhillings into the aliquot parts of twenty, and thereof make two or three products as need ſhall require: which done, adde them together, and bring them into Pounds. And here for thy furtherance I haue ſet downe a note of the order of their parts, as they are to be taken.

ſ		ſ		ſ		ſ
3		2 € 1		13		10.2 € 1
6		4 € 2		14		10. € 4
7		5 € 2		15		10: € 5
8	of	4 € 4	of	16	of	10 5. 1
9		5 € 4		17		10.5.2
11		10 € 1		18		10.4.4
12		10 € 2		19		10.5.4

For 3 ſ *according to the tennor that you ſee* is expreſſed in the Table, you muſt firſt take for 2 *ſhillings* the $\frac{1}{10}$ of the number that is to be multiplied. Then for one *ſhilling* you muſt take the $\frac{1}{2}$ of the product which did come of the ſame $\frac{1}{10}$ part: which two ſummes added together produceth the effect deſired.

Item, for 6 *ſhillings according to the note* ſet forth in the table firſt for 4, ſ I take the $\frac{1}{5}$ of the

number

number that is to be multiplied: Then for 2 ſ, the $\frac{1}{2}$ of the product that came of 4 ſ, and adde them together.

Or elſe as appeareth alſo in the Table, for 5 *ſhillings* you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ part of the product that came of 5 *ſhillings*, and adde them together.

Item, for 7 ſ, *firſt for* 5 ſ, *take* $\frac{1}{4}$ *of the product* that is to be multiplied, then for 2 ſ, take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, &c.

Item, for 8 ſ, *according to reaſon, and the* intent of the *Table*, for the firſt 4 ſ, take the $\frac{1}{5}$ of the product, and the ſame number againe for the other 4 ſhillings, and adde them together.

Item, for 9 ſhillings: firſt for 5 ſhillings, take *the* $\frac{1}{4}$; then for 4 ſhillings take the $\frac{1}{5}$: and adde them together.

Otherwiſe as you ſee by the intent of the Table, worke twice for 4 ſhillings as was taught euen now for 8: and then take the $\frac{1}{4}$ of the laſt product for the 1 ſhilling: but 5 and 4 is the ſhorter.

Item, for 11 ſ, *firſt diſpatch* 10 *ſhillings*: for which you muſt take the $\frac{1}{2}$ of the product, then laſtly, for 1 ſhilling take the $\frac{1}{10}$ part of the ſumme produced of the $\frac{1}{2}$ of the product, and adde them together.

Item, for 12 *ſhillings, where I will end with the firſt part of my Table.* Firſt take the $\frac{1}{2}$ for 10 *ſhillings.* And then for 2 *ſhillings*, take the $\frac{1}{5}$ of the ſumme that came of 10 *ſhillings*, take and adde

adde them together, or else if you please for 2 shillings, you may take the $\frac{1}{10}$ of the whole giuen number.

To write more of the manner of taking the true parts, I omit. The desirous practitioners will no doubt conceiue it. Also the *Table* is some ayde to helpe the vnperfect, whereupon by and by I will set downe three or foure of these notes in *Examples*, and the rest I will leaue to thine owne industrie and practise, to labour vpon.

This is the order most commonly vsed in practise, when the number of shillings is not an *aliquot* part of a pound. But (*louing Reader*) after I haue touched the euen or aliquot parts of a pound that falleth out in pence and shillings, I will deliuer two new rules, that shall drowne this common order quite and cleane: wherein shall be comprehended in one line, or working both of euen and odde parts of shillings vnder 20, without regard whether it be an *aliquot*, or not an *aliquot* part: which two Rules (when they come in place,) I commit to thy Friendly iudgement in working:

Now follow the examples vpon the notes beforesaid.

At 6 shillings the yard, What	3215 yards?
4 shillings	643
2 shillings	321—10
	964—10 s.

Otherwise

Otherwise by Multiplication of 6,

3215

6 *shillings* 1929|0

li——————964——10 *shillings.*

At 7 shillings the Ell, What 4563 Elles?

5 shillings	1140—15
2 shillings	456——6
li	1597–1 shilling.

Otherwise by Multiplication of 7

4563

7 [3194|1

1597—1

At 8 ſ the peece, What 7563 peeces?

4 ſ	1512. 12
4 ſ	1512. 12
Pounds	3025. 4 ſ

Otherwise by Multiplication.

7563

8 ſ 6050|4

Pounds 3025—4 ſhillings.

At 13 ſ the peece, what 401 peeces?

10 ſ	200——10
2 ſ	40——2
1 ſ	20——1
Pounds	260——13

Otherwiſe

Otherwiſe by Multiplication.

	401
13 ſ	1203
	401
	521 | 3
Pounds	260—13 ſ

Theſe and ſuch like queſtions of compound numbers, which I haue here in this fourth rule for orders ſake ſet downe, for that it hath beene heretofore a common courſe of worke, I account but ſuperfluous. For in the eighth and ninth rules of this my ſimple Addition ſhall appeare, that the giuen price of any euen or odde number of ſhillings, either vnder or aboue 20, ſhall be wrought at one or two workings at the moſt, how difficult ſoeuer the queſtion be.

5. Rule. *Item, there reſteth yet a kinde of practiſe, how to bring Pence into Pounds at the firſt working: whereupon you muſt vnderſtand that* 240 *pence maketh one pound, or* 20 ſ. *In conſideration wherof I cut off the laſt figure or* 0, *and there remaineth but* 24 (*of which* 24) 8 d *is the* $\frac{1}{3}$ *part thereof,* 6 d *is the* $\frac{1}{4}$ *part,* 4 d *is the* $\frac{1}{6}$ *part, and* 2 *pence is the* $\frac{1}{12}$ *part thereof.*

To reduce pence into pounds at one operation.

Whereupon if it were demanded what 1486 yards or pounds of any thing commeth to, at 8 pence the yard, in pricking or cutting off the firſt figure towards your right hand, for

the 0 that appertaineth to 240: There is remaining of the ſaid ſumme 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 li, and there reſteth 1, which 1 I put to the 6, that I pricke or cut off, and it maketh 16 peeces of 8 pence, which I double to make into groats, and they make 32, whereof the $\frac{1}{3}$ part maketh 10 ſ, and there remaineth $\frac{2}{3}$ ſ, which is 8 pence, whereby it followeth, that the 1486 yards at 8 pence the yard, maketh 49 li 10 ſ 8 d, as by the example ſhall appeare.

Item, for 6 pence, take $\frac{1}{4}$ part of the number from the prickt figure: and if any vnites do remaine, they are ſo many ſixe pences, whereof taking the $\frac{1}{2}$ they are ſhillings, if there doe remaine yet one, it is in value ſix pence.

Item for 4 d, take the $\frac{1}{6}$ part of the number from the prickt figure: If any vnites remaine, they are ſo many groats, which to conuert into ſhillings, take the $\frac{1}{3}$ part: And if any thing yet remaine, they are thirds of ſhillings, each one in value being worth 4 pence:

Item for three pence, take the $\frac{1}{8}$ part from the prickt figure, if any vnites remaine, they are ſo many peeces of 3 pence, whereof in taking the $\frac{1}{4}$ part, maketh ſhillings: If any thing yet remaine, they are fourth parts of ſhillings each one being in value 3 pence.

Item, for 2 pence, as appeareth alſo by the table, take the $\frac{1}{12}$ part of the number from the prickt figure: if any thing remaine, they are

ſo

ſo many peeces of 2 pence, which by taking the $\frac{1}{6}$ part, you ſhall turne into ſhillings: and if any vnites remaine, they are ſo many ſixt parts of ſhillings, or peeces of two pence, whether you will.

If one coſt 8 pence, what maketh pounds	148 | 6? 49-10--8 d,
If one coſt 6 pence, what maketh pounds	786 | 5? 196-12--6 d
At 4 pence the yard, what maketh pounds	873 | 6 yards? 145--12—0 d
If one coſt 3 pence, what maketh pounds	987 | 4 worth? 123--8—6 d.
At 2 d the Elle, what maketh pounds	789 | Elles to? 65—15—8 d:

BVt if your number of pence bee not an aliquot or euen part of 24: then muſt you bring them into the aliquot parts of 24, and make thereof diuers products which muſt be added together, as by the queſtion hereafter following ſhall appeare. 6 Rule.

Item for 5 d, firſt take for 3 d, then for 2 d, and adde them together, according to the inſtruction of the *ſecond Rule*: Or elſe firſt take for 4 d, then for 1 d.

Item for 7 d, firſt take for 4 d: then for 3 d, and adde them together.

Item for 9 d, firſt take for 6 d, then for 3 d,

and adde them together.

Item for 10 d, firſt take for 6 d, then for 4 d, and adde them together.

Item for 11 d, firſt take for 8 d, then for 3 d, & adde them together: as by theſe examples.

Examples.

1. If one yard coſt 5 d, What 759|6 ?

4 pence	126-12
1 peny	31-13
maketh pounds	158—5

Otherwiſe.

1——5——759|6

3 pence	94-19
2 pence	63—6
maketh pounds	158—51

2. If one coſt 7 d, what 98|7 worth

4 pence	16-9
3 pence	12-6-9
maketh pounds	28-15-9 d.

Otherwiſe.

1——7——98|7

6 pence	24-13-6
1 penny	4—2-3
maketh pounds	28-15-9 d

3. If one coſt 9 d, what 98|7 worth

6 pence	24-13-6
3 pence	12-6-9
maketh pounds	37-0-3 d

Otherwiſe.

Otherwise.

	1——9——98|7
6 pence	24-13-6
3 pence	12-6-9
maketh pounds	37 00 3

4. If one coſt 10 pence, what 98\|7?

6 pence	24-13-6
4 pence	16--9-0
maketh pounds	41—2--6

5. If one coſt 11 pence, what 98\|7?

8 pence	32-18--0
3 pence	12--6--9
maketh pounds	45--6--9

But if you haue any ſhillings and pence to be multiplied together: Then are you to take for the ſhillings according to the inſtruction of the third rule: and for the pence according to the firſt rule before mentioned: vnleſſe you can ſpie the aduantage thereof, and thereby helpe your ſelfe: as appeareth in this ſecond example, where firſt I worke for 6 d: which is to be rebated out of the giuen number, and I haue 719 li 11 ſ my deſire.

At 19ſ 6d the yard, what 738 yards?

	738		*Otherwiſe by Rebating.* 738
10ſ	369--		
5ſ	184-10		
4ſ	144-12	6d	18-9ſ
6d	18--9		719-11ſ
pounds	719-11ſ		

The like againe is done by rebating, as by theſe two examples appeareth.

At 18ſ the Elle, what 418 Elles?

2ſ	41---16
pounds	376----4ſ

At 16ſ the Elle, what 517 Elle?

4ſ	103---8
pounds	413--12ſ

7. Rule. *And now I will touch a little the euen part of a pound, that falleth out in pence and ſhillings, whereof for thoſe parts you ſhall take ſuch like part out of the giuen number that is to be multiplied, as the price of that giuen number beareth in proportion to a pound, which alſo for their better aide is here ſet downe.*

1ſ	8d		$\frac{1}{12}$
2	6	*is the*	$\frac{1}{8}$ *part of a pound*
3	4		$\frac{1}{6}$
6	8		$\frac{1}{3}$

Item, firſt for 1 ſhilling 8 pence take the $\frac{1}{12}$ part

part of the giuen number, and if any thing do remaine, they are twelue parts of a pound, each one being in value 1 ſhilling 8 pence.

Item, for 2 ſhillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplied. And if any thing doe remaine, they are eight parts of a pound, each one being in value 2 ſhillings 6 pence.

Item, for 3 ſhillings 4 pence, as appearth by the Table, you muſt take the $\frac{1}{4}$ part of the giuen number, and if any thing doe remaine, they are 6 parts of a pound, each one being in value 3 ſhillings 4 pence.

Item, for 6 ſhillings 8 pence, take the $\frac{1}{5}$ part of the number that is to bee multiplied: And if any vnites doe remaine, they are thirds of a pound, euery one being worth 6 ſhillings 8 pence.

Other infinite numbers there are, that may be reduced by abreuiatiō into the proportionate parts of a *pound*, as 16 ſhilling 8 pence maketh $\frac{5}{6}$: which 16 ſhillings 8 pence is eaſily reduced into groats, by multiplying 16 by 3. and thereto adde 2, which maketh 50 groats.

Then ſet 60 the groats of a pound vnder 50, cutting off the 2 ciphers, as is here performed.

And then haue you brought 16 *ſhillings* 8 d into the knowne parts of a *pound*, which maketh $\frac{5}{6}$.

16 — 8
3
5 | 0
6 | 0

But

But yet Gentle Reader, for thy further inſtruction, I haue hereunto annexed in a Table, how pence and ſhillings beareth proportion to a pound, which I commit to thy friendly beneuolence; it will be ſome aid vnto the vngrounded Practitioner: but I count him the beſt workeman that can preſently reduce his giuen price into the knowne and proportionate parts of a pound.

A Table of the aliquot parts of a pound, or 20 ſhillings.

s	d	£
0	2	1/120
0	3	1/80
0	4	1/60
0	6	1/40
0	8	1/30
1	0	1/20
1	3	1/16
1	8	1/12
2	0	1/10
2	6	1/8
3	0	3/20
3	4	1/6
3	9	3/16
4	0	1/5
5	0	1/4
6	0	3/10
6	8	1/3
7	0	7/20
7	6	3/8
8	0	2/5

s	d	£
8	4	5/12
8	9	7/16
9	0	9/20
10	0	1/2
11	0	11/20
11	3	9/16
12	8	3/5
12	0	3/5
13	0	13/20
13	4	2/3
13	9	11/16
14	0	7/10
15	0	3/4
16	0	4/5
16	8	5/6
17	0	17/20
17	6	7/8
18	0	9/10
18	4	11/12
18	9	15/16
19	0	19/20

Here follow foure examples vpon the foure notes deliuered.

At 1 ſ 8 d the yard, what 3884 yards?
maketh pounds 323—13-4 d

At 2 ſ 6 d the yard, what 4563 yards?
maketh pounds 570--7--6 d

At 6 ſ 8 d the Ell, what 7562 Ells?
maketh pounds 2520--13-4 d

Now by cuſtome you are able to worke by all ſorts of ſummes being deliuered in ſhillings and pence, as one ſhilling one penie, two ſhillings two pence, three ſhillings three pence, and ſo of all o-ther: wiſhing you to haue ſome conſiderations of your queſtions, when they are ſet downe, for there are many ſubtill abbreuiations, and great aduan-tages to bee gotten, and eaſily to bee percei-ued.

As of 3 ſ--8 d, of 2 ſ, and 1 ſ 8 d.
Of 4 ſ--2 d, of 3 ſ---4 d, and 10 d, which 10 d is $\frac{1}{4}$ of 3 ſ-4 d.
Of 5 ſ-8 d, of 4 ſ, 1 ſ-8 d:
Of 5 ſ 10 d, of 5 ſ and 10 d, which 10 d is $\frac{1}{6}$ of 5 ſ.

And by this meane when you haue taken one product, you may oftentimes vpon the ſame take another more briefly then vpon the ſumme

ſumme which is to be multiplied,&c.

NOw (Gentle Reader) that you haue ſeene the vertue of the euen or aliquot parts of a pound in ſhillings alone, and alſo in the aliquot parts of ſhillings and pence: according to my promiſe hereafter followeth a briefe and eaſier method for any euen number of ſhillings, either vnder or aboue 20, then euer yet hath beene publiſhed: Notwithſtanding. M. Humfrey Baker, *whoſe trauell is worthy commendation, and whom for knowledge ſake I reuerence, hath in ſome part touched this firſt part, though not in this method. The worke of the rule both pleaſant, readie and briefe, as by the variety of the examples deliuered thereupon ſhall appeare. And firſt I will ſet forth a queſtion, thereby the better to expreſſe or teach you the order thereof: which is this.* 8 Rule.

If one coſt 6 ſ, what 8574?

1	6	8574
maketh pounds		2572--4 !

To the vnderſtanding of this example, after you haue ſet downe your giuen number in forme of the rule of 3, with a line drawne vnder it, you ſhal preſently ſet a prick vnder your firſt figure 4, towards your right hand drawing from the pricke, as heretofore hath beene practiſed, a little ſhort line, thereto ſet downe the Mr: IOHN MELLIS *his firſt rule*

the ſhillings anon, which done, multiply the firſt figure 4 by 6, the value of your price, (which here you ſee ſtandeth in ſight aboue the line) it maketh 24, which is one pound foure ſhilling. The one pound keepe to carry to the next place, and the foure ſhillings ſet downe at the end of the preſcribed line towards your right hand. Thus haue you done now with 6 aboue the line, and alſo with 4 in the firſt place, (for the pricke vnder 4 doth ſignifie that 4 hath done his office.) Then ſecondarily for a generall rule take but the ½ of the giuen price, which here is 3, which 3 is the number that ſhall now continue the reſt of the multiplication and end the worke. whereupon I multiply 3 into 7, ſtanding in the ſecond place it maketh 21, and with the one pound I kept in minde 22; ſet downe 2 and keepe 2 in minde, working according to the rule of multiplication, deliuering the tenths in minde in their due place, which done, the product from the pricke to your left hand repreſenteth the pounds, and the other at the end of the line the ſhillings, as appeareth by the examples.

Note a generall rule.

If one yard coſt 2 ſ, what 7536?

1 2 7536

maketh pounds 753 — 12 ſ

If one yard coſt 4 ſ, what 8792?

1 4 8792

maketh pounds 1758 — 8 ſ

If

If one peece coſt 6 ſ, what 9537 ?

1	6	9537
maketh pounds		2861——2 ſ

If one peece coſt 8 ſ, what 7509 ?

1	8	7509
maketh pounds		3003–12 1

If one coſt 12 ſ, what 5794 ?

1	12	5794
maketh pounds		3476——8 ſ

If one coſt 14 ſ, what 3705 ?

1	14	3705
maketh pounds		2593——10 ſ

If one coſt 18 ſ, what 5703 ?

1	18	5703
maketh pounds		5132——14 ſ

If one coſt 22 ſ, what 953 ?

1	22	953
maketh pounds		1048——6 ſ

Let theſe ſuffice (gentle Reader) for an entrance into euen numbers. And now I will ſhew the like rule for any odde or vneuen part of a pound.

TO helpe you to the vnderſtanding of theſe other queſtions that hereafter follow: wherein my firſt example the giuen number is 6487 *at* 3 ſ 9 Rule.

Mr IOHN MELLIS his second rule.

3 ſ the yard : I multiply 3 aboue the line into 7, it maketh 21. The one ſhilling is ſet downe, and the 1 pound I keepe. Now am I to take the ½ of three, which becauſe it is an odde number I cannot.

Therefore I ſhall keepe and continue my multiplication by three ſtill, and worke by the ½ of the reſt of the giuen figures or number, to wit 648. And firſt the ½ of 8 which is 4 multipied into 3, maketh 12, there to ioyne the 1 li in minde, it maketh 13 : ſet downe 3, keepe one. Then againe multiply by two the ½ of foure, it maketh ſix, and with one in minde it maketh 7. Then laſtly take the ½ of ſix, which is 3, ſaying 3 times 3 is 9, which 9 ſet down, and ſo is the queſtion anſwered, as appeareth by the practiſe, and examples following.

At 3 ſ the yard, what 6487 ?

1	3	6487
maketh pounds		973--1 ſ

If one yard coſt 5 ſ, what 4269 ?

1	5	4269
maketh pounds		1067--5 ſ

At 7 ſ the Ell, what 6489 ?

1	7	6489
maketh pounds		2271---3 ſ

If one Ell coſt 9 ſ, what 2807 ?

1	9	2807
maketh pounds		1263--3 ſ

If

At 11 ſ the Piſtolet, what 8263 ?

I	11	8263
maketh pounds		4544-13 ſ

If one piece coſt 13 ſ, what 4629 ?

I	13	4629
maketh pounds		3008 - 17 ſ

But now note (gentle Reader) when the giuen price falleth vpon an odde number, as 3,5,7,11,13,&c. then it is to be preſuppoſed that the giuen ſumme to be multiplied, muſt be a ſumme made of euen numbers, as 2 4, 6, 8, 10, &c. elſe cannot that queſtion bee wrought at one line or working.

Prouiding alwaies that it may beare an odde figure in the firſt place towards your right hand, as appeareth in theſe ſixe examples, which laſt were wrought, and ſuch like, &c. which may beare an odde number for the price, and bee done at one line or working very well.

But if the giuen price be an odde number, and the ſumme to bee multiplied, odde numbers alſo; then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithſtanding to bring to paſſe, take this for a generall rule. Firſt worke for the euen number, contained in that queſtion, or giuen price, according as you haue learned, and then

A generall rule.

afterwards for the one odde ſhilling, take the ½ of the ſumme giuen to be multiplied, omitting the firſt prickt place, as was taught for the working of one ſhilling in my firſt rule of Practiſe, and adde thoſe two together, and you ſhall haue your deſire.

Example.

At 3 ſ the yard, what	7539 yards ?	
2 ſ	753	18
1 ſ	376	19
maketh pounds	1130	17 ſ

At 7 ſ the Ell, what 7539 ?		
1 7	7539	
6 ſ	2261	14
1 ſ	376	19
maketh pounds	2638	13

At 13 ſ the yard, what	7534 ?	
12 ſ.	4520	8
1 ſ	376	14
maketh pounds	4897	2

Note this well.

And thus haue I abridged into theſe two Rules, how to bring any number of ſhillings whatſoeuer they be into pounds, with a briefer Method, then euer yet hath beene publiſhed, which I commend vnto thy friendly cenſure and iudgement in the vſe and practiſe thereof.

If

If one coſt 6 ſ, 5 d, what 1231 ?

6 ſ	369— 6
4 d	20-10-4
1 d	5— 2-7
maketh pounds	394-18-11

At 14 ſ 2 d, what 2825 ?

14 ſ	1977—10
2 d	23—10-10
maketh pounds	2001---0-10

At 16 ſ 4 d, what 2531 ?

16 ſ	2024-16
4 d	42 —3-8 d
maketh pounds	2066 —19-8

At 3 ſ the Piſtolet, what 8325 ?

maketh pounds	1248—15 ſ

At 7 ſ the crowne, what 6529 ?

	2285—3 ſ

At 9 ſ the piece, what 6567 ?

maketh pounds	2955—3 ſ

Theſe three laſt queſtions may ſeeme ſomething hard, yet they are eaſie enough, if you marke them well: if I ſhould explaine them, then are they too eaſie. Therefore I leaue them to whet the mindes of the deſirous.

10 Rule. *ITem, when any one of the ſummes, which is to bee multiplied, is compoſed of many denominations, and the giuen number but of one figure alone; then ſhall you multiply all the denominations of the other ſumme by the ſame one figure, beginning firſt with that ſumme which is leaſt in value toward your right hand, and bring the product of thoſe pence into ſhillings, and the product of the ſhillings into pounds, as by this example appeareth.*

At 3 li, 7 ſ, 4 d a yard, what are 9 worth?
maketh pounds 30—6ſ—0 d

11 Rule. *BVt if in any of the ſummes that are to bee multiplied, there be a broken number, Firſt worke for the whole according to the inſtructions that you haue learned, and then take ſuch part of the giuen price, as that broken number beareth in proportion to the price, as in the examples following. After you haue wrought for 3 ſ. and for 6 d, then are you to take the $\frac{1}{2}$ of 3 ſ 6 d, for the $\frac{1}{2}$ yard, and adde that to the ſumme: So adding all the 3 prodncts together, which makes 43 li- 2 ſ- 9 d, the iuſt price of 246 $\frac{1}{2}$ elles, and thus muſt you doe of all other.*

At 3 ſ, 6 d, the elle, what
3 ſ
6 d.
$\frac{1}{2}$
maketh

246 $\frac{1}{2}$
36—18
6—-3
1—9
43-2-9

At

At 16 ſ, 4 d, the peece what 14 $\frac{3}{4}$?

16 ſ	11—4
4 d	0—4-8
$\frac{3}{4}$	12-3
maketh pounds	12-0-11

If one peece coſt 4 li. 3 ſ, 6 $\frac{1}{2}$ d, what 12 peeces?

4 li	48—
3 ſ	1—16-
6 d	6—6
$\frac{1}{2}$	6
maketh pounds	50--2—6

The proofe.

If 12 peeces coſt 50 li 2 ſ 6 d, what one peece?

Maketh pounds 4--3—6 $\frac{1}{2}$.

ITem, touching the manner how to vnderſtand the order of this queſtion, and others the like, firſt ſeeke how many times 12 *is contained in* 50, *which is* 4 *times, and ſo reſteth* 2 *pound, which* 2 *pound conuerted into ſhillings, and ioyned with the other* 2 *ſhillings, maketh* 42 *ſhillings: wherein is found* 12, *three times, reſteth* 6 *ſhillings, which turned into pence, putting thereto the* 6 *pence in the firſt place, it maketh* 78. *wherein* 12 *is found* 6 *times, reſteth* 6 *pence: which containeth* 12, *but* $\frac{1}{2}$ *a time: Put that* $\frac{1}{2}$ *to the* 6 *pence: and then the ſolution is* 4 li, 3 ſ, 6 $\frac{1}{2}$ d, *as appeareth by the praſtiſe thereof.* 12 Rule.

Item, the like is to be done of any thing that is bought or ſold after fiueſcore to the hundreth, or the Quintall. As for example.

If 100 pound coſt 27 li, 13 ſ, 4 d, what one pound?

27 li—13 ſ—4 d

20

ſ 5|53

|12

d 3|10

5|3

d 6|4|0

|10|0 or $\frac{2}{3}$

Maketh 5 ſ, 6 $\frac{2}{3}$ d.

I haue wrought this at length for the aid of the yong learner, becauſe he ſhould vnderſtand how all the Multiplication is ſet downe.

But to worke it more neately it is by a little vnderſtanding ended thus.

27 li,--13 ſ --4 d

20

ſ 5|53

|12

d 6|40

|100

Maketh 5 ſ, 6 $\frac{2}{3}$ d.

☞ *Item, to the vnderſtanding of this and ſuch like queſtions, the right downe line is all the guide, which is pulled downe cloſe by 20, as you ſee in the example, where 27 Pound 13 ſhillings is reduced all into ſhillings, and maketh 553 ſhillings.*

The 5 towards the left hand being ſeparated with the hanging or right downe line, is the iuſt

number

number of ſhillings that anſwereth to the queſtion. Nextly, 53 *ſhillings is multiplied by* 12 *to reduce them to pence, putting to the* 4 d, *it yeeldeth for the multiplication of the firſt figure two* 110: *the one beyond the line towards the left hand, is* 1 *penny towards the reſt of the price: then* 53 *alſo multiplied by* 1 *yeeldeth* 53: *but the* 5 *behinde the line toward the left hand, is alſo* 5 *pence more, towards the price, which* 1 *and* 5 *I adde together vnder the line: it maketh* 6 d. *So is there found now, as appeareth by the Titles of ſhillings and pence,* 5 *ſhillings* 6 *pence.*

Finally, I come now on this ſide the line towards the right hand, and vnder 12 *I finde firſt* 10, *and then* 3, *which added together maketh* 40, *vnder which* 40 *you muſt put the* 100. *and it maketh* $\frac{40}{100}$, *which abreuiated, commeth to* $\frac{2}{5}$. *So the iuſt price of one pound after* 5 *ſcore to the hundred, makes* 5 ſ $6\frac{2}{5}$ d. One example more, and ſo I will leaue this rule.

If 100 coſt $10\frac{3}{4}$ d, what 9874?

6 d		246	17
4 d		164	11 4
$2\frac{1}{4}$ d		20	11 5
$2\frac{1}{4}$ d		10	5 $8\frac{1}{2}$
		5	5 $\frac{1}{2}$

Maketh (li, ſ, d)

4	42		
	20		
8	45		
	12		
5	$45\frac{1}{2}$	91	
	100	100	

parts of a peny.

Alſo the like may bee done of the vſuall weights here in England (which is 112, for euery hundreth weight) in caſe you know the aliquot parts of a hundreth weight, which are theſe, 56 li, 28 li, 14 li, and 7 li. For 56 li, is the $\frac{1}{2}$ of 112 li, 28 li, is the $\frac{1}{4}$ of 112 li, 14 li, is the $\frac{1}{8}$ and 7 li, is $\frac{1}{16}$ part.

Therefore for 56 li, take the $\frac{1}{2}$ of the ſumme of money that 112 li, weight is worth.

For 28 li, take the $\frac{1}{4}$ of the ſumme of money that 112 li, weight is worth.

For 14 li, take the $\frac{1}{8}$ of the ſumme that 112 li, is worth.

And for 7 li, the $\frac{1}{16}$ of the ſumme of money that 12 li, is worth.

As for example: At 17 li, 19 ſ, the hundreth pounds weight, that is to ſay, the 112 li, what ſhall 3 quarters and 7 pound coſt?

1 C.—17 li,—19 ſ			3q;—7 li,
2 quarternes	8—	19—	6
1 quarterne	4—	9—	9
7 pounds	1—	2—	$5\frac{1}{4}$
Maketh pounds	14—	11—	$8\frac{1}{4}$

The

The ſecond Chapter intreateth of the Reduction of diuers meaſures to others value by Rules of Practiſe.

NOw will I ſhew a few examples of Practiſe in reducing of meaſures, as Elles, Yards, Braces, Pawnes of Genes, &c. Much more would I haue touched, but that I feare the booke will riſe to too great a volume. 14 Rule.

In 864 Elles of Antwerpe, how many yards of London?

864		864	
432		216	
216		648	
maketh 648	yards of London.		

ITem in theſe and ſuch like queſtions of Flemmiſh meaſure, to be brought into yards Engliſh: firſt take the $\frac{1}{2}$ of the giuen number, as appeareth in the firſt example towards your left hand. Then take the $\frac{1}{2}$ of that product, or the $\frac{1}{4}$ of the giuen number, and adde thoſe 2 products together, they ſhall bee yards Engliſh: as by the Example you may perceiue.

The

The second example toward your right hand is yet briefer then the first, whose worke is this; Take the $\frac{1}{4}$ of the deliuered number, and that product subtract out of the giuen number, and the rest sheweth your desire. Of these two wayes vse which you thinke best.

The proofe.

In 648 yards of London,
How many Elles of Antwerpe?

648
216

maketh 864—Elles of Antwerpe.

15 Rule. *ITem for the vnderstanding of this worke: first take the $\frac{1}{3}$ part of the yards of* London; *which found, adde that $\frac{1}{3}$ part and the yards together, as appeareth by the practise, and the product sheweth the Elles of* Antwerpe.

Item, in 320 yards of London,
How many Elles of Antwerpe?
maketh $426\frac{2}{3}$ Elles.

	320 yards.		Proofe.
$\frac{1}{3}$	$106\frac{2}{3}$		$426\frac{2}{3}$ Elles.
	$426\frac{2}{3}$ Elles	$\frac{1}{3}$	$106\frac{2}{3}$
			320 yards.

Other Reductions.

16 Rule. *ITem, you shall vnderstand, that forasmuch as sixe braces of Millan make fiue Elles of Antwerpe, whereupon according to the* rules of Practise *you may reduce the one into the other, by the like reasons aforesaid, in taking the $\frac{1}{6}$ part, and then subtract*

ſubtract the ſame, to make Elles of Antwerpe. And againe by the contrary, taking the $\frac{1}{5}$ part with adding the giuen number, to turne the Elles to Braces, as for Example.

In 876 Braces, how many Elles of Antwerp?

876 Braces

146

Els 730 Antwerp

The contrary.

730 Elles Flemiſh.

146 Braces.

876 Braces.

Elles 730 Antwerpe.

182 $\frac{1}{2}$.

Yards 547 $\frac{1}{2}$ Engliſh.

Thus appeareth, that 876 Braces by Practiſe, make 730 Elles Flemiſh. which Elles Flemiſh reduce into Engliſh yards.

So againe vpon the ſame firſt queſtion of Braces, I would know how many yards Engliſh they make.

After the rate that 100 Braces are worth 62 $\frac{1}{2}$ yards.

876 Braces.

438

109 $\frac{1}{2}$.

I anſwer, 547 $\frac{1}{2}$ yards.

ITem, to the vnderſtanding of this work, and ſuch like, firſt take the $\frac{1}{2}$ of the giuen Braces, and after take the $\frac{1}{4}$ of that halfe, or the $\frac{1}{2}$ of the giuen number, and adde them together, and the products are alſo yards Engliſh. 17 Rule.

Item

Item, three Elles of Rochell make 5 Elles at Lisbone. So likewise three Elles at Lyons make 5 Elles at Antwerpe.

To worke these and such like, double the Elles of Lyons, and the Elles of Rochel, and from their products subtract the $\frac{1}{6}$: and the rest shall bee the Elles of Antwerpe, or the Elles of Lisbone.

Example.

In 63 Elles of Lions how many Elles of Antwerpe?

63
63
126
$\frac{1}{6}$) 21
Ans. 105 Elles Ant.

In 100 Elles of Rochell, how many Elles of Lisbone?

100
200
$\frac{1}{6}$) $33\frac{2}{3}$.
Ans. $166\frac{1}{3}$ els of Lisb.

18 Rule. Touching the proofe or returne of these and such like questions, for a general rule, you shall first take the $\frac{1}{5}$ of the giuen number: and adde that $\frac{1}{5}$ and the giuen number together, and the $\frac{1}{2}$ of that product shall be your desire.

Example.

In 105 Elles of Antwerp, how many Elles of Lyons?

105
$\frac{1}{5}$) 21
$\frac{1}{2}$) 126
An. 63 Els of Lions.

In $166\frac{2}{3}$ Elles of Lisbone, how many Elles of Rochell?

$166\frac{2}{3}$
$\frac{1}{5}$ $33\frac{1}{3}$
$\frac{1}{2}$) 200
An. 100 Els of Roch.

The

The third Chapter teacheth of the order and worke of the Rule of 3 in broken numbers after the Trade of Merchants, digreſſing ſomething from M. *Records*, which is comprehended in three *Rules*.

Ow that I haue ſomewhat intreated of the Rules of Practiſe, I will giue a few inſtructions, after my ſimple order, for the working of the Rule of three in broken numbers, wherein I ſhall need to ſay the leſſe, becauſe I hope the ſtudious learner, that hath trauelled any thing in the Grounds of Arts, *is not vnfurniſhed of knowledge capable to vnderſtand me.*

But before I deliuer any inſtructions for broken numbers, I will propone a queſtion which ſhall bee wrought three ſundry waies, thereby to ſhew, as it were, three degrees of compariſon: how farre the Rule of three in broken, for more ſpeed of worke, differeth from the whole, which I rather ſet downe for a view, that the ſtudious herein may be more deſirous to attaine broken, leauing any more to diſcourſe in dialogue forme, but onely to giue inſtructions where need is: and in the reſt to put forth the queſtions with their anſwer.

My

My first queſtion is this.

The firſt way. If one yard coſt 6ſ, 8 d, what are 789 worth at that rate?

1——6ſ——8d——789
12 80
80 63120d

Here the product of the ſumme are pence, according to the nature of the middle number

x x
x 3 7 0 x
6 3 2 2 0 (5 2 6 0 (2 6 3
x 2 2 2 2 2 2 2 0
x x x
I anſwer——263 li:

The ſe cond way.

1——6 2/3 ſ——789
——— ——— 20
3 20 ——— ——— ———
15780 ſ

Here the product of the ſumme are ſ, according to the nature of the middle number.

x
x 5 7 8 0 (5 2 6 0 (263 li.
3 3 3 3 2 2 2 0

li,
The third way. 1——1/3——789
3 1 1
789

Here

Here the product is pounds, according to the title of the second number.

x
789 (263
333

I answer. 263 li.

Now that you haue seene the three former vertues of the Rule of three, whose products haue first brought forth d, next s, and lastly li, I will deliuer three notes in order following: and with them a dozen questions that shall shew the worke of the Rule of three in broken numbers or Fractions.

1 The first foure, shall be sundry questions of a fraction comming in the second place. Note these three rules

2 The second foure, shall bee of two fractions comming in the second or third place.

3 The third foure, of Fractions in all three places.

Notes vpon the first Rule for a fraction comming in the second place.

My first question is this.

If one yard cost mee 3 s 4 d: *what are* 756 *worth at that price?* 1 Rule.

In setting downe the question to performe the worke, I turne foure pence into the part of a shilling, which is $\frac{1}{3}$ and then the question standeth thus: The first varietie.

I — 3 $\frac{1}{3}$ — 756.

To

To the ready working of this Queſtion, and all ſuch other like, my firſt note is this, which take for a generall rule: that when any one Fraction ſhall come, either in the ſecond or third place, that the Denominator of that fraction or fractions, muſt alwaies bee brought vnto the Number, or Numerator of the firſt place; and thereby multiply the one into the other.

A general Rule.

And this benefit is alwaies gotten by the vertue of bringing the Denominator of the ſecond Numbers Fractions vnto the firſt place; For the Fraction in the middle Number is now releaſed: and the product that commeth of the multiplication, is of the nature and like Denomination of the whole Number in the ſecond place which here are ſhillings.

Note this.

Whereupon now to worke the queſtion, I bring 3, the Denominator of the Fraction in the ſecond place, vnto my firſt Number 1. with a line ſet vnder thus $\frac{1}{}$ and the 3 vnder it thus $\frac{1}{3}$: ſaying, once 3, is 3 my Diuiſor: that done, reduce $3\frac{1}{3}$ ſaying, 3 times 3 is 9, and the other 1 ouer 3 make 10: my ſecond number in the rule of three: by which 10 I doe multiply my laſt number 756, as appeareth by the worke thereof, and it yeeldeth 7560 ſhillings my Dividend.

Then diuiding 7560 by 3 my Diuiſor, it yeeldeth in quotient 2520 pounds, which maketh 126 pounds, as appeareth here moſt plainely, both by the example and the worke.

At

At 3 ſ 4 d the yard, what 756 yards?

$$\frac{1}{3} \quad \frac{3\frac{1}{3}}{10} \quad \frac{756}{10}$$

7590 ſ

℔	℔	
7560	2520	(126.
3333	2220	I anſwer 126 li.

Yet otherwiſe vpon the ſame queſtion, altering the price now into the proportion it beareth to a pound, for the 3 ſ 4 d *is* $\frac{1}{6}$ *part of a pound: which example firſt ſtandeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.*

$$1 \quad \frac{1}{6} \quad 756$$

$$1 \quad \frac{1}{6} \quad 756 \qquad 6 \qquad \frac{1}{1} \qquad 1$$

756 pounds.

The ſecond varietie.

As ſoone as I haue carried 6 the denominator of my middle number vnto my firſt place, as before hath beene taught, I pull downe one the numerator of 6, with a line vnder 6 thus, $\frac{6}{1}$, and that one in cuſtome I pull downe in light, being the figure that I will multiply my third or laſt number by, according to the tenor of the rule of three. And becauſe one can neither multiply nor yet diuide (though here it is ſet downe in forme of multiplication, the rather for your vnderſtanding) the product of the multiplication according to the decla-

ration of this my first Rule or note, is conuer-ted into the title of my second number, which here are pounds. Now followeth the diuision performed in my Diuisor 6, to make an end of that question.

x 3
7 5 6 (1 2 6 ; *which maketh* 1 2 6 li, *as before*:
6 6 6.

And thus much for the variety in working that question.

And now followeth another.

My second question.

If one yard of Cotton cost $8\frac{1}{4}$ d, what 859?

1	$8\frac{1}{4}$	859
4	33	33
	2577	
	2577	
	28347	

x

2 | 3	2 x	x | 1	li	s	d
2 8 3 4 7	(7 0 8 6	(5 9 0	(2 9	- 1 0	- $6\frac{3}{4}$
4 4 4 4	x 2 2 2	2 2 0			

This question was also wrought like the first, and bringeth forth 29 li - 10 s - $6\frac{3}{4}$ d, *the price of* 859 *yards.*

My

My third queſtion.

If 7 pounds of any thing coſt 3 li--10ſ, what comes 987 pounds to?

li		
7	$3\frac{1}{2}$	987
2	7	7
14		6909

00
X 4 X
2 3 4 | 7
6 9 0 9 (493 $\frac{7}{14}\frac{1}{2}$
X 4 4 4
X X

I anſwere, 493 li--10ſ.

Notes vpon my ſecond Rule for two Fractions comming in the ſecond and third place.

My firſt queſtion is this.

IF one Elle coſt 13 ſ--4 d: what halfe a quarter or $\frac{1}{8}$ of an Elle?

Anſwer. *Firſt bring* 13 ſ--4 d, *into the parts of a pound, which is* $\frac{2}{3}$, *and then will the queſtion ſtand thus:*

1--$\frac{2}{3}$ li--$\frac{1}{8}$

ITem for the performance of this worke, doe as before was taught in the firſt Rule: firſt *bring* 3 *the* denominator *of the ſecond* fracti-

on vnto your *first number* 1, ſetting a line vnder it thus: 1 Saying once 3 is 3: that done, bring 8 the *denominator* of the *third fraction*, ſetting it vnder 3, and multiply them together, ſaying, 3 times 8 maketh 24, which 24 is your *diuiſor*. (Now haue you done with the *denominator* 8.) Therefore you ſhall put a line vnder thus 3 And the like line alſo vnder 8, ſetting or pulling downe vnder them their owne *numerators*, that is 2 vnder 3, and alſo 1 vnder 8 as appeareth in the example, which *numerators* for a generall rule euermore to be pulled down of cuſtome in ſight, to multiply the one by the other, according to the tenor of the *Rule of three*. Then I multiply the one by the other, ſaying: once 2 is two, which ſignifieth 2 li, being of the nature and like denomination of the middle number, which 2 li is to be reduced into ſhillings, otherwiſe it cannot be diuided by my firſt number 24.

Then diuiding 40 by 24, the quotient bringeth forth 1 $\frac{2}{3}$. So much is $\frac{1}{8}$ of an elle worth after that rate. Otherwiſe although 2 pound could not be diuided by 24, yet it might haue been abreuiated to $\frac{1}{12}$ of a pound: which is worth 1 ſ, 8 d, as before.

li

1	$\frac{2}{3}$	$\frac{7}{8}$	(1	
3	2	1	2 (6	
8	20		40 (	$1\frac{2}{3}$
24	40		24	

Second queſtion.

IF one pound of any weight coſt 13 *ſhillings* 4 *pence what are* $\frac{7}{8}$ *of the pound worth after that rate?*

Anſwer. Reduce the 13 ſhillings 4 pence into the parts of a pound: which is $\frac{2}{3}$, and then will the queſtion ſtand thus.

li

1 —— $\frac{2}{3}$ —— $\frac{7}{8}$

ITem for the underſtanding of this, if you marke well the laſt example this and the reſt lieth open, and need ſmall inſtruction. For as you did laſt, ſo againe, bring the denominator *of the ſecond and third* Fraction *vnto the firſt figure* 1, *multiplying the one into the other, which maketh alſo* 24, *your* Diuiſor.

Then making a line vnder 3, *thus* $\frac{3}{}$, *and a line vnder* 8, *thus* $\frac{8}{}$, *and pulling downe their* Numerators *vnder each figure, that is* 2 *vnder* 3, *and* 7 *vnder* 8, *which as I ſaid before for a generall rule I pull downe of cuſtome in ſight, to be the two numbers that of duty ought to be multi-* Note.

plyed together, which done, I bring 2, being the leſſer figure vnder 7, multiplying them together, it maketh 14, which are of the nature of the middle number: that is to wit, pounds: which 14 cannot aptly bee diuided among 24, therefore are reduced into *ſhillings*, as is plainely to be ſeene in the example: then 280 *ſhillings* parted among 24, yeeldeth for his *quotient* 11 ſ 8 d, your deſire, and the iuſt price of $\frac{7}{2}$ of an *Elle*. Otherwiſe 14, though it could not bee diuided by 24, might by *mediation* or *diuiſion* in broken *numbers* haue beene diuided or abbreuiated to $\frac{7}{12}$, which in effect being reduced to his knowne parts, maketh 11 ſ 8 d, as before. But my good will and meaning is to aid young beginners: therefore haue I reduced the 14 *pound* into *ſhillings*, which is the eaſier way.

Now followeth the Example.

1	2	7	l.
3	3	8	x
8	2	7	x (6
24		2	x 80 (11 $\frac{2}{3}$ l.
		14	x 44
		20	x

x 80 ſ I anſwer, 11 $\frac{2}{3}$ l.

The

The third Example.

If one yard coſt me 2 ſ 6 d, what 345 $\frac{1}{4}$ yards?

Anſwer. Firſt put 6 d into the parts of a ſhilling, and then the queſtion ſtandeth thus.

1 —— 2 $\frac{1}{2}$ —— 345 $\frac{1}{4}$

Item, to the ready vnderſtanding of this, and all ſuch like, according as before hath beene declared, bring the *Denominators* of the ſecond and third *Fractions* vnto the firſt place, multiplying them the one into the other, all which make 8 for the common *Diuiſor*. Then next reduce your ſecond number: ſaying, two times 2 is 4, and 1 is 5; as was taught in the example aforeſaid. Laſtly, reduce your third number 345 $\frac{1}{4}$ all into fourths, and they make 1381, which 1381 is to bee multiplyed by 5, according to the tenor of the *Rule of three*: which done maketh 6905 ſ: and diuided by 8, your Diuiſor yeeldeth in Quotient 863 $\frac{1}{8}$ ſ, which maketh in pounds 43 li, 3 ſ 1 $\frac{1}{2}$ d: and ſo much are the 345 $\frac{1}{4}$ yards worth at that price.

The ſame queſtion wrought againe by 2 ſhillings 6 pence, is now conuerted into the parts of a pound, and ſtandeth thus:

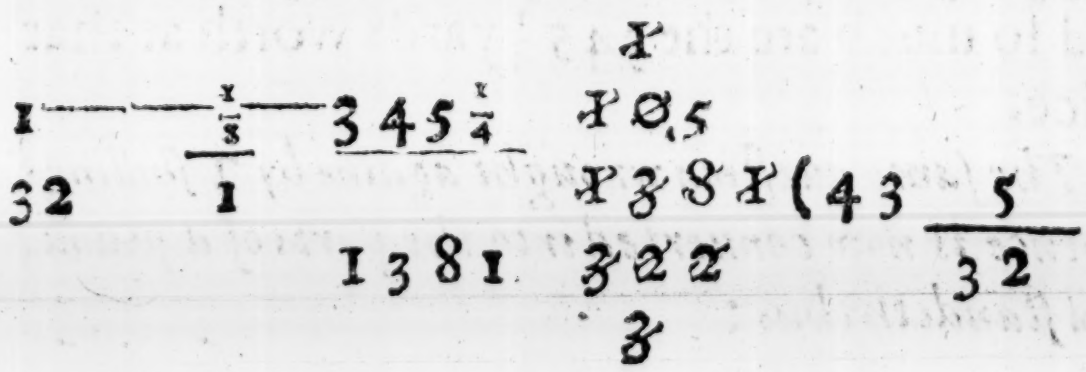
I —— $\frac{1}{8}$ —— 345 $\frac{1}{4}$

Item, after I haue brought here my ſecond and third *denominator* vnto my firſt place, and found 32 to be my *Diuiſor*; hauing thus finiſhed

ſhed my firſt place with al things vnto him be-longing (which is meant of bringing & multiplying the Denominators of the ſecond and third fractions into him) I then goe in hand to ſee what is to do in my ſecond place, where preſently of cuſtome I pull downe my Numerator 1 vnder 8 : being the figure in ſight that ſhall multiply my third number.

Then laſtly I reduce $345\frac{1}{4}$ all into fourths, as afore was practiſed, which maketh 1381, the which 1381 I am to multiply by 1 my ſecond number, they are nothing increaſed, but by the *Metamorphoſis* of my worke they are now 1381 pound, being of the nature of the middle number as I haue often ſhewed you, which diuided by 32 my diuiſor, yeeldeth 43 pound and $\frac{5}{32}$ which $\frac{1}{32}$ of a pound reduced into knowne numbers, make 3 ſhillinge $1\frac{1}{2}$ d, as before.

Example.



NOw follow 4 other queſtions, which are in all three places broken numbers, or whole and broken together.

Item, firſt for the finding out of your *Diuiſor*,

niſor, you ſhall take this for a moſt certaine and generall rule: That you muſt multiplie the *Numerator* of the firſt *number* in the queſtion, by the *Denominator* of the ſecond: And that *Product* againe, by the *Denominator* of the third: And the totall thereof ſhall be your *Diuiſor*.

Secondly, for a generall rule to finde out your *Diuidend*, *Multiply* the *Denominator* of the firſt number by the *Numerator* of the ſecond, and the whole thereof by the *Numerator* of the third: And the totall thereof ſhall euermore be your *Diuidend*.

Now for an example, I propone this queſtion, thereby to make my meaning the more plaine, and to ſhew you, as I haue done in the reſt, the maner and order of the worke.

If $\frac{2}{3}$ *of any weight or meaſure coſt* $\frac{5}{6}$ *of a pound, or* 20*ſ. what are* $\frac{3}{4}$ *of the like weight or meaſure worth after that rate?*

Example.

$\frac{2}{3}$ — $\frac{5}{6}$ — $\frac{3}{4}$

ITem for the more plainer vnderſtanding hereof, and all other the like, in broken numbers: Firſt you ſhall pull downe 2, the *Numerator* of the firſt *Number* or Fraction, with a line vnder thus, $2\,\frac{3}{4}$: that done, according as you haue learned before, bring 6, the *Denominator* of the ſecond *Fraction*, and ſet it vnder 2, multiplying the one into the other,

other, which maketh 12. Then lastly bring 8, the *Denominator* of the third *Fraction*, and set it vnder 12, multiplying that 12 by 8, which amounteth to 96, or else for more briefe, multiply 6 by 8, saying: 6 times 8 makes 48, which 48 set vnder 2, and multiply the one into the other, it maketh 96, as before. And this 96 is the first *number* in the Rule of three. That shall alwaies for a most generall rule be your *Deuisor*:

Secondly to worke for your *Diuidend*, you shall (as hath beene sufficiently declared before) pull downe 5, the *Numerator* of your second *fraction*, and set it vnder 6, with a line vnder thus, $\frac{5}{6}$.

That done, (as you know,) you are to pull downe 3, the *Numerator* of the third *Fraction*, and set it vnder 8, with a line vnder it thus $\frac{3}{8}$, multiplying the one into the other, according to the tenour of the Rule of three; which maketh 15. Then according to my note; forget not to bring the *Denominator* of the first *Fraction* which is 3, vnder 15, and multiply them together, which maketh 45, which 45 is your *Diuidend*, and are of the nature or *Denomination* of the Middle *number*, as I haue taugh you before: And therefore are 45 l, which aptly cannot be diuided by 96. Therefore you shall reduce the 45 li into s, as you see performed in the Example, which amounteth to 900 s which diuided by 96 your diuisor,

uiſor, it yeeldth 9 ſ, and $\frac{36}{96}$ of a ſhilling, which in leſſer termes is $\frac{3}{8}$: which $\frac{3}{8}$ in money maketh $4\frac{1}{2}$ d : and ſo much will the aforeſaid $\frac{3}{8}$ coſt, as by the worke following ſhall appeare.

The example.

2	5	3	
3	6	8	
2	5	3	
6		5	3
		15	9 6
12		3	9 0 0 0 (9 $\frac{36}{96}$ $\frac{3}{8}$
8		45	96
96		20	
		900	

Otherwiſe though 45 could not be diuided by 96, yet by Diuiſion in broken numbers it might haue beene abbreuiated to $\frac{15}{32}$ of a li; which reduced into knowne parts, will make 9 ſ $4\frac{1}{4}$ d, as before.

Now my ſecond example ſhall be the proofe of this queſtion.

If $\frac{3}{8}$ yards coſt $\frac{15}{32}$ of a pound or 20 ſhillings what ſhall $\frac{2}{3}$ coſt?

Anſwer. Worke as was taught you before, and you ſhall haue your deſire.

Here

$\frac{3}{8}$	$\frac{15}{33}$	$\frac{2}{3}$
3	15	2
32	2	
96	30	
3	8	
288	240	

Here as appearth by the worke, the multiplication being ended, 240 is to be diuided by 288, which to ſome perchance may ſeeme hard, yet notwitſtanding is the worke good. Therefore abreuiate 240 by 288, as you ſee here is practiſed: and the end of your abreuiation ſhall come to $\frac{5}{6}$ your deſire, $\frac{240}{288}|\frac{30}{36}|\frac{5}{6}$.

Otherwiſe. 240|120|60|30|5 / 288|144|72|36|6

Otherwiſe. 240|40|5 / 288|48|6

The third queſtion.

If $\frac{3}{4}$ *Elles coſt* 13 ſ–4 d, *what* 156 $\frac{1}{2}$ *Els?*

Anſwer. To worke this queſtion the ſhorteſt way: reduce 13 ſ–4 d into the parts of a pound, which is $\frac{2}{3}$.

Then as you did afore, after you haue ſet downe the queſtion, the numerator of the firſt fraction 3 is pulled downe vnder 4, and *Denominators* of the other, 2 *Fractions* multiplied

plied into him, which maketh 18, your *Diui-sor*.

Then the *Numerators* of the second *Fraction*, is pulled downe, vnder 3 of cuſtome now in ſight, ready to *multiply* my third number by which is performed as ſoone as the laſt numbers $156\frac{1}{2}$ is reduced into halfes.

Then laſtly, I multiply that product by 4. the *Denominator* of the *Fraction*: it yeeldeth 1504, which I diuide by 18, and my quotient is 139 li, and $\frac{1}{9}$ of a pound remaining, which is worth 2 ſ–2 $\frac{2}{3}$ d: And ſo much will $156\frac{3}{4}$ ells coſt as by the worke following doth appeare.

3	2	$156\frac{1}{2}$	x
4	3	313	47
3	2	2	x76|2 li.
6		626	2504 (139$\frac{1}{9}$
18		4	x888
		2504	xx

The fourth queſtion.

If $2\frac{1}{2}$ *Elles coſt* $1\frac{2}{3}$, pounds, *What commeth* $29\frac{1}{4}$ *Elles to?*

Item, to the workmanſhip of this queſtion, firſt reduce your ſecond *Number* in ſaying, three times 1 is 3, and 2 is 5: Then bring the *multiplication* of the *Denominators* of the ſecond and third *Fractions* which maketh 12: and *multiply* that 2 by 5 your firſt *numerator*,

and

and it maketh 60, which is your *diuiſor*.

Then the *reduction* of the ſecond number, which is 5, *multiplied* by 117 the product of the laſt *numbers* reduction, make 585, which 585 yet reſteth to be *multiplied* by 2 the *denominator* of the Fraction in the firſt place, yeeldeth 1170, which diuided by your *Diuiſor*, 60, yeeldeth 19 pound, 10 ſ as appeareth by the worke thereof.

Thus hauing now touched the 12 queſtions whereof I firſt pretended, which with diligence and oft Practiſe, I truſt are ſufficient to aide the deſirous vnto the working of any broken numbers, I will now intreate of diuers neceſſary Rules incident vnto trafficke, as hereafter followeth.

The 4 Chapter treateth of Loſſe and gaine in the trade of Merchandiſe.

IF one yard coſt 6 ſ—8 d, and the ſame is ſold againe for 8 ſ—6 d: the queſtion is, what is gained in 100 Pounds laying out on ſuch commodities?

Anſwer. The Rule of three direct, applied two manner of wayes to doe the ſame: the one is to ſay, If $6\frac{2}{3}$ giue $8\frac{1}{2}$, what giueth 100? *Multiply* and diuide, and looke what your quotient bringeth foorth aboue your laying out, is the neate gaines, and ſolution

to

to your queſtion: If you follow the worke, your ſolution will bring forth 127 li,—10 ſ, which is 27 li 10 ſ more then your princiall, and ſo much is gained in the 100 pounds laying out.

Item to worke it the other way, which I take the neareſt ſeeke the difference betwixt the iuſt, price and the other price, which is one ſhilling 10 pence; then ſay by the rule of three:

If $6\frac{2}{3}$ ſ. gaine $1\frac{5}{6}$ ſ, what ſhall 100 li gaine? *Multiply* and diuide, and you ſhall find 27 li-10 ſ, and ſo much is gained in 100 li laying out.

You may vſe which of theſe two wayes you thinke good.

The proofe.

If a yard of Cloth be deliuered for 8 ſ 6 d whereupon was gayned after the rate of 27 li 10 ſ, in 100 pound laying out: The queſtion is, what the yard coſt at the firſt hand?

Anſwer. Put your Gaine 27 pounds 10 ſ to 100 pounds, all maketh 127 li–10 ſ: Then ſay, If 127 li, 10 ſ, giue but 100 pound, what giueth $8\frac{1}{2}$ ſ? Worke and you ſhall finde 6 ſ–8 d, the true ſolution to your queſtion.

Yet an other Example, or proofe vpon the firſt queſtion.

If one yard coſt 6 ſ 8 d, the queſtion is, what

what price the ſame is to be ſold againe, for to gaine 27 li 10 ſ, in 100 pounds laying out?

Anſwer. *Say by the Rule of three, if* 100 li *gaine* 127 li-10 ſ, *what giueth* $6\frac{2}{3}$ ſ? *Multiply and diuide, and you ſhall finde* 8 ſ 6 d, *your true ſolution.*

If one Elle coſt 7 ſ 8 d, and be ſold againe for 8 ſ 6 d: *The queſtion is.* What is gained in 20 pounds laying out in ſuch commodities?

Anſwer. *Seeke the difference betwixt the iuſt price, and the other price which is* 10 *pence, and then apply the* Rule of three, *as before is taught, ſaying: If* $7\frac{2}{3}$ ſ *giue* $\frac{5}{6}$ *ſhillings what giueth* 20 li? *Multiply and diuide, and you ſhall find* 2 li $3\frac{11}{23}$ ſ, *and ſo much is gained in* 20 pound *laying out.*

The proofe alſo by an example of loſſe.

A *Merchant hath bought Holland cloth at* 8 ſ 6 d *the* Elle, *which proueth not to his expectation, whereupon hee is content, to loſe* 2 li $3\frac{11}{23}$ ſ *in* 20 *pounds laying out. The queſtion is, what price ought to be made of the Cloth, abating this loſſe?*

Anſwer. Doe as before in Gaynes hath beene taught, putting 2 li $3\frac{11}{23}$ ſ to your 20 pound, all together maketh 22 li-$3\frac{11}{23}$ Then ſay by the Rule of three: If 22 li-$3\frac{11}{23}$ ſ, giue but 20 li, what ſhall come of $8\frac{1}{2}$ ſ? worke, & you ſhall finde 7 ſ-8 d, the iuſt price that the Elle ought

to

to be sold for after the rate of this losse.

Thus it appeareth euidently, as in company the *Rule* is appliable as well to gaine as losse.

If 20 $\frac{1}{4}$ *yards cost* 36 li,- 10 s, *how shall I sell the same againe to gaine* $\frac{1}{3}$ *of the principall, or to make of* 3,4; *which is all one.*

Answer. By the *Rule of three*, if 3 doe giue 4, what will 36 $\frac{1}{2}$ giue? Multiply and diuide, and you shall finde 48 $\frac{2}{3}$ li. Then say againe, if 20 $\frac{1}{4}$ yards doe giue 48 $\frac{2}{3}$ pounds, as well principall as gaine, what will one yard bee worth at that price? Multiply and diuide, and you shall finde 2 li,- 8 $\frac{16}{243}$ s.

If one Elle of Cloth cost mee 8 s, 8 d, *and afterwards I sell* 10 $\frac{1}{2}$ *Elles thereof for* 5 li, 13 s, 4,d, *I would know whether I winne or lose: and how much vpon the* 100 *pounds of money.*

Answer. See first at 8 s 8 d the Elle, what 10 $\frac{1}{2}$ Elles come to, and you shall finde 4 li, 11 s, and I sold the same for 5 li- 13 s-4 d; so that I did gaine vpon the 10 $\frac{1}{2}$ *Elles* 22 shillings 4 pence. Then if you would know how much is gained in 100 pounds, I say by the *Rule of three*, If 4 li, 11 s did gaine 22 s 4 d, what will 100 pounds gaine? Multiply and diuide, and you shall finde 24 pound-10 s -10 d $\frac{10}{91}$. and so much is gained in the 100 pound of money.

If 12 $\frac{1}{2}$ *yards cost mee* 11 *pound fiue shil-*

lings,

lings, and I ſell the yard againe for 16 *ſhillings, the queſtion is, whether I doe win or loſe, and how much in or vpon the pound of money?*

Anſwer. Looke what the 12 $\frac{1}{2}$ yards come to at 16 ſ the yard, and you ſhall finde 10 pound. But they coſt 11 pound 5 ſhillings. So there is loſt vpon the whole 1 li, 5 ſ. Then to know how much is loſt in the pound, ſay by the rule of 3, if 11 $\frac{1}{4}$ pound doe loſe 1 $\frac{1}{4}$ pound, what will 1 pound loſe? Multiply and diuide, and you ſhall finde 2 ſ 2 d $\frac{2}{3}$, and ſo much is loſt in the pound of money.

If I ſell the 100 *weight of any commodity for* 4 *pound, whereupon I doe loſe after* 10 *pound in the* 100 *pound, I demand how much I ſhall loſe or gaine in the* 100 li? *if in caſe I had ſold the ſame for* 4 *pound* 10 *ſhillings.*

Anſwer. Say, if 90 pound yeeld 100, how much will 4 giue? multiply and diuide, and you ſhall finde 4 $\frac{4}{9}$. Then ſay againe, if 4 $\frac{4}{9}$ giue me 4 $\frac{1}{2}$ what will 100 come to? Multiply and diuide, and you ſhall finde 101 pound $\frac{1}{4}$ which is more then 100 pound by 1 pound 5 ſhillings: & ſo much is gained in the 100 pound.

A Merchant hath ſold Currans for the ſumme of 430 *pound, and hee hath gained therein after* 10 *pound in the* 100 *pound. The queſtion is to know how much he gained in all.*

Anſwer. Say by the rule of three. If 100 pound doe gaine 10 pound, what will 430 pound gaine? Multiply and diuide, and you ſhall finde 43, and ſo much hath he gained in all.

If

If one yard be worth 28 ½ ſ, *for how much ſhall* 10 *yards be ſold to gaine after* 8 li 6 ſ 8 d *in the* 100 *Pound.*

Anſwer. Firſt adde 8 li—6 ſ—8 d, to 100. Then ſay, if 100 li doe giue 108 ⅓ for principall and gaine, what will 28 ½ ſ principall yeeld? Multiply and diuide, and you ſhall find 30 ⅞ ſ. Then ſay againe by the Rule of three, if 1 yard doe giue 30 ⅞ ſ (which is as well the principall as the gaine) what ſhall 10 yards giue? Multiply and diuide, and you ſhall finde 15 li 8 ſ 9 d. And for the ſame price ſhall the 10 yards be ſold, for to gaine after the rate of 8 li—6 ſ—8 d vpon the 100.

A branch or proofe out of this Queſtion.

A Merchant hath ſolde clothes for 15 li—8 ſ—9 d, *and he hath gained in the whole, the ſumme of* 1 li—3 ſ—9 d. *The queſtion is, to know how much he hath gained in the* 100 *pounds?*

Anſwer: To know this, firſt rebate the gaines from the price, and there will remaine 14 li 5 ſ 0 d. Then ſay by the rule of three direct, if 14 li ¼ giue me 1 li 3 ¾ ſ, what will 100 li giue? Multiply and diuide, and you ſhall finde 8 li 6 ſ-8 d, the effect deſired, the proofe is apparant in the queſtion before.

Yet another Branch or Proofe of the firſt Queſtion.

If 10 *yards bee deliuered for* 15 li, 8 ſ, 9 d, *whereupon was gained after the rate of* 8 li, 6 ſ, 8 d, *vpon the* 100 *pound, the queſtion is, what the yard did coſt at the firſt hand.*

Anſwer. Firſt, ſay by the Rule of three, if 10 with principall and gaine yeeld 15 li, 8 $\frac{3}{4}$ ſ, what ſhall 1 yeeld? Multiply and diuide and you ſhall finde 30$\frac{7}{8}$ ſ. Then ſay againe by the rule of three, if 108 $\frac{1}{3}$ principall and gaine giue but 100, what ſhall 30$\frac{7}{8}$ ſ of principall and gaine yeeld? Worke, and you ſhall finde 28$\frac{1}{2}$ ſ. And ſo much did the yard coſt at the firſt Penny.

If one yard coſt 36 ſ, *how much ſhall* 12 *yards be ſold for to gaine after the rate of* 10 li, *in the* 100?

Anſwer. Firſt, ſay, If 100 giue 110 pound principall and gaine, what will 36 ſ giue? Multiply and diuide, and you ſhall finde 39 $\frac{3}{5}$ ſ. Then ſay againe by the Rule of three. If one yard of principall & gaine yeeld 39 $\frac{3}{5}$ ſhillings, what ſhall 12 yards gaine? Multiply and diuide, and you ſhall finde 23 li, -15 $\frac{1}{5}$ ſ, which $\frac{1}{5}$ ſ in knowne number, is 2 $\frac{2}{5}$ d. And for the ſame price ſhall the 12 yards bee ſold, to gaine after the rate of 10 in the 100.

The proofe.

If 12 *yards be ſold for* 23 li, -15-2 $\frac{2}{5}$ d, *where-*

upon

upon is gained after 10 li *in the* 100. *The queſtion is, what the yard coſt at the firſt penny?*

Anſwer. Firſt ſay, if 12 giue 23 li, 15 $\frac{2}{7}$ ſ, what one yard? Multiply and diuide, and you ſhall finde 39 $\frac{3}{5}$ ſ. Then ſay againe by the rule of 3, if 110 pounds giue but 100, what ſhall 39 $\frac{3}{5}$ ſ giue? Worke and you ſhall finde 36 ſ, the iuſt price of the yard at the firſt hand.

Item, when one Merchant ſelleth wares to another, and he giueth to the buyer 1 li 6 ſ, 8 d, *vpon the ſcore, or* 20 li: *The queſtion is, How much ſhall the buyer gaine vpon the* 100 *pound after that rate.*

Anſwer. Firſt adde 1 li- 6 ſ- 8 d, vnto 20 li, and they are 21 $\frac{1}{3}$. Then ſay, if 20 pound giue 21 $\frac{1}{3}$, what ſhall 100 giue? Multiply and diuide, and you ſhall finde 106 $\frac{2}{3}$. So the buyer getteth after the rate of 6 $\frac{2}{3}$ li, vpon the 100 pound.

Gentle Reader, other neceſſary queſtions appertaining to Losse and Gaine, you ſhall haue in the eight Chapter of this Treatiſe.

The fifth Chapter entreateth of Loſſe and Gaine vpon time, wrought by *the double rule of three, or by the Rule* compoſed: which is contained in 4 ſpeciall ſelected branches or queſtions of diuers formes; *each one of them ſpringing from the firſt* queſtion, and each one of them alſo, being a proofe to other, &c.

IF *one yard coſt mee* 2 ſ— 8 d, *ready money, and after I ſell the ſame againe for* 2 ſ— 10 d, *to be paid for it at the end of* 3 *moneths: the queſtion is, what I gaine vpon the* 100 li, in 12 *moneths?*

Anſwer. Firſt ſay, if $2\frac{2}{3}$ gaine $\frac{1}{6}$, what ſhall 100 li gaine? Multiply and diuide, and you ſhall finde $6\frac{1}{4}$ li. Then ſay againe, by the Rule of three, if three moneths gaine $6\frac{1}{4}$ pound, what ſhall 12 moneths gaine? Worke, and you ſhall finde 25 li, and ſo much ſhall I gaine in 12 moneths after that rate.

Item, you may alſo worke it all at one working by the firſt part of the rule of 3 compoſed, ſaying, if $2\frac{2}{3}$ d in three months doe gaine $\frac{1}{6}$ of a ſhilling, (which is 2 d,) what will 100 li gaine in

in 12 moneths: which for thy further encouragement the worke of this one example I haue here put downe, to verifie that I affirme in the first part of this *Ground of Arts*, that this rule and so all others, more reioyceth in *Broken* then n *Whole*.

ſ	*moneths*	ſ	li	mo:
$2\frac{2}{3}$	3	$\frac{1}{6}$	100	12
8		1	20	
3	2		2000	
24	72000	1	3	
6	14444	(500	(25 6000	
144	144	220	12	
	1		72000	

Where the multiplication and the diuision being ended, maketh 25 li your desire.

If a yard be deliuered for 2 ſ — 10 d, *to be paid at* 3 *moneths, whereupon was gained after the rate of* 25 li *in the* 100 *for* 12 *moneths, the question is now what the yard cost at the first hand?*

Answer. First say, if 12 moneths gaine 25 li, what shall three moneths gaine? Worke and you shall finde $6\frac{1}{4}$ li: Then say againe the second time, if 106 $\frac{1}{4}$ li, giue but 100, what shall $2\frac{5}{8}$ ſ giue? Worke, and you shall find 2 ſ 8 d, which is the iuſt price that the yard cost at the first hand.

If one yard of cloth cost mee 2 ſ — 8 d *ready*

money, for what terme ſhall I ſell the ſame againe for 2 ſ– 10 d, ſo that I might gaine after the rate of 25 pound vpon the 100 pound in 12 moneths.

Anſwer. Firſt ſay, if $2\frac{2}{3}$ gaine $\frac{1}{6}$ what ſhall 100 pound gaine? Multiply and diuide, and you ſhall finde $6\frac{1}{4}$ pound. Then ſay againe for the ſecond worke, if 25 pound become of 12 moneths, what ſhall come of $6\frac{1}{4}$? Worke, and you ſhall finde three moneths, the iuſt terme of time that the cloth ought to be deliuered at 22 ſ, 10 d, to gaine 25 pound vpon the 100 pound in 12 moneths.

If one yard coſt me 2 ſ & d, ready money, for what price ſhall I ſell the ſame againe to be paid at the end of three moneths, ſo that I may gaine after the rate of 25 pound in the 100 pound for 12 moneths.

Anſwer. Firſt ſay, if 12 gaine 25 li, what ſhall 3 moneths gaine? Multiply and diuide, and you ſhall finde $6\frac{1}{4}$ li. Then ſay for the ſecond worke, if 100 li, giue $6\frac{1}{4}$, what glueth $2\frac{2}{3}$ ſ? Worke and you ſhall finde 2 ſ– 10 d, and for that price muſt the yard bee ſold to gaine after 25 pound in the 100 pound for twelue moneths.

Many other of theſe queſtions I might here haue deliuered, but for feare the Booke would riſe to too thicke a volume, and ſo to make the price ſo much the dearer, whereby it might not bee ſo portable to my Country men as I wiſh it. But theſe 4 I haue of purpoſe framed in this order, hauing relation one to another,

aſſuring

assuring you that what question soeuer may be proposed within the compasse of this Rule, you shall finde by one of these 4, to make a solution. And moreouer, diuers others are yet to be deliuered: where the Creditor giueth diuers dayes of payment, which can neuer bee well wrought, nor yet vnderstood, vnlesse you can first finde by art the iust times that all those payments, how different soeuer they be, ought to be paid at once: whereupon first I thinke good here to giue some instructions into such a rule, for it is the onely aide for the finishing of such questions as hereafter shall follow.

The sixth Chapter intreateth of Rules of Payment, which is a right necessary Rule, and one of the chiefest hand-maides *that attendeth vpon buying and selling, &c.*

Example.

A Merchant doth owe a summe of money, whereof, the $\frac{1}{3}$ is to bee paid at 6 moneths, and the $\frac{1}{2}$ at 8 moneths, & the rest at a yeere. If hee would pay all at one payment, the question is what time ought to be giuen him.

Answer. I haue omitted the quantity of the

the ſumme, for you ſhall vnderſtand, the Rule is appliable, and yeeldeth a true ſolution to what ſumme ſoeuer ſhall be proponed. But now for order ſake in teaching, I doe imagine the ſumme to be 60 pounds, whereupon the manner of this worke is to multiply the proportionate part of the money by the time, as in company. Then 20 being the firſt payment, and the $\frac{1}{3}$ of 60, which $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the example in the operation, maketh two months: next 30, which is the $\frac{1}{2}$ multiplied by his terme 8, yeelds 4 moneths, then the reſt which is 10 li, muſt needs be abreuiated into the proportion it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 moneths, produceth $\frac{12}{2}$, maketh two moneths. All which added together, as appeareth in the operation, maketh eight moneths, which is the iuſt time that all thoſe payments ought to be paid at once.

$\frac{1}{3}$ by $\frac{6}{1}$ $\frac{6}{3}$	2 Mon:
$\frac{1}{2}$ by $\frac{8}{1}$ $\frac{8}{2}$	4 Mon:
$\frac{1}{6}$ by $\frac{12}{1}$ $\frac{12}{6}$	2 Mon:
	8 Mon:

A Merchant hath 800 li *to pay, the* $\frac{1}{6}$ *thereof ready money, the* $\frac{1}{4}$ *at two moneths, the* $\frac{1}{2}$ *at foure moneths, and the reſt at a yeere. The queſtion is, if he would pay all at one payment, what time ought to be giuen him?*

Anſwer.

Answer. The ready money is neuer multiplied: then $\frac{1}{4}$ multiplied by two moneths as you did before, maketh $\frac{1}{2}$, then $\frac{1}{2}$ by 4 produceth 2 moneths as appeareth here in the operation. But now for the reſt of the money you cannot multiply it vntill you haue ſought what proportion it beareth to 800 pounds. Therefore you muſt ſubtract the readie money, the $\frac{1}{4}$ and the $\frac{1}{2}$ out of the principall. The reſt will be 66 $\frac{1}{3}$ li, which you muſt looke what part it beareth to the principall, which you ſhall finde to bee $\frac{1}{12}$, the ſame you muſt alſo multiply by his time 12 moneths, and it yeeldeth 1 moneth, ſo all make 3 $\frac{1}{2}$ moneths, as appeareth in the operation.

$\frac{1}{4}$ $\frac{2}{1}$ $\frac{2}{4}$ $\frac{1}{2}$ $\frac{4}{1}$	0 $\frac{1}{2}$
$\frac{4}{2}$	2
$\frac{1}{12}$ $\frac{12}{1}$ $\frac{12}{12}$	1
	3 $\frac{1}{2}$

A Merchant is to pay 1200 li *in three termes, that is to wit,* 400 li *at two weekes, and* 600 li *at foure moneths, laſtly,* 200 li *at fiue moneths: The queſtion is, in what time they ought to bee paid at once?*

Answer. Proportionate the parts, and you ſhall finde that 400 is $\frac{1}{3}$ part, and for 600 you ſhall finde $\frac{1}{2}$, and likewiſe 200 is the $\frac{1}{6}$ part, which multiply by their times as before, and you ſhall haue $\frac{2}{3}$ weekes, more eight weeks, and laſtly three $\frac{1}{3}$ weekes, which together maketh twelue weekes, or three moneths your deſire.

A Merchant is to pay 600 *pound in three termes, whereof* 100 *pound is paid preſent, more* 300 *pound at twenty dayes, and the reſt at fiue moneths,*

neths, accounting thirty daies to a moneth. The queſtion is, what time ought theſe payments to bee paid at once.

Anſwer. Worke, and you ſhall finde two moneths.

The ſeuenth Chapter intreateth of buying and ſelling in the Trade of Merchandiſe, wherein is taken part readie Money, and diuers daies of payment giuen for the reſt, & what is won or loſt in the 100 li,

forbearance for 12 moneths more or leſſe, according to the quantity of money, or proportion of time, &c.

A Merchant hath bought Sattens which coſt eight ſ the yard ready money. And hee ſelleth the ſame againe to another man for ten ſ the yard, but he giueth two dayes for the payment, that is to ſay, three moneths for the one halfe, and fiue moneths for the other halfe. The queſtion is to know how much the ſeller doth gaine vpon the 100 pound in 12 moneths after that rate.

Anſwer. Seeke firſt by the Rules of payment, at what time thoſe two payments ought to be paid at once, and you ſhall finde foure moneths, at which time the ſecond Merchant ought to haue paid the whole entire paiment:

And

And therefore ſay by the firſt part of the Rule of three compoſed: If 8 ſhillings in 4 moneths doe gaine 2 ſ, what will 100 li, gaine in 12 moneths?

ſ m ſ li m.
8 -- 4 -- 2 -- 100 -- 12
4
32
20
2000
2
4000
12
48000

X
X6
48000 (X500 (75
322 220
3

Multiply and diuide, and you ſhall finde 75 pounds, as appeareth in the example, and ſo much doth the firſt Merchant gaine vpon 100 pounds in 12 moneths.

A Merchant hath ſold fifty clothes, at $9\frac{1}{2}$ li, *the peece, to be paid the one* $\frac{1}{2}$ *at foure moneths, the* $\frac{1}{3}$ *at fiue moneths, and the* $\frac{1}{6}$ *at 7 moneths, and the ſellers minde is to take no more but after* 8 *pounds in the* 100 *for* 12 *moneths. The queſtion is now what the firſt Merchant gaineth in the ſale of theſe cloathes after that rate.*

Anſwer. Firſt looke what the 50 cloathes come to at that price, and you ſhall finde 475 pounds. Then ſecondly, according to your direction in the rules of paiment, ſeeke at what time all the payments are to bee performed at once. And you ſhall finde $4\frac{5}{6}$ moneths. Then thirdly ſay by the firſt part of the Rule of three compoſed, If 100 li in 12 moneths gaine 8 li, what will 475 li gaine in $4\frac{5}{6}$ moneths? Worke and you ſhall finde 15 li

and

and $\frac{11}{36}$ of a pound, which is the neat gaines that the firſt Merchant hath after the rate aforeſaid.

A Merchant hath bought Holland at 7 ſ 3 d *the Elle ready money, and hee ſelleth the ſame againe for* 8 ſ 4 d *the Elle to be paid* $\frac{1}{4}$ *part in ready money, more* $\frac{1}{3}$ *part at* 2 *moneths, and the reſt at* 4 *moneths. The queſtion is now to know how much the firſt Merchant doth gaine vpon the* 100 *pounds in twelue moneths after that rate?*

Anſwer. According to the direction deliuered you in the rule of payment, the ready money is not to be multiplied. Then working for the other two payments to finde out the true proportion at what time they ought to be paid at once, you ſhall finde for $\frac{1}{3}$ at two moneths, $\frac{2}{3}$ of a moneth. And the reſt of the money which is $\frac{5}{12}$ multiplied by his terme 4 moneths, yeeldeth 1 $\frac{2}{3}$ moneths, both which added together make 2 $\frac{1}{3}$ moneths, the iuſt time that both the payments ought to bee performed at once. And therefore ſay by the firſt part of the Rule of 3 compoſed, if $7\frac{1}{4}$ in 2 $\frac{1}{3}$ moneths doe gaine $\frac{13}{240}$ of a pound, what ſhall 100 pounds gaine in 12 moneths after that rate? worke and you ſhall finde 76 $\frac{172}{203}$ pounds. And ſo much doth hee gaine vpon 100 pounds in twelue moneths.

A Merchant hath bought 30 *cloathes at* 6 *pounds the piece for ready money. Afterward hee ſelleth tenne of them for* 7 *pound the piece, for three*

three moneths terme. And the other twenty hee ſelleth for 8 *pound the piece for* 4 *moneths terme. The queſtion is now what hee gaineth vpon* 100 *pounds in twelue moneths?*

Anſwer. Firſt finde the value of the thirty clothes, which amount to 180 pounds. Secondly, ſeeke what the tenne pieces come to at 7 pounds, and what the twenty pieces come to at 8 pounds; the one comes to 70, & the other to 160: both which together make 230, which is 50 pounds more then they coſt. Thirdly, as I haue taught you in the Rule of payment, proportionate the firſt and ſecond prices vnto the proportion they beare vnto 230, the product of their two prices, and you ſhall find $\frac{7}{23}$ for the firſt, and $\frac{16}{23}$ for the latter. Then fourthly, multiply thoſe parts by their times, and you ſhall haue $\frac{21}{23}$ and $\frac{64}{23}$: both which together maketh three whole moneths, and $\frac{16}{23}$ of a moneth, which is the iuſt time that both thoſe payments ought to be paid at once.

Then ſay by the firſt part of the Rule of three compoſed. If 180 pounds in $3\frac{16}{23}$ moneths doe gaine 50 pounds, what ſhall 100 gaine in twelue moneths? Multiply and diuide, and you ſhall finde $90\frac{10}{51}$ pound, and ſo much doth hee gaine vpon 100 pounds in twelue moneths.

A Merchant hath bought cinamon which coſt him 9 *ſhillings the pound ready money. The queſtion is now at what price hee ought to ſell the* 100 *weight. To wit,* 112 *pounds to bee paid the* $\frac{1}{4}$ *at two moneths, and the reſidue*

at the end of three moneths, ſo that he may gaine after the rate of ten pounds vpon 100 *pounds for twelue moneths.*

Anſwer. Seeke firſt by the Rules of payment at what terme both the payments ought to be paid at once, where the $\frac{1}{4}$ multiplied by his terme two moneths, maketh $\frac{2}{4}$ moneths.

Likewiſe the next paiment, which is $\frac{2}{4}$ multiplied by his terme three moneths, maketh $2\frac{1}{4}$ moneths, both which added together, maketh $2\frac{3}{4}$ moneths, which is the time, that both the payments ought to be paid at once. Then ſay by the Rule of three, if 12 moneths doe giue mee 10 pounds, what will $2\frac{3}{4}$ moneths giue? Multiply and diuide, and you ſhall finde $2\frac{7}{12}$ pounds. Then ſay againe by the Rule of three. If one pound coſt mee 9 ſ, what will 112 pound coſt? Multiply and diuide, and you ſhall finde 50 li --8 ſ. Then ſay once againe: If 100 pound doe giue $102\frac{7}{24}$, what will $50\frac{2}{5}$ pounds giue? Multiply and diuide, and you ſhall finde 51 li-- 11 ſ-- $1\frac{1}{5}$ d, and for that price ought I to ſell 112 pound of Cinamon to be paid at the two ſeuerall payments aforeſaid, to gaine thereby after the rate of 10 pounds vpon the 100 pound in twelue moneths.

Briefe

Briefe Rules for our hundreth waight here at London, which is after 112 pound for the 100.

Item, *who that multiplieth the pence that one pound weight is worth by* 7, *and diuideth the product by* 15, *ſhall finde how many pounds in money* 112 *pound weight is worth.*

And contrariwiſe, he that multiplieth the pounds that 112 *pounds weight is worth by* 15, *and diuideth the product by* 7, *ſhall find how many pence in money the one pounds weight is worth.*

Example.

At 10 pence the pound weight, what is 112 pounds weight worth?

Anſwer. Multiply 10 by 7, and thereof commeth 70, the which diuided by 15, and you ſhall find $4\frac{2}{3}$ pounds. And thus the 112 pounds is worth 4 li, 13 ſ, 4 d, after the rate of 10 pence the pound aforeſaid.

At 6 pounds the 112 pounds weight, what is one pound worth?

Anſwer. Multiply 6 by 15, and thereof commeth 90: the which diuide by 7, and you ſhall finde $12\frac{6}{7}$ d. So much is one pound worth when the 112 pounds did coſt 6 pounds:

The eight Chapter intreateth of Tares and allowances of Merchandize sold by weight, and of losses and gaines therein, &c.

AT 16 pound the 100 Suttle, what shall 895 pound suttle bee worth, in giuing 4 pound weight vpon euery 100 for treat?

Answer. *Adde* 4 *vnto* 100, *and you shall haue* 104. *Then say by the Rule of three: If* 104 *be worth* 16 *pounds, what are* 895 *pounds worth? Multiply and diuide, and you shall finde* 237 li, 13 s, 10 $\frac{2}{13}$ d, *and so much shall the* 895 *pounds weight be worth.*

Item, at 3 s 4 d the pound weight shall 754 $\frac{1}{2}$ pound be worth, in giuing 4 pounds weight vpon euery 100 for treat?

Answer. See first by the rule of three what the 100 pound is worth, saying: If one cost 3 $\frac{1}{3}$ s, what 100? Multiply and diuide, and you, shall find 16 $\frac{2}{3}$ pounds. Then adde 4 vnto 100, and they are 104. Then say againe by the rule of three, if 104 be sold for 16 $\frac{2}{3}$ pounds, for how much shal 754 $\frac{1}{2}$ be sold for? Multiply and diuide, & you shall finde 120 li, 18 s, 3 $\frac{3}{13}$ d. And for so much shall the 754 $\frac{1}{2}$ pound bee sold for at 3 s 4 d the li, in giuing 4 vpon the 100.

Other necessarie briefe rules there are for the finding of treats, or casting vp of chests of Sugar, &c.

&c. which for that it is a myſtery, I omit: If any lacke inſtruction that way, they ſhall finde me ready to pleaſure them.

Item, if 100 pounds be worth 36 ſ 8 d, what ſhall 860 pounds be worth in rebating 4 pounds vpon euery 100 for tare and cloffe?

Anſwer. Multiply 860 by 4, and thereof commeth 3440, the which diuide by 100, and you ſhal haue $34\frac{2}{5}$ pounds, abate $34\frac{2}{5}$ from 860 and there will remaine $825\frac{3}{5}$ pounds. Then, ſay by the rule of three, If 100 pound coſt $36\frac{2}{3}$ ſ, what will $825\frac{3}{5}$ coſt after that rate? Multiply and diuide, and you ſhall finde 15 li, 2 ſ, $8\frac{16}{25}$ d. And ſo much ſhall the 860 coſt in rebating 4 pounds vpon euery 100, for tare and cloffe.

Item, *whether doth he loſe more that giueth 4 pounds vpon the* 100. *or he that rebateth 4 pounds vpon the* 100?

Anſwer. Firſt note, that he that giueth 4 pounds on 100, giueth 104 for 100. And he which rebateth 4 pounds vpon the 100, giueth the 100, for 96. Therefore ſay by the rule of three, If 104 bee deliuered for 100, for how much ſhall the 100 be deliuered? Multiply & diuide, and you ſhall find $96\frac{2}{13}$, and he which rebateth 4 in the 100, maketh but 96 pounds of 100, ſo that hee loſeth 4 pounds in the 100 and the other which giueth 4 li, vpon the 100 looſeth but $3\frac{11}{13}$ pounds vpon the 100. Thus you may ſee, that he which rebateth 4 pounds

in the 100, loseth more by $\frac{11}{13}$ pound in the 100, pounds, then the other which gaue 4 pounds vpon the 100, for tare and cloffe.

If 100 *pounds of any thing coſt me* 23 ſ, 4 d: *the queſtion is, how I ſhall ſell the pound, to gaine after the rate of* 10 *pounds, vpon the* 100. *pound.*

Anſwer. Say by the rule of three, if 100 pounds giue 116 pounds, what ſhall $23\frac{1}{3}$ ſ giue? Multiply and diuide, and you ſhall finde $1\frac{17}{60}$ pounds. Then ſay againe, If 100 pound be worth $1\frac{17}{60}$ pounds, what is one Pound worth? Multiply and diuide, and you ſhall find 3 d $\frac{2}{25}$. And ſo much is the pound worth in gaining 10 pounds vpon the 100.

Item, *a Grocer hath bought C. weight of commoditie, for* 6 li 10 ſ. *The queſtion is now, to know how many pounds thereof hee ſhall ſell for* 33 ſ 4 d, *to gaine* 20 ſ *in the C. weight.*

Anſwer. Adde 20 ſ vnto 6 li 10 ſ, and they make 7 li-10 ſ. Then ſay, if $7\frac{1}{2}$ pound yeeld me 112 pound, what ſhall $1\frac{2}{3}$ pounds yeeld? Multiply and diuide, and you ſhall find $24\frac{8}{9}$ pound. And ſo many pound ought he to ſell to gaine 20 ſ in his C weight.

If one pound weight coſt 3 ſ 4 d, *and I ſell the ſame againe for* 4 ſ, *what is gained in a* 100 *pound of mony laid out in that commodity?*

Anſwer. You may ſay, If $3\frac{1}{3}$ ſ giue 4 ſ, what

what will 100 pound giue? But then when you haue found, you must subtract 100 pounds out of the Product, the rest is your neat againe, or else to produce the neat againe in your worke at the first. Subtract the iust price out of the ouerprice, as I taught before in the first beginning of Losse and Gaine, and your conclusion shal be all one. Multiply and diuide, by which of the two wayes you thinke good, and you shall finde that he gaineth 20 pounds, in the 100 pound.

Item, if the pound weight which cost 4 ſ *be sold againe for* 3 ſ—4 d, *I demand what is lost in the* 100 *pounds of money?*

Answer. Say, If 4 ſ lose $\frac{2}{3}$ ſ, what shall 100 lose? Multiply and diuide, and you shall find 16 li, 13 ſ, 4 d, and so much is lost vpon the 100 of money.

Item, if C weight of any commoditie cost 45 *pounds, & the buyer repenting, would lose* 5 *pounds in the* 100 *of money, I demand how the pounds, may be sold, his losse to be neither more nor lesse then after the rate aforesaid of* 5 *by the* 100?

Answer. By the Rule of three, if 100 lose 5, what shall 45 lose? Worke, and you shall find $2\frac{1}{4}$ pound, which rebated from the principall 45, resteth 42 li, 15 ſ. Lastly say, if 112 yeeldeth but 42 li, 15 ſ, what one pound? Multiply & diuide, and shall find 7 ſ, 7 d $\frac{17}{28}$.

And ſo much is the pound worth after that loſſe.

A Grocer hath bought 3 peeces of raiſins, weighing 175 $\frac{1}{2}$ pounds, 182 $\frac{1}{4}$ pounds: 191 pound: tare for each frayle 2 $\frac{1}{4}$ pounds, as 25 $\frac{1}{2}$ ſ the C weight. The queſtion is, what they amount to in money?

I anſwere, 6 li——3 ſ—4 $\frac{23}{28}$ d.

A Grocer hath bought three ſacks of Almonds waying 267 $\frac{1}{2}$ pound, Tare 2 pound, 257 $\frac{1}{2}$ pounds tare 2 $\frac{1}{2}$ pound, 252 pound, tare 3 pound, at 2 ſ, 10 $\frac{1}{2}$ d the pound, what amount they to in money?

I anſwere, 110 li—12 ſ—3 $\frac{3}{4}$ d.

The ninth Chapter intreateth of lengths and breadths of Arras and other clothes, with other queſtions incident vnto length and breadth.

IF a peece of Arras be 7 Els and $\frac{3}{4}$ long, and 5 Els and $\frac{2}{3}$ broad, how many Elles ſquare doth the ſame peece containe?

Anſwer. Multiply the length by the breadth, that is to ſay, 7 $\frac{3}{4}$ by 5 $\frac{2}{3}$ And therof will come 43 $\frac{11}{12}$ Elles: ſo many Elles ſquare doth the ſame peece containe.

Item more, a peece of Arras doth containe 22 Els ſquare and if the ſame were in length 3 $\frac{1}{4}$ Els,

I demaund how many Elles in breadth the ſame peece doth containe?

Anſwer. Diuide 22 Els by $3\frac{1}{4}$ and thereof commeth $6\frac{10}{13}$. So many Els doth the ſame containe in breadth.

Item *more, a Merchant hath* $3\frac{1}{4}$ *Elles of Arras, at* $1\frac{2}{3}$ *Ells broad, which hee will change with another man for a piece of Arras, that is* $\frac{7}{8}$ *Ells ſquare. The queſtion is, how many Ells of that ſquareneſſe ought the firſt Merchant to haue?*

Anſwer. Multiply the firſt Merchants piece his length by the breadth, and you ſhall find, it containeth $5\frac{5}{12}$ Ells, which $5\frac{5}{12}$ Ells you ſhall diuide by $\frac{7}{8}$, and you ſhall finde $6\frac{4}{21}$ Ells, and ſo many Ells of that ſquareneſſe ought the latter Merchant to giue the firſt.

Item, *a Student hath bought* $3\frac{1}{2}$ *yards of broad cloth, at 7 quarters broad, to make a gowne, and ſhould line the ſame throughout with Lambe, at a foote ſquare each skin: The queſtion is now how many skinnes hee ought to haue?*

Anſwer. Seeke firſt the number of yards ſquare that his cloth containeth, which to do multiply $3\frac{1}{2}$ his length, by $1\frac{3}{4}$ his breadth, and you ſhall finde $6\frac{1}{8}$ yards ſquare: then ſay by the rule of three, if one yard ſquare giue 9 foot, what ſhall $6\frac{1}{8}$? Worke and you ſhall find $55\frac{1}{8}$ skins.

Item *more, a Lawyer hath a rich piece of ſeeling come home, which is* 24 *foot and three inches long, and* 7 *foot and* 2 $\frac{1}{2}$ *inches high: the Ioyner is to be paid by the yard ſquare: The queſtion is, how many yards this piece containeth?*

Anſwer, Multiply his length by his breadth, that is to wit, 24 $\frac{1}{4}$ foote by 7 $\frac{5}{12}$ foote, and you ſhall finde 174 $\frac{77}{96}$ foote ſquare, which 174 you ſhall diuide by 9 (for ſo many foote make a yard ſquare) and you ſhall finde 19 yards three foor, and $\frac{77}{96}$ of a foote, and ſo many yards doth this piece hold.

Item, *Bought a piece of Holland cloth containing* 36 *Ells* $\frac{1}{3}$ *Flemmiſh. The queſtion is how many ells Engliſh it makes?*

Anſwer. You muſt note, that fiue Ells Flemmiſh, doth make but three Ells Engliſh.

Therefore ſay by the Rule of three, If 5 Ells Flemmiſh, make but three Ells Engliſh, how many Ells Engliſh will 36 $\frac{1}{3}$ Ells Flemmiſh make? Multiply and diuide, and you ſhall finde 21 $\frac{4}{5}$, & ſo many Ells Engliſh doth 36 $\frac{1}{3}$ Ells Flemmiſh containe. The like is to be done of all others.

Item, *more, I haue bought* 342 *Ells Flemmiſh, of Arras worke, at two Ells broad Flemiſh, and I would line the ſame with Ell broad Cannas of Engliſh meaſure. The queſtion is, how many Ells Engliſh will ſerue my turne?*

Anſwer. For as much as three Ells Engliſh

liſh are worth fiue Ells Flemmiſh, theretore put 3 Ells Engliſh into his ſquare, in multiplying 3 by himſelfe, which maketh 9. Likewiſe multiply the Engliſh Ell, which is fiue quarters, euery way into himſelfe ſquarely, and you ſhall finde 25. Then multiply 342 which is the length of the piece, by 2, which is the breadth, & thereof commeth 684: then ſay by the Rule of three, as before: if 25 Ells ſquare of Flemmiſh meaſure, be worth 9 Ells ſquare of Engliſh meaſure, what are 684 of Flemmiſh meaſure? Multiply and diuide, and you ſhall finde 246 $\frac{6}{25}$ Ells Engliſh.

The ſame is alſo wrought by the Backer Rule of three, in ſeeking the ſquares contained in the Flemmiſh Ell of two Ells broad (which are 18) and alſo in ſeeking the ſquares contained in the Engliſh Ell (which are 25) then ſay by the Rule of three backward, If 18 quarters require 342 Ells, what ſhall 25 quarters giue? Multiply and diuide by the Rule of three Reuerſe, and you ſhall finde as before 246 $\frac{6}{25}$ Ells Engliſh.

Item *more, at three ſhillings foure pence the Flemmiſh Ell, what is the Engliſh Ell worth after the rate?*

Anſwer. Say if three quarters giue 3 $\frac{1}{3}$ ſ, what giueth fiue quarters? Multiply and diuide, and you ſhall finde 5 ſhillings 6 $\frac{2}{3}$ d.

Item, *more, at* 8 ſ, 4 d, *the Flemmiſh Ell ſquare,*

ſquare, what is the Engliſh Ell worth after that rate?

Anſwer. According to the reaſon of the laſt Queſtion, conſider that a Flemmiſh Ell ſquare is equal to nine quarters of a yard Engliſh, and an Engliſh Ell ſquare is equall to 25 quarters of a yard. Therefore ſay by the rule of three, if 9 quarters giue $8\frac{1}{3}$ ſ, what 25 quarters? Worke and finde 23 ſ, $1\frac{7}{9}$ pence. And ſo much is the Engliſh Ell worth.

Item *more, at* 6 ſ, 8 d *the Ell ſquare: what ſhall a piece of cloth coſt that is* $7\frac{1}{2}$ *Ells long, and* $3\frac{1}{4}$ *Ells broad?*

Anſwer. Multiply the breadth by the length, and you ſhall find $24\frac{3}{8}$ Ells ſquare. Then ſay by the Rule of three, if 1 Ell ſquare coſt $6\frac{2}{3}$ ſ, what $24\frac{3}{8}$ Ells. Multiply and diuide, and you ſhall finde 8 pounds, 2 ſhillings 6 pence, and ſo much the ſame piece of cloth coſt.

Item *more, a Mercer ſold three pieces of Silke. To wit* $24\frac{1}{4}$, $23\frac{1}{3}$, *and* 25 *yards, at* $9\frac{3}{4}$ ſ *the yard, and was glad to receiue in part of paiment again, a cloth containing* $34\frac{1}{2}$ *yards at* $7\frac{2}{3}$ *ſhillings the yard. The queſtion is now, what the Debitor is in the Creditors debt? Worke, and you ſhall finde hee oweth the Mercer* 22 *pounds* 3 *ſhillings* $2\frac{1}{4}$ *pence.*

The

The tenth Chapter entreateth of reducing of Pawnes of Geanes into Engliſh yards.

NOte, that 100 *Pawnes doe make* 26 *yards, whereupon three Pawnes* $\frac{11}{13}$ *doe make one yard, and one Pawne after that rate and proportion is* $\frac{13}{50}$ *of a yard.*

In 4563 *Pawnes of Geanes, how many yards Engliſh?*

Anſwer. Say by the rule of three, if 100 Pawnes doe make 26 yards, what will 4563 Pawnes make? Multiply and diuide, and you ſhall finde 1186 yards $\frac{19}{50}$. So many yards doe 4563 Pawnes make.

Otherwiſe, take ſome other number at your pleaſure, as 10 Pawnes, which is the $\frac{1}{10}$ part of 100, then to find his proportion, take the $\frac{1}{10}$ part of 26, which is 2 $\frac{3}{5}$, and then ſay alſo by the Rule of three, if 10 Pawnes giue 2 $\frac{3}{5}$ yards, what will 4563 Pawnes giue? Worke, & you ſhall find 1186 $\frac{19}{50}$ yards, as before.

More, at 2 ſ — 6 d *the Pawnes of Geanes, what will the Engliſh yard bee worth after the rate?*

Anſwer. Say by the Rule of three, if $\frac{13}{50}$ of a yard coſt 2 $\frac{1}{2}$ ſ, what one yard? Multiply and diuide, and you ſhall finde 9 ſ, 7 $\frac{5}{13}$ d.

More,

More, if 346 $\frac{1}{2}$ Pawnes cost 30 li, 13 ſ, 4 d ſterling, what is that the Engliſh yard after the rate?

Anſwer. Say by the Rule of three, If 346 $\frac{1}{2}$ Pawnes coſt 30 $\frac{2}{3}$ pounds, what are 3 $\frac{11}{13}$ Pawnes worth (for ſo many Pawnes make a yard) Multiply and diuide, and you ſhall find $\frac{9200}{17827}$ parts of a Pound, which in knowne numbers is worth 6 ſ 9 d $\frac{6271}{9009}$.

The eleuenth Chapter intreateth of Rules of Loane and Intereſt, with certaine neceſſary Queſtions and Proofes incident thereunto, &c.

ITem, lent my friend 226 pounds for 5 $\frac{1}{2}$ moneths ſimply without any Intereſt, vpon condition, to haue the like curteſie againe when I need. But when I came to borrow, hee could ſpare mee but 149 li, 8 ſ, 4 d. *The queſtion is now how long time I ought to haue the vſe thereof, to counteruaile my friendſhip before time ſhewed him?*

Anſwer. Say by the backer Rule of three, If 326 pounds giue 5 $\frac{1}{2}$ moneths, what time will 149 $\frac{5}{12}$ pounds giue? Multiply and diuide, and you ſhall finde twelue moneths, and ſo long time ought I to vſe his money.

The proofe.

Item, lent my friend 149 li--8 ſ--4 d, *for* 12 *moneths.*

moneths. The queſtion is now how much money hee ought to lend me againe for 5 ½ moneths to recompence my friendſhip ſhewed him?

Anſwer. Say by the backer or reuerſe Rule of three, If 12 moneths giue 149 $\frac{5}{12}$, what ſhall 5 ½ moneths giue? Worke, and you ſhall find 326 pounds, and ſo much ought he to lend me to requite my gentleneſſe or good turne.

Two other Branches, yet more, for proofe out of the ſame queſtion.

Item, lent my friend 149 li, 8 ſ, 4 d, *for* 12 *moneths, to haue the like friendſhip againe when I need. And comming to borrow of him, hee very curteouſly tooke me* 326 *pounds, (for thas hee could well then ſpare the ſame.) The queſtion is now, how long I ought to occupy it, not vſurping friendſhip but in his due time to reſtore it againe.*

Anſwer. Say by the Rule of three Reuerſe, if 149 $\frac{5}{12}$ pounds giue 12 moneths, what ſhall 326 pounds giue? Multiply and diuide, and you ſhall find, that at 5 ½ moneths terme, I ought to reſtore it againe.

Proofe.

Item, *Lent my friend* 326 *pounds for* 5 ½ *moneths. The queſtion is now, how many pounds hee ought to lend me for* 12 *moneths, to recompence*

pence this pleaſure againe?

Scholer. Worke by the Rule of three, Reuerſe, as you haue done before, and you ſhall find 149 li--8 ſ---4 d.

Againe, foure other ſelected Queſtions of Loane and Intreſt, all out of one branch, and each one alſo a neceſſarie Queſtion, and a particular proofe to other.

ITem, *Lent my friend* 430 *pounds at Intereſt for three moneths, to receiue after the rate of* 8 *pounds, in the* 100 *pounds for* 12 *moneths. The queſtion is, what the Intereſt commeth to?*

You may, if you pleaſe, worke it at two workings by the Rule of three direct, in ſaying: If 12 moneths giue 8 pounds, what giueth three moneths? Multiply and diuide, and it giueth 2 pound.

Then for the ſecond worke ſay: If 100 pound yeeld 2 pounds, what yeeldeth 430 pound? Multiply and diuide: and you ſhall find 8 li, 12 ſ, and ſo much comes the loane of 430 pounds to, for 3 moneths, after the rate of 8 pounds in the 100 pounds of 12 moneths.

Otherwiſe wrought thus by the Rule of three at twice alſo.

If 100 pound giue 8 pounds, what giueth 430 pounds? Multiply and diuide, and you ſhall finde 34 pounds $\frac{2}{5}$. Then againe for the ſecond worke ſay: If 12 moneths giue 34 pounds

pounds $\frac{2}{5}$, what giueth three moneths? Worke and finde 8 li, 12 ſ as before.

Otherwiſe yet at one working: By the firſt part of the rule of fiue numbers forward, in ſaying, if 100 pounds in 12 moneths, gaine 8 pounds, what ſhall 430 pounds gaine in three moneths? Multiply the firſt by the ſecond for your Diuiſor, and the other three the one into the other for the Diuidend, and you ſhal find eight pounds 12 ſhillings as aforeſaid.

Proofe.

Item, *a friend of mine receiued of me* 8 *pounds* 12 *ſhillings for the Intereſt and vſe of* 430 *pounds for three moneths terme: The queſtion is now, what he tooke in the* 100 *pounds for* 12 *moneths after that rate?*

Anſwer. For moſt briefe, ſay by the firſt part or rule of fiue numbers forward: If 430 pounds in three moneths did pay 8 li, 12 ſ, what doth 100 pounds in 12 moneths take after the rate? Worke, and you ſhall finde 8 pounds, and ſo much he tooke vpon the 100 pounds for twelue moneths.

A third Queſtion and proofe alſo by the Backer Rule of 5 Numbers.

Item, *lent my friend* 430 *pound to receiue for the intereſt thereof, after the rate of* 8 *pounds in*

in the 100 *for* 12 *moneths: The queſtion is now how long time my friend ought to giue the vſe thereof, that it may bee returned with* 8 li, 12 ſ, *gaines?*

You may worke it, if you pleaſe, by the Rule of three direct at twice, in ſaying: If 100 pound yeeld 8 pounds, what yeeldeth 430 pound? Multiply and diuide, & finde 34 pound and $\frac{2}{5}$.

Then againe for the ſecond worke ſay, If $34\frac{2}{5}$ pounds, giue 12 moneths, what giueth $8\frac{3}{5}$ pounds? Multiply & diuide, & you ſhall find three moneth, & ſo long time ought my friend to vſe it to returne with 8 li, –12 ſ gaine.

Otherwiſe at one working by the Backer Rule of 5 numbers, in ſaying: If 100 pounds in 12 moneths doe gaine 8 pounds, how long time ſhal 430 pounds be a gaining of 8 pounds 12 ſ? Multiply the firſt and the ſecond into the laſt for your diuidend, and the third and fourth multiply together for your diuiſor, and then diuide, and you ſhal find 3 moneths, the iuſt time that my friend ought to vſe it, to returne it with 8 li, 12 ſ gaine.

A fourth deriued queſtion out of this Branch, which is a proofe of this laſt, and alſo of the other two going before.

ITem, *how much mony ought a Merchant to deliuer after* 8 *pounds in the* 100 *for* 12 *moneths, that in three moneths he may gaine* 8 li, 12 ſ?

Anſwer

Anſwer. You may alſo, if you pleaſe, worke it by the Golden Rule of three at twice: firſt ſaying, If three moneths giue $8\frac{3}{5}$ pound, what 12. moneths gaine? You ſhall finde $34\frac{2}{5}$. Then ſay againe, If 8 pounds be come of 100 pounds what ſhall come of 34 li 8 ſ? Worke, and you ſhall finde the anſwer to the queſtion, which is 430 pounds, and ſo much ought the Merchant to deliuer.

But moſt briefly it is anſwered by the Backer Rule of fiue numbers, where I argue thus, ſaying: If 100 li, be 12 moneths a gaining of 8 li, then but for three moneths terme only to take 8 li, 12 ſ, muſt needs be a good round ſumme: to worke it, ſet your numbers thus; 100—12 8——2——$8\frac{3}{5}$: multiplying the firſt into the ſecond, & alſo by 43 the product of the fifth, for your diuidend, and the third and fourth together with 5 the Denominator of your fraction for your Diuiſor: then diuide, and you ſhall find as before, 430 pounds: the true ſolution to your queſtion.

The twelfth Chapter intreateth of the making of Factors, which is taken in two ſorts.

THe firſt is, when the eſtimation of the Factor is, taken vpon the ſending of the Merchant, as if the eſtimation of his perſon be $\frac{1}{4}$ it is vnderſtood that he ſhall haue $\frac{1}{4}$ of the gaine, and the Merchant the other $\frac{3}{4}$.

The other ſort is, when the eſtimation of

his making is out of the ſending of the Merchant, as if the order and agreement between them were ſuch, that the Merchant ſhall put in 800 li, and the Factor for his making ſhall haue $\frac{3}{4}$: neuertheleſſe he ſhall haue but $\frac{1}{5}$ of the gaine or profit, for the $\frac{1}{4}$ of 800 is 200 (for the eſtimation of his making) which with the 800 pounds in all make 1000 pounds, whereof the 200 pounds, is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, vnder ſuch condition that the ſaid Factor ſhall haue $\frac{1}{4}$. And after certaine time, they finde in profit 124 li, 6 ſ 8 d, I demand how much the Merchant ſhall haue hereof, and how much ought the Factor to haue?

Anſwer. *When the eſtimation of the Factor is out of the ſending of the Merchant, it maketh,*

li	ſ	d		
99	9	4	*for the*	*Merchant.*
24	17	4		*Factor.*

But if that his eſtimation be at the ſending of the Merchant, then it maketh but,

li	ſ	d		
93	5	0	*for the*	*Merchant.*
31	1	8		*Factor.*

For the Merchant is then to haue $\frac{3}{4}$, *and the Factor* $\frac{1}{4}$

A Merchant doth put into the hands of his Factor 800 pounds, and the Factor 400 pounds to haue the $\frac{1}{5}$ part of the profit: I demand now for how much his perſon is eſteemed, when the ſame is counted vpon the ſending of the Merchant?

Anſwer. *According to the tenor and order before preſcribed in the firſt Rule, that is, if his eſtimate be $\frac{1}{4}$ he ſhall haue thee $\frac{1}{4}$ of the gaine. Therefore ſay by the Rule of three direct, If $\frac{1}{4}$ taken put in 400 pound, what is the eſtimate, or putting in of $\frac{1}{5}$ taking? Multiply and diuide, and you ſhall finde 320 pounds, and ſo much is the perſon of the Factor eſtimated.*

Otherwiſe:

To finde the eſtimation of the perſon of the Factor, you ſhall conſider, that ſeeing it was agreed betweene them, that the Factor ſhould take the $\frac{1}{5}$, then the Merchant ſhall haue the reſidue, which are $\frac{4}{5}$: wherefore the gaine of the Merchant vnto that of the Factor is in ſuch proportion as 5 vnto 4. Then if you will know the eſtimation of the perſon of the Factor: ſay, if 5 giue 4, what will 400 giue? Multiply and diuide, and you ſhall find 320 pound. And ſo much is the perſon of the Factor eſteemed to be worth.

Other conditions then theſe aforeſaid, may alſo bee betweene Merchants and Factors, without reſpect either of ſending,

or not ſending of the Merchant, where moſt commonly the eſtimation of the body of the Factor is in ſuch proportion of the ſtocke which the Merchant layeth in, as the gaine of the ſaid Factor is vnto the gaine of the Merchant. As thus, If a Merchant doe deliuer into the hands of his Factor 400 pound and hee to haue halfe the profit: the perſon of the ſaid Factor ſhall bee eſteemed to be worth 400 pound: and if the Factor doe take but $\frac{1}{3}$ of the gaine, hee ſhould haue but $\frac{1}{2}$ ſo much of the gaine as the Merchant taketh, which muſt haue $\frac{2}{3}$, wherefore the perſon of the Factor is eſteemed but the $\frac{1}{2}$ of that which the Merchant layeth in, that is to ſay, 200 pound.

And if the Factor did take the $\frac{2}{5}$ of the gaine, then the Merchant ſhall take the reſidue which are $\frac{3}{5}$, wherefore the gaine of the Merchants vnto the Factor is then in ſuch proportion as 3 vnto 2: whereupon if you will then know the eſtimation of the perſon of the Factor, ſay, If three giue 2, what ſhall 400 giue? Worke, and you ſhall finde 266 $\frac{2}{3}$ pounds. And ſo much is the perſon of the Factor eſteemed to be worth.

And if the Merchant ſhould deliuer vnto his Factor 400 pound, and the Factor would lay in 80, and his perſon, to the end he might haue the $\frac{1}{2}$ of the gaine, I demand how much ſhall his perſon be eſteemed?

Anſwer. *Abate* 80 *from* 400; *and there will remaine*

remaine 320. *And at ſo much ſhall his perſon bee eſteemed.*

A Merchant hath deliuered vnto his Factor 900 pounds to gouerne in the trade of Merchandize, vpon condition that hee ſhall haue the $\frac{1}{3}$ of the gaine, if any thing be gained, and alſo to beare the $\frac{1}{3}$ of the loſſe if any thing be loſt. Now I demand how much his perſon was eſteemed at ?

Anſwer. *Seeing that the Factor taketh the* $\frac{2}{3}$ *of the gaine, his perſon ought to bee eſteemed as much as* $\frac{1}{2}$ *of the ſtocke, which the Merchant layeth in: that is to ſay, the* $\frac{1}{2}$ *of* 900 *pound, which is* 450. *The reaſon is, becauſe* $\frac{1}{3}$ *of the gaine that the Factor taketh, is the* $\frac{1}{2}$ *of the* $\frac{1}{3}$ *of the gaine that the Merchant taketh, and ſo the Factor his perſon is eſteemed to be worth* 450 *pounds.*

A Merchant hath deliuered vnto his Factor 600 pound, and the Factor layeth in 250 pound and his perſon. Now becauſe he layeth in 250 pounds and his perſon, it is agreed betweene them, that hee ſhall take the $\frac{2}{3}$ of the gaine. I demand for how much his perſon was eſteemed?

Anſwer. *For as much as the Factor taketh* $\frac{2}{3}$ *of the gaine, hee taketh* $\frac{2}{3}$ *of that which the Merchant taketh, for* $\frac{2}{3}$ *are the* $\frac{2}{3}$ *of* $\frac{3}{3}$. *And therefore the Factors laying in ought to be* 400 *pound, which is* $\frac{2}{3}$ *of* 600 *pound that the Merchant layed in. Then ſubtract* 250, *which the Factor did lay in, from* 400 *pound, which*

ſhould haue beene his whole ſtocke, and there remaineth 150 *pound for the eſtimation of his perſon.*

More, a Merchant hath deliuered vnto his Factor 800 pound, vpon condition that the Factor ſhall haue the gaine of 160 pound, as though he laid in ſo much ready money: I demand what portion of the gaine the ſaid Factor ſhall take?

Anſwer. *See what part the* 160 *(which the Factor laid in) is of* 960, *which is the whole ſtocke of their company, and you ſhall finde* $\frac{1}{6}$. *And ſuch part of the gaine ſhall the Factor take.*

But in caſe, that in making their couenants, it were ſo agreed betweene them, that the Factor ſhould haue the gaine of 160 *pound of the whole ſtocke which the Merchant layeth in, that is to ſay, of the* 800 *pound: then ſhould the Factor take* $\frac{1}{5}$ *of the gaine: for* 160 *is* $\frac{1}{5}$ *of* 800 *pound.*

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandize, which is diſtinct into ſeuen Rules, with diuers other neceſſarie queſtions incident thereunto.

The firſt Rule.

Two Merchants willing to change their Merchandize the one with the other: The one

one hath 24 broad cloathes at 10 li–10 ſ, the peece: The other hath Mace at 12 ſ, the pounds. The queſtion is, how many pounds of Mace he ought to giue for his Cloth, to ſaue himſelfe harmeleſſe and be no loſer?

Anſwer. *Seeke firſt by the Rule of three, what the 24 clothes coſt at 10 pound 10 ſhillings the peece, and you ſhall finde 252 pound. Then to finde the quantitie of Mace, ſay againe by the Rule of three, If 12 ſhillings buy one pound, what ſhall 252 pound buy me? Worke, and you ſhall find 420 pound of Mace: and ſo many pound ought he to giue for his Cloathes.*

The proofe.

Two barter. The one hath 420 pounds of Mace at 12 ſ the pound, to barter or change broad cloathes at 10 pounds 10 ſhillings the peece. The queſtion is, how many broad cloathes he ought to giue for all his Mace.

Anſwer. *Firſt ſay, if one coſt 12 ſ, what 420? you ſhall finde 5040 ſ. Then ſay againe, If 10 $\frac{1}{2}$ pounds giue 1 cloth, what ſhall 5040 ſhillings giue? Worke and you ſhall finde 24 cloathes your deſire.*

The ſecond Rule.

Two change Merchandize for Merchandize: The one hath Pepper at 2 ſhillings 4 pence the pound, to ſell for ready money.

But in barter he will haue leſſe then three ſhillings the pound. And the other hath Holland at fiue ſhillings 6 pence the Ell ready mony. The queſtion is now at what price hee ought to deliuer the Ell in the barter to ſaue himſelfe harmeleſſe.

Anſwer. *Say by the Rule of three direct If* $2\frac{1}{3}$ *ſ ready money giue* 3 *ſ, in barter, what ſhall* $5\frac{1}{2}$ *giue in barter? You ſhall finde* $7\frac{1}{14}$ *ſ, and at that price ought the ſecond Merchant to ſell his Holland in barter.*

The proofe.

Two barter. The one hath Holland at 5 ſ 6 pence the Ell to ſell for ready mony. And in barter he will haue $7\frac{1}{14}$ ſ. The other hath Pepper at 2 ſ—4 d, the pound, to ſell for ready mony. The queſtion is now, how hee ought to ſell in barter?

Anſwer. *Say by the Rule of three direct, if* $5\frac{1}{2}$ *ready mony giue* $7\frac{1}{14}$ *ſ in barter, what ought* $2\frac{1}{3}$ *ſ to take in barter? Multiply and diuide, and you ſhall find* 3 *ſ your deſire.*

The third Rule.

Two barter: The one hath cloth of Arras at 30 ſ the Ell ready mony, but in barter he wil haue $35\frac{1}{2}$ ſ. And the other hath white wines, which he deliuered in barter for 16 pounds the Tunne. The queſtion is now, what his

wines

wines coſt the Tunne in ready mony.

Anſwer. *Say by the Rule of three direct, If* $35\frac{1}{2}$ *ſ, in barter, giue but* 30 *ſ ready mony, what did* 16 *pound in barter coſt? Worke, and you ſhall finde* 13 li–10 ſ $\frac{30}{71}$. *And ſo much coſt his wines for a Tunne ready money.*

The proofe.

Two barter merchandiſe for merchandiſe: The one hath white wines at 13 li,--10 : $\frac{30}{71}$ (the Tunne to ſell for ready money. But in barter he deliuered it for 16 pounds. The other, to make his match good and ſaue himſelfe harmleſſe, deliuereth Arras at $35\frac{1}{2}$ ſ the Elle. The queſtion is now, what an Elle of his Arras coſt in ready money.

Anſwer. *Say by the Rule of three direct: If* 16 *pounds in barter giue but* 13 li–10 $\frac{30}{71}$ ſ *in ready mony, what ſhall* $35\frac{1}{2}$ ſ *yeeld in barter? worke and you ſhall finde* 30 ſ, *your deſire.*

The fourth Rule.

Two barter. The one hath Kerſeys at 14 *pounds the peece ready mony. But in barter he will haue* 18 *pounds. And yet he will haue the* $\frac{1}{3}$ *part of his ouerprice in ready money. And the other hath Ginger at eight groates the pound, to ſell for ready money. The queſtion is, how hee ought to deliuer the Ginger by the pound in barter to ſaue himſelfe harmleſſe, and make the barter equall.*

Anſwer.

Anſwer. Item, for the working of this queſtion, and ſuch other the like, you muſt vnderſtand, if the party ouerſelling his wares, require to haue alſo ſome portion in ready money, as $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$, &c. Then ſhall you firſt rebate the ſame demanded part, whatſoeuer it bee, from the ouerprice, and alſo from the iuſt price. And thoſe two numbers that ſhall remaine after the ſubtraction is made, ſhall bee the two firſt numbers in the Rule of three. And the iuſt price of the ſame Merchandiſe ſhall be the third number, which by the operation of the Rule of three direct, ſhall yeeld you a true ſolution, how, and at what price you ſhall ouerſel that your merchandiſe, to ſaue your ſelfe harmeleſſe, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighteene) which is the ouerprice of his cloath, which $\frac{1}{3}$ of eighteene is ſixe, which you muſt ſubtract from 18, there reſt 12. And alſo abate it from 14, which is the iuſt price of the cloath, and there remaineth 8, which 8 and 12 are the two firſt numbers in the Rule of three. Then take eight groates or $2\frac{2}{3}$ ſhillings for the third number. Then ſay by the Rule of three direct. If 8 pounds giue 12 pounds,

14	18
6	6
8	12

what

what ſhall 2 $\frac{2}{3}$ ſ giue? Multiply and diuide, and you ſhall find 4 ſ. And for ſo much ſhall the ſecond Merchant ſell his Ginger, or his commodity in barter, to ballance the ſame equall.

The proofe.

Two barter: The one hath fine Kerſeyes at 14 *pounds the peece ready mony: But in barter he will haue* 18 *pounds, and yet he will haue the* $\frac{1}{3}$ *part of his ouerprice in ready money: And the other hath Ginger, which hee hauing cunning enough to make the barter equall, deliuered in barter for* 4 ℓ *the pound. The queſtion is now, what his Ginger coſt him in ready money?*

Anſwer. After you haue made the ſubtraction, abating ſ the $\frac{1}{7}$ part of 18, both from 18 and 14, (as before was taught you;) then wil there remaine 8 & 12 for your two firſt numbers in the Rule of three. Then ſay. If 12 giue 8, what ſhall come of 4 the ouer-price of the pound of Ginger, multiply and diuide, and you ſhall find 2 ſhillings 8 pence your deſire.

Two Merchants barter merchandiſe for merchandiſ. The one hath Denſhire *whites at* 7 li–13 ſ 4 d *the peece ready money: but in barter he doth them away for* 8 li–3 ſ–4 d, *and yet he will haue the* $\frac{1}{3}$ *part of his price in ready money. And the other hath Cottens at* 3 *pounds the peece ready money. The queſtion is now, at what price he*

ought

ought to sell or exchange his Cottens in barter to saue himselfe harmelesse, and make the barter equall?

7	——	13	——	4	
2	——	14	——	$5\frac{1}{3}$	
4	——	18	——	$10\frac{2}{3}$	

8	——	3	——	4	
2	——	14	——	$5\frac{1}{3}$	
5	——	8	——	$10\frac{2}{3}$	

Answer. First seeke the $\frac{1}{3}$ part of 8 l—3 s 4 d, which is, 2 li—14—$5\frac{1}{3}$ d, which rebated from 8 l—3 s—4 d, there resteth, as appeareth by the example aboueſaid, 5 l—8 s—$10\frac{2}{3}$ pence, which is $\frac{2}{3}$ parts of 8—3—4 d, also rebated from 7—13—4 d, there resteth, 4 l—18 s—$10\frac{2}{3}$ d, the two first numbers in the Rule of three, and the 3 pounds, which is the neat price of the peece of Cotten, is the third number. Then say by the Rule of three direct, as was taught before: If 4 li—18 s—$10\frac{2}{3}$ d giue 5 li—8 s—$10\frac{2}{3}$ d, what shall 3 pounds giue? multiply and diuide, and you shall find 3 li 6 s—$\frac{72}{89}$ pence, the iust price that he ought to deliuer his Cottens in barter.

The fift Rule.

Two Merchants will change merchandise for merchandise. The one hath Kerseys at 40 s the peece to sell them for ready money. And in barter, he will sell them for 56 s 8 d, *and he will gaine after* 10 *pounds vpon the* 100 *pounds. And yet hee will*

will haue the $\frac{1}{2}$ of his ouerprice in ready money. The other hath flaxe at 3 d *the pound ready money. The question is now, how he ſhall ſell the pound of of his flaxe in barter?*

Anſwer. See firſt at 10 pounds vpon the 100 pounds what the $56\frac{2}{3}$ ſ commeth to, in ſaying (by the Rule of three direct) If 100 pounds giue 110 pounds, what $56\frac{2}{3}$ ſ? Multiply and diuide, and you ſhall find 3 li—2 ſ—4d of which the $\frac{1}{2}$ that hee demandeth in ready money, is 1 li,——11 ſ——2 d the ſame 31 ſ——2 d abated from 40 ſ, and alſo from 56 ſ—8 d, there will remaine 8 ſhillings 10 d, and 25 ſ—6 d, for the two firſt numbers in the Rule of three, and 3 pence the price of the pound of flax for the third number. Then multiply and diuide, and you ſhall find $8\frac{15}{53}$ d. And for ſomuch ſhall hee ſell the pound of flax in barter.

The ſixt Rule.

Two are willing to exchange merchandiſe: the one hath Norwich Grograns at 25 ſ *the peece ready money: and in barter hee will haue* 30 ſ *and hee will haue the $\frac{1}{4}$ part of his ouerprice in ready money. The other hath Norwich ſtockings at* 40 ſ *the dozen to ſell for ready money. But in as much as the firſt Merchants Grogranes are no better, hee would deliuer them ſo to ballance the barter, that he may gaine after* 10 *pounds in the* 100 *pounds.*

The

The queſtion is now, how he ſhall ſell his Hoſe the dozen in barter according to his requeſt.

Anſwer. Say if 100 giue 110; what ſhall 40 ſ giue, which is the iuſt price of the dozen of ſtockings? Multiply and diuide, and you ſhall finde 44 ſ. Then take the $\frac{1}{4}$ of 30 ſ, which is 7 ſ 6 d, and ſubtract it from 25 ſ, and alſo from 30 ſ, and there will remaine 17 ſ——6 d, and 22 ſ——6 d, for the two firſt numbers in the Rule of three, and 44 ſhillings, which is the iuſt price (with his gaine in the dozen of ſtockings) for the third number. Then multiply and diuide, and you ſhall finde 56 ſ 6 $\frac{6}{7}$ d, and for ſo much he is to ſell his dozen of ſtockings in barter.

The ſeauenth Rule.

Two Merchants will change their merchandiſe one with the other. The one hath 720 Elles of Cambrick at 5 ſ the Elle to ſell for ready money, but in barter he requireth 6 ſ 8 d. And yet notwithſtanding he loſeth by it after 10 pounds vpon the 100 pounds whereupon he requireth one halfe of his ouerprice in ready money: and the other Merchant hauing skill enough to make the barter equall, deliuered Engliſh Saffrons at 30 ſ the pound. The queſtion is now, what his Saffrons coſt the pound in readie money?

Anſwer. You muſt firſt ſeeke what is loſt

vpon

vpon the 100 pound, which to doe, you may ſay, (if you pleaſe) If 100 pound loſe 10, what ſhall 6 ſ loſe? Worke, and you ſhall finde ſ (or 8 d) which muſt be rebated from 6 ſ 8 d ſo reſteth 6 ſ ſtill. Or you may ſay, if 100 pound giue me but 90 pounds, what ſhall 6 ſ 8 d giue? Worke this way either, and you ſhall find alſo as before directly in your quotient 6 ſ your deſire. Then are you next to caſt vp what the 720 Elles of Cambrick commeth to at 6 ſ 8 d the Ell, & you ſhall finde 240 pounds: the $\frac{1}{2}$ whereof the Cambricke Merchant will haue in ready mony, (which is 120 pounds): Nextly you muſt caſt what the Cambrik commeth to after his loſſe in the 100 pound, which, as you found, is but 6 ſ an Ell, & you ſhall find 216 pounds. Now muſt you ſubtract his ready mony (which is 120 pounds in all,) out of 240 pound, and alſo out of 216 pound and there will remiane 120 pounds, and 96 pounds for your firſt two numbers in the Rule of three, and 30 ſhillings is the ouer-price of your Saffron for the third number: Then multiply aud diuide, and you ſhall find 24 ſhillings. And ſo much did his Saffrons coſt in ready money.

Two Merchants barter. The one hath 50 *cloaths to put away for ready mony at* 11 *pounds, the cloth, & in barter putteth them away for* 12 *pounds, taking Holland Cloth at* 20 d *the Flemmiſh Ell, which was worth no more but* 18 d. *The queſtion is now, what Holland payeth for the Cloath,*

and

and what hee winneth or loſeth by the bargaine?

Anſwer. 50 Colthes at 11 pounds the cloth commeth to 550 pounds, and put away at 12 pounds the peece, maketh 600 pound. Then to find what Holland payth for the Cloth, ſay by the Rule of three direct, If 20 d buy one Ell what 600 pounds? Worke and you ſhall find 7200 Elles. Now to find the eſtate of his gaine or loſſe, you muſt ſeeke what his 7200 Elles commeth to at 18 d the Elle: Worke by the Rule of Proportion direct, and you ſhall find 540 pounds, which is not ſo much as his cloathes were worth in ready money by 10 pounds: and ſo much loſt the firſt Merchant by his exchange.

A Venetian hath in London 100 *peeces of ſilke, to put away for ready money at 3 pounds the peece. But in barter hee deliuered them for 4 pounds the peece, taking Wools of a Felmonger at 7 li–10 ſ the C: weight, which was worth no more but 6 pounds the C. ready money. The queſtion is now, what Wooles payeth for the ſilkes, and which of them winneth or loſeth by the barter?*

Anſwer. 100 peeces of ſilke at 3 pound, is in all 300 pounds, & 4 pounds, is 400 pound. Then to find what woles payeth for the ſilke, ſay by the Rule of three direct: If $7\frac{1}{2}$ buy me 100 weight, what 400 pound? Worke, and finde

finde 53 $\frac{1}{3}$ C weight of wooll. Now to finde the eſtate of their gaine, or loſſe, caſt vp his wooll at 6 li the C (for ſo much they were worth ready money) and you ſhall finde 320 pound, which is 20 pound more then the ſilkes were to bee ſold for ready money, whereupon the Venetian gained 20 pounds by the barter.

A Merchant hath 53 $\frac{1}{3}$ *C waight of wooll at 6 pounds the C, to ſell for ready money, but in barter he will haue 7 pounds* 10 *l: and another doth barter with him for ſilkes, which are worth* 3 *pounds a peece ready money. The queſtion is now, how he ought to deliuer his ſilkes the peece in barter, and how many payeth for the wools?*

Anſwere. Say by the Rule of proportion, (or by the Rule of three direct) If 6 pounds for C waight ready money yeeld me 7 li 10 ſ, what will 3 pound yeeld, which is the iuſt price of a peece of ſilke in barter, to make the Truck equall? Worke, and finde 3 li 15 ſ, the price of a peece of ſilke in barter: Then ſay, If 3 li 15 ſ require one peece of ſilke, how many peeces of ſilkes are bought with 400 pound, which is the value of the 53 $\frac{1}{3}$ C waight of wooll at 7 li 10 ſ? Worke by the rule of three direct, and you ſhall finde 160 peeces of ſilke and $\frac{2}{3}$ of a peece, and ſo many peeces of ſilke payeth for the wooll, and neither party hath aduantage of other.

Two men will change merchandiſe the one with the other. The one of them hath Beere at 6 ſ 8 d

the barrell to ſell for ready money, but in barter he will ſell the barrell for 8 *ſ, and yet he will gaine moreouer after* 10 *pound, vpon the* 100 *pounds. And the other hath white Spaniſh wooll at* 20 *ſ the Roue, to ſell for ready money. The queſtion is now, how he ſhall deliuer the Roue of Wooll in barter to ſaue himſelfe harmeleſſe.*

Anſwere. Say, If $6\frac{2}{3}$ ſ, which is the iuſt price of the barrell of beere, bee ſold in barter for 8 ſhillings : for how much ſhall 20 ſhillings (which is the iuſt price of the Roue of Wooll be ſold in barter ? Worke by the rule of three direct, and you ſhall finde 24 ſ. Then for becauſe the firſt Merchant will gaine after 10 pounds vpon the 100 pounds, he maketh his 100 pounds, 110 pounds. And therefore ſay by the rule of three, If the ſecond Merchant of 110 pounds doe make but 100 pounds, how much ſhall he make of 24 ſ ? Multiply and diuide, and you ſhall finde 21 ſ, 9 d $\frac{9}{11}$ of a penny. And for ſo much ſhall hee ſell the Roue of wooll to be deliuered in barter, to the end the firſt Merchant may giue 10 in th 100.

Two Merchants will change their commodities the one with the other. The one of then hath white Paper at 4 ſ *the reame, to ſell for ready money. And in barter he will do it away for* 5 ſ, *and yet he will gaine moreouer after the rate of* 10 *pounds vpon the* 100 *pounds. And the other hath Mace at* 14 ſ 6 d *the pound waight to ſell in barter. Now I demand what the pound did coſt in ready money.*

Anſ.

Answere. Say, If 5 ſ, (which is the ouer-price of the Paper in barter) bee come of 4 ſ the iuſt price, of how much ſhall come 14 ½ ſhillings, which is the ſurpriſe of the pound of Mace in barter? Multiply and diuide, and you ſhall finde 11 ⅗ ſ. Then for becauſe the firſt Merchant of Paper will gaine after 10 vpon the 100. Say, If 100 doe giue 110, what ſhall 11 ⅗ ſhillings giue? Worke, and you ſhall finde 12 ſ — 9 $\frac{13}{25}$ d, and ſo much did the pound of Mace coſt in ready money.

The fourteenth Chapter entreateth of exchanging of money from one place to the other

EXchange is no other thing, then to take or receiue money in one City, to render or pay the value thereof in another City: or elſe to giue mony in one place, and receiue the value thereof in another, at tearme of certaine dayes, moneths, or faires, according to the diuerſitie of the place.

But this practice chiefely conſiſteth in the knowledge of the mony or Coines in diuers places, of which for thy benefit (after a few examples gizen to the introduction of this worke) I wil ſet down certaine notes of the diuerſitie of the common and vſuall coines in moſt places in Chriſtendome for trafique.

And firſt I will begin at *Antwerpe*, where they vſe to make their accounts by *Deniers de*

groſſe, that is to ſay, pence Flemmiſh, whereof 12 doe make 1 ſ Flemmiſh, and 20 ſ doe make one pound *de groſſe*.

Item, a Merchant deliuered at Antwerpe 400 *pounds Flemmiſh to receiue in* London 20 ſ *ſterling, for euery* 23 ſ — 4 d *Flemmiſh: The queſtion is now, how much ſterling money is to be receiued at* London *for the* 400 *pounds of Flemmiſh?*

Anſwere. Say by the rule of three, If 23 $\frac{1}{3}$ Flemmiſh giue 20 ſ, ſterling, what 400 pounds Flemmiſh? Worke, and you ſhall finde 342 li — 17 ſ — 1 $\frac{5}{7}$ pence, and ſo much ſterling ſhall I receiue in *London* for the ſaid 400 pounds Flemmiſh.

Otherwiſe alſo wrought by Rules of Practice in taking the $\frac{1}{7}$ of the Flemmiſh money deliuered, and abating the ſame from the principall, the reſt is Engliſh money, as before.

400 li — 0 ſ — 0 d
57 — 2 — 10 $\frac{2}{7}$

342 — 17 — 1 $\frac{5}{7}$ ſterling.

A Merchant at London *deliuered* 200 li *ſterling for* Antwerpe, *at* 23 ſ — 5 d *Flemmiſh the pounds ſterling. The queſtion is, how much he muſt receiue at* Antwerpe.

Anſwere. Say by the Rule of three, if 1 pound ſterling giue 23 ſ 5 d Flemmiſh, what

200

200 li ſterling? Worke, and you ſhall finde 234 li — 3 ſ — 4 d. So many pounds Flemmiſh ſhall he receiue at *Antwerpe* for the ſaid 200 pounds ſterling.

Otherwiſe by practiſe.

1	23	5	200
3 ſ — 4 d			33 — 6 — 8
1 d			16 — 8
maketh ſterling			234 l — 3 ſ — 4 d.

In London 20 *pound ſterling is deliuered by Exchange, for* Antwerpe, *at* 23 ſ 9 d *Flemmiſh the pound ſterling. The queſtion is, at what rate the Flemmiſh money ought to be returned to gaine* 4 *pounds vpon the* 100 *pound ſterling at* London.

Anſwere. Firſt ſay by the Rule of three direct: If 1 pound ſterling giue 23 $\frac{3}{4}$ Flemmiſh, what 200 pounds ſterling? Multiply and diuide, and you ſhall finde 237 pounds 10 ſhillings. The which to returne to gaine 8 pounds ſterling in *London*, ſay by the backer Rule, If 200 pounds ſterling require the exchange 23 ſ 9 d Flemmiſh, what the exchange to make 208 l ſterling? Worke by the Rule, and finde 22 ſ — 10 $\frac{1}{16}$ d Flemmiſh, the effect in the queſtion required.

If I take vp money at Antwerpe *after* 19 ſ 4 d *Flemmiſh, to pay for the ſame at* London 20

ſhillings ſterling, and when the day of payment is come, I am forced to returne the ſame mony againe in London, *to pay my Bill of Exchange: ſo that for* 20 *ſhillings which I take vp here at* London, *I muſt pay* 19 ſ - 6 d *at* Antwerpe : *I demand whether I doe win or loſe, and how much in or vpon the* 100 *pounds of money?*

Anſwer. Say by the Rule of three: If 19 $\frac{1}{2}$ giue 19 $\frac{1}{3}$, what will 100 pounds giue? Multiply and diuide, and you ſhall finde 99 li 2 $\frac{106}{117}$ ſ, which being abated from 10 pounds, there will remaine 17 ſhillings $\frac{11}{117}$, and ſo much I doe loſe vpon the 100 pounds of money.

If I take vp at London 20 *ſhillings ſterling, to pay at* Antwerpe 22 ſ 4 d, *and when the day of payment is come, my Factor is conſtrained to take vp money againe at* Antwerpe, *wherewith to pay the aforeſaid ſumme, and there hee doth receiue* 23 ſ 4 d *Flemmiſh, for the which I muſt pay* 20 ſ *at* London: *The queſtion is now, whether I doe win or loſe, and how much vpon the* 100 l *of money after that rate.*

Anſwer. Say by the Rule of Proportion, If 22 $\frac{1}{3}$ ſ giue 23 $\frac{1}{3}$ ſ, what will 100 pounds giue? Multiply and diuide, and you ſhall finde 104 pounds 9 ſhillings $\frac{37}{67}$, from the which abate 100 pounds, and there will remaine 4 pounds 9 ſhillings $\frac{37}{67}$, and ſo much is there gained vpon

on the 100 pounds of money.

In Antwerpe *is deliuered* 200 *pounds Flemmiſh by exchange for* London *at* 20 *ſhillings ſterling for euery* 23 *ſhillings* 4 d *Flemmiſh. The queſtion is, at what rate the ſame is to be returned to gaine* 10 *pounds vpon the* 100 *pounds Flemmiſh in* Antwerpe.

Anſwer. Firſt ſay by the Rule of three, if 23 $\frac{1}{3}$ Flemmiſh giue 20 ſ, what ſhall 200 pounds gaine? Worke, and you ſhall finde 171 pounds 8 ſ 6 $\frac{6}{7}$ d. Then ſay againe by the Rule of three direct, if 171 l – 8 ſ – 6 $\frac{6}{7}$ ſ ſterling giue me 210 pounds Flemmiſh, what ſhall 20 ſ ſterling giue? Worke, and you ſhall finde 24 ſ 6 d Flemmiſh. And at the ſame rate ought the ſame to be rerurned at *Antwerpe* to gaine 10 pounds vpon the 100 Flemmiſh.

A Merchant of Antwerpe *deliuereth* 234 *pound* 3 ſ 4 d *Flemmiſh, to receiue at* London 200 *pounds ſterling: The queſtion is now, how the exchange goeth after this rate?*

Anſwer. Say by the Rule of three direct, If 200 giue 20, what 234 $\frac{1}{2}$? Multiply and diuide, and you ſhall finde 23 ſ — 5 d, And for ſo much goeth the exchange.

Item, the exchange from London *into* France, *is not like as it is in* Flanders, *but is deliuered by the French Crowne, which is worth* 50 *ſoulx Turnois the peece.*

Whereupon alſo you muſt note, that in

France they make their accounts by Francks, Soulx, and Deniers Turnois, whereof 12 Deniers make one Soulx Turnois, and 20 Soulx maketh 1 pound Turnois, which they call a Liure or Franke. But the Merchants, to make their accounts, doe vſe French Crownes, which is currant among them for 51 Soulx Turnois. But by exchange it is otherwiſe, for it is deliuered but for 50 Soulx Turnois the Crowne; or as the taker vp of the money can agree with the deliuerer. And note that this ∆ Charecter repreſenteth the Crown by exchange, and is euer 50 ſoulx Turnois or Freuch money.

A *Merchant deliuereth at* London 240 *pounds ſterling, after* 5 *ſhillings* 6 *pence the crowne, to receiue at* Paris 50 *Soulx Turnois for euery Crowne. I demand how much Turnois or French money payeth the Bils for the ſaid* 240 *pounds ſterling.*

Anſwer. Say by the Rule of three, If $5\frac{1}{2}$ ſ ſterling giue me 50 ſ Turnois, what ſhall 240 pounds ſterling giue? Reduce the pounds into ſhillings, then multiply and diuide, and you ſhall finde 2181 Liures, 16 Soulx, 4 Deniers, and $\frac{4}{11}$ Turnois, and ſo much payeth the Bils at *Paris* for the 240 pounds ſterling.

A *Merchant deliuereth at* Roan *or elſewhere in* France 1430 *pounds or francks, the which*

francke

francke or pound is 20 *ſoulx, or a pound Turnois, to receiue in* London 6 ſ 4 d *ſterling for euery* Δ *of* 50 *ſoulx Turnois. The queſtion is, how much ſterling mony I ought to receiue at* London *for my* 1430 *pound Turnois.*

Anſwer. Say if 2 $\frac{1}{2}$ pounds giue mee 6 $\frac{1}{3}$ ſ, what will 1430 giue me? Worke, and you ſhall finde 3622 $\frac{2}{3}$ ſhillings ſterling, which maketh 181 pounds 2 ſ 8 d, and ſo much money is to be receiued at *London*, for the ſaid 1430 Liuers Turnois, after 6 ſ 4 d for euery Δ of 50 ſoulx.

In London *is deliuered* 200 *pound ſterling by exchange for* Paris, *at* 5 ſ 9 d *the* Δ *of* 50 *ſoulx Turnois. The queſtion is, at what price the ſaid* Δ *is to be returned to gaine* 6 *pounds vpon the* 100 *pound ſterling at* London?

Anſwer: Firſt ſay (by the Rule of three direct) if 5 $\frac{3}{4}$ ſ ſterling, giue 50 ſoulx Turnois: what ſhall 200 pound ſterling giue? Worke, and you ſhall finde 1739 franks or liuers, 2 $\frac{14}{23}$ ſoulx Then, the which to returne and gaine 6 pounds vpon the 100 pounds in *London*, ſay by the rule of three direct, if 1739 franks 2 $\frac{14}{23}$ ſoulx yeeld 212 pounds, what the Δ of 50 ſoulx? worke and finde 6 ſ 1 $\frac{7}{50}$ d, the effect required in the queſtion.

A *Merchant deliuered in* London 160 *pounds ſterling, to receiue in* Biskay *for euery* 5 ſ 6 d, *one Ducket of* 374 *Maruides. The queſtion is, how many Maruides ought I to receiue at Biskay?*

Anſwer.

Anſwer. Say, if $5\frac{1}{2}$ ſ ſterling giue 374 Maruides: what ſhall 160 pounds ſterling giue? Multiply and diuide, and you ſhall finde 217600 Maruides, and ſo many I ought to receiue at *Biskay* for my 160 pounds ſterling.

A Merchant deliuered in Baion 40000 *Maruides, to receiue in* London 5 ſ–8 d *ſterling for euery Ducket of* 374 *Maruides. The queſtion is now, how much ſterling money payeth the Bils of Exchange for the ſaid* 40000 *Maruides?*

Anſwer. Say, if 374 Maruides make one Ducket, what 40000 Maruides? Multiply & diuide, and finde 106 Duckets $\frac{178}{187}$.

Then ſay againe, if 1 Ducket giue $5\frac{2}{3}$ ſ, what giueth $106\frac{178}{187}$ Duckets? Worke, and find 30 li—6 ſ, $\frac{34}{561}$.

Otherwiſe it is wrought more briefe at one working, as in the laſt queſtion before, in conſidering that 5 ſ–8 d containeth one Ducket, or 374 Maruecides. Therefore ſay by the Rule of three, if 374 Maruecides giue $5\frac{2}{3}$ ſ, what 40000 Maruecides? Worke, and you ſhall alſo finde in your quotient 30 li—6 $\frac{14}{561}$ ſ. And ſo many pounds ſterling is to be receiued for the 40000 Maruecids.

In London 200 *pounds deliuered by Exchange for* Vigo, 374 *Maruecids the Ducket of* 5 ſ, 10 d *ſterling,*

ſterling, maketh 256457 $\frac{1}{17}$ *Marueides: the which to returne and gaine* 10 li. *vpon the* 100 *pounds in* London, *ſay by the Rule of three direct, if* 220 *pounds require* 256457 $\frac{1}{7}$ *Marueids, what* 5 ſ 10 d? *Worke, and finde* 340 *Marueids, the price of euery Ducket in returne, which is the effect in the queſtion required.*

Theſe may ſeeme ſufficient for inſtructions.

NOtwithſtanding for the further aid and beuefit, hereafter follow ſix ſpeciall and moſt briefe Rules of Practiſe, for Engliſh, French, and Flemmiſh money.

1	teacheth	How to turne Flem. to Eng. ſterling.
2		How to turne Eng. ſterling to Flem.
3		How to turne Flemmiſh to French.
4		How to turne French into Flēmiſh.
5		How to turne ſterling into French.
6		How to turne French into ſterling.

The

The fifteenth Chapter intreateth of the ſaid ſixe Rules of Breuity, and of valuation of Engliſh, Flemmiſh, and French money, and how each of them may eaſily bee brought to others value,

Rule 1. *How briefely to reduce pounds, ſhillings and pence Flemmiſh, into pounds, ſhillings, and pence Engliſh ſterling.*

IT is to bee noted, that 7 pounds Flemiſh maketh but 6 pounds ſterling : : 7 (ſlemiſh maketh 6 ſ ſterling, and 7 d, ſlemiſh 6 d, ſterling : ſo that 7 yeeldeth but 6. Wherein is euident that then is loſt $\frac{1}{7}$, (if it may bee ſo called) when it is reduced into Engliſh money: wherefore to know how much 233 l–13 ſ–4 d, Flemiſh maketh Engliſh, you muſt ſubtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which reſteth after this ſubtraction, is the ſumme required : ſo that 233 li–13 ſ 4 d, ſlemmiſh, maketh 200 li––5 ſ––8 $\frac{4}{7}$ d, ſterling.

Example

li	ſ	d
233	13	4
$\frac{1}{7}$ 33	7	$7\frac{3}{7}$
200	5	$8\frac{4}{7}$ ſter.

Another example.

li	ſ	d
311	0	0
$\frac{1}{7}$ 44	8	$6\frac{6}{7}$
266	11	$5\frac{1}{7}$

To

To reduce pounds, ſhillings, and pence ſterling, into pounds, ſhillings, and pence Flemmiſh. Rule 2.

Note that a pound ſterling maketh 1 li–3 ſ 4 d Flemmiſh: that is, 1 $\frac{1}{6}$ li, 1 ſ ſter. maketh 1 $\frac{1}{6}$ ſ Flemmiſh, and 1 d, ſter. maketh 1 $\frac{1}{6}$ d, Flemmiſh. So that there is gained (if it may be ſo called) $\frac{1}{6}$ of the ſumme being thus reduced to Flemmiſh, for of $\frac{6}{6}$ is made $\frac{7}{6}$, which is one whole and $\frac{1}{6}$. Then to know how much 237 li--7 ſ--6 d, ſterling maketh Flemmiſh ſubtract from your ſterling, the $\frac{1}{6}$ of the whole ſumme, and adde it to the ſame ſumme, and it maketh 276 li--18 ſ--4 d, which is the ſumme required.

Example.

li,	ſ	d	
237	7	6	ſter.
$\frac{1}{6}$ 39	11	3	
276	18	9	Flem.

Another Example.

li	ſ	d
337		
$\frac{1}{6}$ 56	3	4
393	3	4

To reduce pounds, ſhillings, and pence Flemmiſh, into pounds, ſhillings, and pence French. Rule 3:

Ye ſhall note, that the equality of Flemmiſh and French money is this, that is to ſay, the pound Flemmiſh, maketh 7 pound $\frac{1}{5}$ French, or Turnois, 1 ſ Flemmiſh maketh 7 $\frac{1}{5}$ ſ French and a groat Flemmiſh, maketh 7 $\frac{1}{5}$ d French.

Wherefore to know how much 143 li-4 ſ 9 d,

9 d, Flemmiſh maketh French, ye muſt multiply the whole number twice by 6, beginning at pence, and ſo forward, and the product of your ſecond multiplication diuide by 5, ſo the worke is finiſhed. Or multiply the ſaid ſumme by 7. and take out of it $\frac{1}{7}$, adding it to the product of your multiplication by 7, and that is your number required. So that as well by the one as by the other, 143 li——4 ſ——9 d, Flemmiſh, maketh 1031 li——6 ſ——$2\frac{2}{3}$ d, French or Turnois.

Example.

li	ſ	d	
143	4	9	*Flem.*
		6	
859	8	6	
		6	
5156	11	0	*Fren.*
$\frac{1}{5}$ 1031	6	$2\frac{2}{3}$	*Fren.*

The ſame otherwiſe

li	ſ	d
143	4	9
		7
1002	13	3
$\frac{1}{5}$ 28	12	$11\frac{2}{5}$
1031	6	$2\frac{2}{3}$

Another example.

143 l, *Flem.*

6

858

6

$\frac{1}{5}$ 5148

Fren.

1029 li $\frac{3}{5}$ *or* 12 ſ, *Fren.*

Or thus:

143

7

1001

$\frac{1}{5}$ 28—12

1029 li, —12

To reduce pounds, ſhillings, and pence French, into pounds, ſhillings, and pence Flemmiſh. 4. Rule.

Multiply 233 li—8 ſ—4 d French by 5, and diuide the Product twice by 6, that is, the ſaid number by 6, and the product or quotient againe by 6, and the quotient of this ſecond Diuiſion is the thing required. So that 233 li-8 ſ-4 d French, maketh 32 li-8 ſ-$4\frac{5}{9}$ d Flemmiſh.

Example.

li	ſ	d		
233	8	4	*Fren.*	
		5		
1167	1	8		
$\frac{1}{6}$ 164	10	$3\frac{2}{6}$		
$\frac{1}{9}$ 32	8	$4\frac{5}{9}$	*Flem.*	

Another.

li	ſ	d	
753			*Fren.*
		5	
3765			
$\frac{1}{6}$ 627	10		
$\frac{1}{6}$ 104	11	8	*Flem.*

To reduce pounds, ſhillings, and pence ſterling, into pounds, ſhillings, and pence, French, or Turnois. Rule 5.

The pound ſterling maketh 8 l-. 8 ſ, French that is to ſay, $8\frac{2}{5}$ pounds: the ſhillings, maketh $8\frac{2}{5}$ ſhillings, and the peny $8\frac{2}{5}$ d French. Wherefore to know what 231 li—13 ſ—4 d ſterling maketh French, ye muſt multiply your whole ſumme by 42, that is, by 7, and the product of it by 6, and diuide this ſecond product by

by 5, and that is the ſumme required,

Otherwiſe, multiply the ſumme ſterling by 8, and adde twice to the product $\frac{2}{5}$, and it ſhall produce the ſumme required. So that both wayes 231 li—13 ſ—4 d ſterling, maketh 1946 li French, as hereunder followeth.

	Example. li	ſ	d *ſter.*	The ſame othewiſe. li	ſ	d
Sterl.	231	13	4	231	13	4
			6			8
	1390	0	0	1853	6	8
			7	46	6	8
				46	6	8
	$\frac{1}{5}$ 9730	0	0			
				1946	0	0
French.	$\frac{1}{5}$ 1946	0	0 *Fren.*			

	Another example.		The ſame.
Sterl.	753	*Ster.*	753
	6		8
	4518		6024
	7		$\frac{1}{5}$ 150——12
	31626		$\frac{1}{5}$ 150——12
French.	$\frac{1}{5}$ 6325——4	*Fren.*	6325——4

To reduce pounds, ſhillings, and pence, French, into pounds, ſhillings, and pence, ſterling.

To

To know how much 1256 li, 12 ſ, 6 d, *French* maketh in ſterling money: multiply the ſumme by 5, and diuide the product by 7 and 6 at twice, and the laſt quotient ſhall be the thing required, that is to ſay, 1256 li, 12 ſ, 6 d, maketh 149 pounds, 11 ſ, 11 $\frac{4}{7}$ d ſterling. Rule 6.

Example.

li	ſ	d	*French.*
1256	12	6	
		5	
6283	2	6	
$\frac{1}{6}$ 1047	3	9	
$\frac{1}{7}$ 149	11	11 $\frac{4}{7}$	*Sterl.*

Another Example.

li	ſ	d	
2531	0	0	*fren.*
		5	
12655			
$\frac{1}{6}$ 2109	3	4	
$\frac{1}{7}$ 301	6	2 $\frac{2}{7}$	*ſter.*

Note, that when any money is giuen by exchange at *London* for *Roan* at 71 $\frac{1}{2}$ d, or rather 71 $\frac{1}{7}$ for the Crowne of 50 ſ, French, there is neither gaine nor loſſe: for it is one money for another, accounting 8 li—8 ſ French, for one pound ſterling. So the giuer loſeth the time of paiment, which is about 15 dayes, and he that taketh it, hath the gaine of the ſame.

They of *Roan* that put forth or take money by exchange for *London*, ought to haue like conſideration.

Item, when any man giueth at *London* 64 pence $\frac{1}{3}$, or rather 64 $\frac{2}{7}$ d, to haue at on of the Fayres of *Lyons* a Crowne *de Marc*, hee

that ſo giueth the money, loſeth the time, and he that taketh it, gaineth the ſame: for 62 pence ½, is equall in value to 45 ſ Freneh. He that putteth or taketh money at *Lyons* for *London*, ought to conſider the ſame.

Item, when any deliuer in *Antwerpe* 75 pence, to receiue at *Lyons* a Crowne *de Marc*, he that putteth it forth, loſeth the time, and he that taketh it gaineth the ſame. For 75 groates Flemmiſh, is equall in value to 45 ſ French.

Thus for this time I make an end of the practiſe of Exchange, and the inſtructions thereunto belonging, and according to my promiſe: yet further to gratifie ſuch as are deſirous to know the common Coynes vſed for trafficke among Merchants in theſe Cities following, a briefe declaration of their Moneyes, and the reckonings, and account of them.

The

The sixteenth Chapter containeth a declaration of the valuation and diuersitie of Coynes of most places of Christendome for trafficke; And the manner of exchange in those places from one Citie or Towne to another: which knowne, is right necessarie for Merchants, by meanes whereof they doe finde the gaine or losse vpon the exchange.

Item, for as much as the greatest diuersity of money of exchange is at *Lyons*; therefore I will begin duely of the money of that place.

At *Lyons* they vse Franckes, Soulx, and Deniers Tournois. A Franke maketh 20 Soulx, and one Soulx 12 Deniers. But the Merchants, to keepe their bookes of accounts, do vse French Crownes of the Marke at 45 Soulx the piece, and doe diuide it into 20 Soulx, 1 Soulx, is 12 Deniers.

Item, a Marke of Gold maketh 65 Δ of the Marke, which serueth for exchange, and diuide it into 8 ounces. The ounce into 24 pence or deniers, the denier into 24 graines, and so the summe or whole by imagination or gesse.

Δ This Marke standeth for a Crowne.

Also at *Lyons* there are 4 Faires in a yeere, at the which they doe commonly exchange, which are from three moneths to 3 months:

At *Geanes* they vſe the Soulx: one Ducket maketh 3 pound.

At *Naples* they vſe Duckets, Taries, and Graines, the Ducket maketh 5 Taries, and one Tarie 20 Graines: but they take 6 Duckets (which maketh thirtie Taries) for the ounce.

A Ducket maketh ten Carlins, and a Carlin ten Graines, ſo that 2 Carlins make a Tarie and 100 Graines make a Ducket.

At *Rome* they vſe the Duckets of the Chamber: one Ducket is worth 12 Guilis, and one Guilis 10 Soulx.

At *Venice* they vſe Duckets currant at 124 ſoulx a peece, or 24 Deniers, and one Denier maketh 32 Picolis.

At *Palerme* and *Meſſine* they write, after ounce, tarie, and graines, and one ounce is worth 6 Duckets of 30 taries, and 1 tarie is 20 graines, and 1 graine 6 Picolis, 1 ducket is alſo worth 24 Carlins.

At *Millan* they vſe li, ſ, d, of ducket Imperials, and △ of exchange is worth 4 li.

At *Lucques, Florence, and Ancone*, they vſe the △ of Gold: in Gold the French Crowne is worth 7 li, but at *Boloigne* 3 li, 10 ſ.

At *Barcelone* they vſe the Soulx: the ducket of exchange is worth 22 Soulx.

At *Valence* and *Saragoſſe* they vſe the Liuer, Soulx, and denier: the French Crowne of exchange is worth 20 Soulx, and one ſoulx is twelue Deniers.

At

At the Faires of *Castill* they vse the Marueides, the Ducket is worth 375 Marueides.

At *Lisbone* they vse the Raies, one Ducket of exchange is worth 400 Rayes.

At *Noremburge*, *Franckford*, and *August* in *Germany*, they vse the *Krentzars*, whereof 60 make a Floren.

At *Antwerpe* they vse li—sand d, de Gros and they exchange into the denier de Gros, to wit, our Englisk Peny.

At *London* they vse the li the s and d sterling, and they exchange in pence sterling.

The exchange of Lyons at sundry places.

Item, at *Lyons* there is exchange in three sorts, at the Cities and Townes following.

First they deliuer at *Lyons* one Marke to haue or receiue at *Naples* almost 41½ Duckets, at *Venice* 70 Duckets currant: at *Rome* 63 Duckets of the Chamber: at *Lucques* and *Florence* 65 Δ of gold at *Millan* 82 Δ.

And contrariwise, at the said Cities aforesaid, they doe giue so much of mony, to haue a Marke at *Lyons*.

Secondly, they giue at *Lisbone* one Δ of Marke of 45 soulx Turnois a peece, to haue at *Geanes* almost 68 soulx, at *Palerme* and *Messine* almost 24 Carlins, at *Barselone* 22 soulx, at *Valence* or *Saragosse* 20 soulx, at the Faire at *Castill* 350 Marueides, at *Lisbone* 360 *Rayes*, in *Antwerpe* 57. Deniers *de Gros*, and at *London* 70 d sterling.

And contrariwiſe, they giue in the ſaid Cities almoſt as much of their money to haue a French Crowne of the Marke at *Lyons*.

Thirdlv, they doe giue at *Lyons* a △ of the Sun, to haue almoſt 93 *Krentzars* at *Franckford*, *Augsburg*, *Noremberg*, or other Cities in *Almaine*.

Alſo at *Lyons* only they do pay the change the $\frac{2}{3}$ in Gold, and $\frac{1}{3}$ in Money, or elſe all in Money, in giuing $1\frac{1}{2}$ for the hundreth.

Changes at Naples and other Townes.

Item, at *Naples* they giue or deliuer almoſt 112 Duckets, to receiue at *Rome* 100 Duckets of the Chamber at the old value.

Through *Lucques* and *Florence* they deliuer 100 Duckets Carlins, to receiue there almoſt 86 △ of Gold.

Through *Palerm* & *Meſſine*, one Ducket of 5 Tary, to receiue there almoſt 164 graines.

Through *Millan*, one Ducket, to receiue there almoſt 90 Soulx.

Through *Geans* one Ducket to receiue there almoſt 65 ſoulx. The whole ſumme to be paid within 10 dayes after the ſight of the Bill of Exchange.

Alſo at *Naples* they deliuer one Ducket to receiue in *Antwerpe* almoſt 67 d, or Deniers *de Gros*, within two moneths. At *London* almoſt 60 d ſterling in three moneths. At *Barſelone* almoſt 20 ſoulx within two moneths.

At

At *Valence* almoſt 18 ſoulx within two moneths. At *Lisbone* 333 Rayes within three moneths: and at the Faire at *Caſtill*, almoſt 340 Marueides at the ſame Faire.

Change of Venice to other places.

At *Venice* they deliuer 100 Duckets currant to receiue in *Almaine* almoſt 140 *Florens* at 60 *Krentzers* the peece.

At *Lucques* and *Florence* almoſt 108 △ of Gold in 10 dayes.

Likewiſe at *Venice* they deliuer a Ducket currant to receiue at *Palerme* and *Meſſine* almoſt 21 Carlins, at *Millan* almoſt 93 ſoulx, at *Geanes* almoſt 62 ſoulx, the whole at ten dayes end.

Of the Pair or Pari.

As touching the Exchange, it is neceſſary to vnderſtand or know the *Pair*, which the Italians call *Pari*, which is no other thing then to make the money of the change of one Citie or Towne, to or with the money of another, by meanes whereof they doe finde the gaines or loſſe vpon the Exchange.

Example.

Item, hauing receiued letters of credit of one of *Antwerpe*, that the △ of the Sunne is there worth 7 ſoulx: The queſtion is, what the ſame is worth at *London*, when the *Pair* of Exchange goeth for 23 ſhillings?

Anſwer. *Say, if* 23 *giue but* 20, *what giueth* 7? *Worke, and find* 6 ſ, 1 $\frac{1}{23}$ d, *and ſo much is the* △ *of the Sunne worth at* London.

The ſeuenteenth Chapter contayneth *alſo a declaration of the diuerſity of the waights and meaſures of moſt places of Chriſtendom for traffique. At the end of which diſcourſe are two Tables, the one for weight, and the other for meaſure, proportionate and reduced to an equality of our Engliſh meaſure and weight, by the aid whereof, the ingenious may eaſily by the Rule of three, conuert the one into the other at pleaſure, &c.*

AT *London*, and ſo all *England* thorow, are vſed two kinds of weights and meaſures, as the *Troy* weight, and the *Haberdepoiſe*. From the *Troy* weight is deriued the proportion and quantitie of all kinde of dry and liquid meaſures, as pecks, buſhels, quarters, &c. wherewith is bought and ſold all kind of graine and other commodities met by the Buſh. And in liquid, ale, beere, wine, oyle, butter, hony, &c. vpon theſe grounds & ſtatutes is bread made, and ſold by the *Troy* waight: and ſo is gold, ſiluer, pearle, precious ſtrones, and Iewels. The leaſt quantity of this *Troy* weight is a graine: 24 of theſe graines make a penny weight, 20 penny weights an ounce, and

12 ounces a pound; 2 pounds or pintes of this weight maketh a quart. And so ascending into bigger quantities, is produced the measures whereby are sold our other naturall sustenance: *viz.* Ale or Beere, with all other necessarie commodities, as Butter, Hony, Herrings, Eeles, Sope, &c. All which last before rehearsed, though their measures (wherein they are contained) be framed and deriued from the *Troy weight*, yet are they in traffique with diuers commodities, as Lead, Tinne, Flax, Wax, with all other commodities both of this Realme, and of other forraigne Countries whatsoeuer, bought and sold by the *Haberdepoise weight* after sixteene ounces to the pound and 112 pound to the C. weight. And to euery C. is allowed but 12 pound weight at the common beame. From hence is also deriued the weight of *Suffolke* Cheese, which containeth 32 cloues, 8 pound to a cloue, and waieth in all 256 pounds. And also the Barrel of *Suffolke* Butter, is, or should be of like weight with the weight of Cheese, *viz.* 256 pounds. More 14 of these pounds make a stone, and 26 stone conteineth a Sacke of English wool, Forraigne wools; to wit, French, Spanish, and Estrich, is also sold by the pound or C. weight, but most commonly by the Roue, 25 pound to a Roue: other commodities of Tale, are bought and sold by the C. fiue score to the C. Except headed ware, to wit,

See further of these Waights and Measures in reduction, beginning pag. 133.

wir,Cattell, Nayles, and Fiſh, which are ſold after ſixeſcore to the C. There is alſo two other ſorts of meaſures,to wit the Elle and the Yard. By the Ell is vſually met, Linnen cloth, as Canuas,&c. And by the yard, Silkes, woollen clothes, &c.

Antwerpe.

At *Antwerpe* are alſo 2 ſorts of waights, their gold and ſiluer weight, and their common weight. Gold and ſiluer is weighed by the *Mark*, the *Marke* is 8 ounces, the ounce 20 Eſterlings, and the Eſterling 32, as our graines. The Goldſmiths diuide that into ſmaller but not the Merchants: The proofe of gold is made by Karects, whereof 24 maketh a marke of fine gold, the Karect is 24. graines: the proofe of the money is made by *Deniers*,12 deniers is 1 ſ fine, that is a *mark* of fine ſiluer; the denier alſo is diuided into 24 graines,and the graine into 4 quarters.

Item,100 *Markes in* Antwerpe,Troy *weight, maketh at* Lyons 103 *Markes*,2 $\frac{1}{2}$ *ounces, and* 20 *graines*,23 *p. At* Noremberg 103 *markes* 2 $\frac{1}{2}$ *ounces*,2 *quints*,3 *Deniers*; *at* Franckford, 105 *marks*,*at* Augsburg 104 *markes*,*three ounces*, 1 *quint. At* Venice 103 *markes*,1 *ounce*,7 *deniers*, 18 *Graines. At* London 66 *pounds.*

The marke of Gold or ſiluer at Antwerpe, Troy *weight, which is* 8 *ounces, maketh* 7 $\frac{1}{2}$ *ounces common weight, with which all other merchandiſe*

diſe is weighed. So that the Troy *weight is greater then the common weight by* 6 $\frac{1}{4}$ *in the C. By this weight of* Troy, *they alſo weigh* Musk, Amber, Pearle, *&c.*

All ſilkes are bought at Antwerpe, *by the* Burges Ell, *which is greater then the common meaſure, by which they retaile by two in the hundred. Their common Ell is* $\frac{3}{4}$ *of our yeard, and* $\frac{3}{5}$ *of our Ell.*

Lyons

At *Lyons* is vſed 3 ſorts of waight, whereof the firſt is the common Towne waight, with which they weigh all kinde of Spicery, and diuers other merchandiſe. The ſecond is called *Geneua* weight, which is 8 in the C greater then the common weight, with which they weigh ſilkes, &c. The third is French waight, called commonly the *Marke weight*, and 100 pounds thereof maketh 106 $\frac{1}{4}$ li *Geneua*, and 114 $\frac{3}{4}$ of their common weight: with which French waight, is weighed all things that paied cuſtome or toll.

At *Lyons* is alſo vſed two ſorts of Elles or *Aulnes*. The one wherewith they meaſure groſſe clothes, as canuas, and ſuch like. The other is called the French Ell or *Aulne*, with which they meaſure all other kinde of merchandiſe, whereof ſeuen common towne Elles maketh 11 ordinary French Ells.

Roan.

At *Roan* 6 $\frac{1}{2}$ Muides of ſalt, being the meaſure

ſure of the place, make a C. at *Armuiden* in *Zeland*, and the C. of *Bronage* meaſure of *Armuiden*, maketh at *Roan* 11. Muides, 30 Muides make a laſt of corne, and 16 a laſt of Oates, 100 pound weight there, maketh at *London* 114 $\frac{1}{4}$, and 190 $\frac{1}{4}$ at *Antwerpe*. And 100 ells make at *London* 115 $\frac{1}{4}$.

Noremburge.

A 100 pound weight at *Noremburge* maketh at *London* 111 $\frac{3}{4}$, at *Antwerpe* 107 $\frac{1}{2}$, and 100 ells at *Noremburge* make at *London* 75 $\frac{2}{3}$ at *Antwerpe* 95 $\frac{3}{5}$ &c.

Lisbone.

The C waight at *Lisbone* maketh foure Roues, euery Roue 32 pounds: ſo that their C. weight is 128 pounds, and their pound containeth 14 ounces, and 100 pounds of their weight maketh at *London* 113 $\frac{1}{8}$,

Their Silke, cloth of Gold, and Woollen is meaſured with a meaſure which they call a cubite, containing about $\frac{3}{4}$ of a Varre of *Caſtile*. Howbeit, their common meaſure is called a Varre, which maketh 5 Palmes, and containeth 1 $\frac{1}{4}$ of a Varre of *Caſtile*, our elle of *London* is equall with the Varre of *Lisbone*.

All kinde of Merchandize brought from *Flanders*, *Roan*, or *Brittaine*, payeth at *Lisbone*, as a duety or cuſtome to the King, 20 in the C, which they call the tenth in Merchandize,

and

and the other tenth in money.

Note also, that all kinde of Merchandize comming to *Lisbone* by Land, payeth lesse in custome, then that that commeth by water.

Ciuill.

The Roue of *Ciuill* is 30 pound, 4 Roues make their C weight, which is 120 pounds. The 100 pounds of *Ciuill* maketh at *London* 102 pounds. Their other common measure is a Varre, whereof 100 maketh at *London* 74 Ells, and at *Rome* 40 Canes, &c.

Venice.

At *Venice* bee two sorts of weight; the one called *La Grosse*, the other *La Suttle*. with, the grosse is weighed all kinde of great wares, and with the small all kinde of spicery and such like: 96 pounds of grosse weight there, maketh at *London* 100 pound, and 100 pounds of spicerie there without any tare or allowance, make at *London* 94, and with tare 65.

Their owne common measure are Braces, wherof 100 make at *London* 55 ½ Ells, at *Antwerpe* 92 ½ &c:

Florence.

At *Florence* the 100 li weight maketh at *Aquila*, for Saffron 110, and 145 pounds of *Florence*, make at *Roan* but 100 pounds, the weight of *Florence* & that of *Luke* is all one.

Their

Their other meaſures are Braces, whereof 100 maketh at *Antwerp Burges* meaſure, 81 $\frac{2}{5}$ Ells, 100 Braces there, make at *London* 49 Ells, &c.

Lucque.

The *Lucque* Sattens are commonly ſold at *Lyons* by weight, and 133 $\frac{1}{3}$ pounds maketh at *Lyons* 100 pound, ſo that 1 pound $\frac{1}{3}$ maketh at *Lyons* but one pound.

Their other meaſures are Braces, whereof 100 of them make at *London* 50 Ells, at *Antwerpe* 83 $\frac{1}{3}$ Ells, &c.

Aquila.

At *Aquila* their 100 pounds maketh at *London* 71 $\frac{1}{4}$, their 136 $\frac{2}{3}$ pounds of Saffrom maketh at *Geneua* but 100, and 11 li of *Geneua* maketh 15 li, at *Aquila.*

Valentia.

At *Valentia* be 2 ſorts of weights, a great and a ſmall. The C weight or great weight containeth 4 Roues, the Roue 36 li, ſo the C. great weight is 144 li. and the C. weight ſmall containeth but 120 pounds, and is alſo parted into 4 Roues, which is 30 pounds to a Roue. By the ſmall is ſold the ſcarlet graine, with all other kind of ſpicery, and by the great is ſold wool, withall ſuch like groſſe wares. The 1 $\frac{1}{3}$ pounds of ſilke at *Valentia*, maketh at *Lyons* one pound *Geneua* waight. The charge of great Merchandize at *Valentia* containeth

432 pound, and in ſmall wares 360 pounds. The weight here and at *Barſellone* is all one. Their 100 pound weight maketh at *London* 78 pound, and at *Antwerpe* 75.

Danſicke.

At *Danſicke* or *Spruce-land* the rule is, that whoſoeuer buyeth any Merchandize there, buyeth it by the ſhip-pound, which is 320 li, 20 Liſpounds make a Ship pound, and the Liſpound containeth 16 pound, which Ship-pound of *Danſicke* maketh at *Antwerpe* 266 $\frac{2}{3}$ li. Their 100 li weight, maketh at *London* 86 $\frac{5}{8}$ &c.

Their other common meaſures are Ells, whereof 100 maketh at *London* 72 $\frac{1}{4}$, and at *Antwerpe* 120 $\frac{1}{2}$ Ells.

Toulhouſe.

At *Toulhouſe* 6 Cabes of woad maketh a Charge, two Ceſternes of Corne-meaſure, and all kinde of graine maketh a Charge, the Ceſterne weigheth 160 li weight of that place. Their 100 in weight, maketh at *London* but 91 $\frac{1}{4}$ pound,

Geanes.

At *Genua* or *Geanes*, 100 li of their weight maketh at *London* 71 $\frac{1}{4}$, and at *Antwerpe* 68 $\frac{3}{8}$ 100 li weight at *Genua*, maketh at *Venice*, to wit, Suttle 106 li.

Their other common Meaſures are *Palmes*, whereof 100 make at *London* 20 $\frac{5}{8}$ Ells; and at *Antwerpe* 34 $\frac{5}{8}$

The

The reſt are ſupplied in two Tables, which hereafter followeth: whereby the ingenious may gather his deſire.

The Table of the agreement of the Weights of diuers Countries, the one with the other, being reduced to an equality, as followeth.

112 pounds weight at London make at		112 pounds weight at London, make at	
Antwerpe.	107 5/8	*Venice groſſe weight.*	105 3/8
Franckford.	099	*Venice ſubtle weight.*	166 7/8
Collen and Auſburg.	102 1/4	*Aquila.*	157 1/2
Noremburg.	100 1/8	*Vienna.*	089 3/8
Roan.	098	*Preſlaw.*	134 5/8
Paris.	102 1/4	*Leipſig.*	101 1/4
Lyons.	118 1/2	*Danſicke.*	129 1/4
Diepe.	100 1/4	*Lubecke.*	097 3/8
Geneua.	090 1/8	*Barcellon.*	144 1/8
Towlouſe.	122 1/4	*Lisbon.*	099
Rochel.	124 7/8	*Geanes.*	157 1/8
Marcellis.	124 1/4		
Ciuill, &c.	109 3/4		

The

The other Table of agreement of Measures of diuers Countries reduced vnto an equality, by the aide whereof you may with the vse of the Rule of three, conuert either more or lesse of any one measure vnto the other.

60 Ells, or 75 Yards at London make at

Antwerpe	100	
Norenburg	104½	*Ells.*
Franckford	125	
Leipsig	125	
Preslaw	125	
Dansicke	183	
Vienne in Austri:	87	
Lyons in France	60¾	*Aulnes.*
Paris in France	57	
Roan in Normandy	52	
Lisbone	60	*Varres.*
Ciuill in Spaine	81	
Castile in Spaine	81	
Methera Iles	62	
Venice	108	*Braces.*
Lucques	120	
Florence	122½	
Millane	138	
Rome	90	*Canes.*
Geanes	288 6/23	*Palmes.*

The eighteenth Chapter treateth of Sports and Paſtimes, done by Number.

IF you would know the number that any man doth thinke or imagine in his minde, as though you could diuine, bid them triple it, or put twice ſo much more to it as it is, which done, aske him whether it be euen or odde: if he ſay odde, bid him take one to it, to make it euen, and for that one keepe one in your minde. Now after he hath taken one to it, to make it euen, bid him giue away halfe, and keepe the other halfe for himſelfe, which when he hath done, bid him triple that halfe, and againe after he hath tripled it, aske him whether it be euen or odde: if hee ſay odde, then bid him take one to make it euen again, & for that laſt one, keepe two in your minde: now after hee hath made his number euen, bid him caſt away the one halfe, and keepe the other ſtill, from which halfe that he keepeth cauſe him ſubtilly to put away or giue you 9 out of his number, as oftas he can and for each 9 that he giueth you, keepe, 4 in your minde, and thereunto ioyne the 3 which I bade you keepe, and you ſhall haue your deſire.

Example.

Imagine he thought 7, the triple whereof is

21, and becauſe it is odde, he is to take 1 to make it euen, which firſt 1 giuen, is for you to keepe in minde. Then the halfe of his 22 being caſt away, he reſerueth ſtill 11, which after you haue bid him triple, it maketh 33: then in giuing of him one againe to make it euen, vpon that laſt 1 reſerue 2 in your minde, then his halfe of 34 maketh 17, from whence he can giue you 9 but once. Therefore that yeelding to you 4 and the 3 that you keepe, make 7 your deſire.

Another kinde of Diuination, to tell your friend how many pence or ſingle pieces, reckoning them one with another, hee hath in his purſe, or ſhould thinke in his minde.

Which to doe, firſt bid him double the pieces he hath in his purſe, or the number he thinketh, (if he participate his number or ſecreſie vnto ſome one friend that ſitteth by him that can but multiply, and adde neuer ſo little: if their number be great, then ſhall they worke as you bid them ſo much the ſurer.)

Now after hee hath doubled his number, bid him adde thereunto 5 more; which done, bid him multiply that his number by 5 alſo: which done, bid him tell you the iuſt ſum of his laſt multiplication, which ſum the giuer thinketh it nothing auaileable, becauſe it is ſo great aboue his pretended imagination: yet thereby ſhall you preſently with the helpe of Subtraction tell his propoſed number.

The Rule is this:

Imagine he thought 17, *Double* 17, *and it maketh* 34, *wherevnto if you adde* 5, *it maketh* 39: *which multiplyed by* 5, *as heere is practiſed, it yeeldeth* 195, *which* 195 *is the ſumme deliuered you in the worke: then for a generall Rule you ſhal euermore cut off the laſt figure toward your right hand, with a daſh of your Pen, as here is performed, as a figure nothing auaileable vnto your worke, and then rebate* 2 *from your firſt figure, after* 5 *is cut off, and the reſt ſhall euermore be your deſire, as by this example doth appeare.*

17
2
34
5
39
5
19 | 5
2 |
17

Another of a Ring.

If in any company you are diſpoſed to make them merry by manner of diuining, in deliuering a Ring vnto any one of them, which after you haue deliuered it vnto them, that you will abſent your ſelfe from them, and they to deuiſe after you are gone, which of them ſhall haue the keeping thereof, and that you at your returne will tell them what perſon hath it, vpon what hand, vpon what finger, and what ioynt: Which to doe, cauſe the perſons to ſit downe all in a row, and to keepe likewiſe an order of their fingers: now, after ye are gone out from them to ſome other place, ſay vnto one of the lookers on, that he double the numbers of him that hath the

Ring,

Ring, and vnto the double bid him adde 5, and then cauſe him to multiply the Addition by 5, and vnto the product, bid him adde the number of the finger of the perſon that hath the Ring. And laſtly, to end the worke, beyond that number towards his right hand, let him ſet downe a figure ſignifying vpon which of the ioynts he hath the Ring, as if it be vpon the ſecond ioynt let him put downe 2. Then demand of him what number hee keepeth, from the which you ſhall abate 250, and you ſhal haue three figures remaining at the leaſt. The firſt toward your left hand, ſhall ſignifie the number of the perſon which hath the Ring, the ſecond or middle number ſhall declare the number of the finger, and the laſt figure towards your right hand ſhall betoken the number of the ioynt.

Example.

Imagine the ſeuenth perſon is determined to keepe the Ring vpon the fifth finger, and the third ioynt: firſt double 7, it maketh 14 thereto adde 5, it maketh 19, which multiplyed by 5 yeeldeth 95, vnto which 95, adde the number of the finger, and it maketh 100: and beyond 100 toward the right hand, I ſet downe 3 the number of the ioynt, all maketh 1003 which is the number that it to bee deliuered you, from which abating 205, there reſteth 753. which prefigureth vnto you the ſeuenth perſon,

the fifth figure, and the third ioynt.

But note, that when you haue made your ſubtraction, if there doe remaine 0, *in the place of tens, that is to ſay, in the ſecond place, you muſt then abate* 1, *from the figure which is in the place of the hundreds, that is to wit, from the figure which is next your left hand, and that ſhall be worth* 10 *tenths, ſignifying the tenth finger, as if there ſhould remaine* 803, *you muſt ſay, that the ſeuenth perſon vpon his tenth finger, and vpon his third ioynt, hath the Ring.*

Another of three Dice.

If a man doe caſt 3 Dice, you may know the points of one of euery of them. For if you cauſe him to double the points of one Die, & to the double to adde 5, and the ſame ſumme to multiply by 5, and vnto the product adde the points of one of the other Dice. And behinde the number towards the right hand, to put the figure which ſignifieth the points of the laſt Die, and then to aske what number he keepeth, from which abate 250, and there will remaine 3 figures, which doe note vnto you the points of euery Die.

Another of things hidden.

If three diuers things are to be hidden of three diuers perſons, and you to diuine, which of the three perſons hath the three diuers things, doe thus: imagine the three things to be repreſented *A, B, C.* Then ſecondly keepe well

well in your minde which of the perſons you meane to be the firſt, ſecond, and third. Then take 24 counters or ſtones, and your three things, and giue *A* to the partie whom you imagine to be your firſt man, and therewithall giue him one of your 24 counters in his hand, and *B*, vnto your ſecond man, and therewithall 2 Counters. And *C*, vnto your third man, & therewithall 3 counters: and leaue the reſt, which are 18, ſtil among them: which done, ſeparate your ſelfe from them, and afterwards bid them change the things among them as they ſhall thinke good: which done, after they are agreed, bid him that hath ſuch a thing, as before you haue repreſented by *A*, for euery counter that hee hath in his hand, to take vp as many more. And for him that hath *B*, for euery one in his hand to take vp two. And for him that hath *C*, for euery one in his hand to take vp 4, and the reſt of them to leaue ſtill vpon the board. Theſe 3 things and the three perſons being fully printed in your minde, come to the Table, & you ſhall euermore finde one of theſe 6 numbers, 1, 2, 3, 4, 5, 6, or 7. If therefore one remaine ſtill vpon the board, then haue they made no exchange, but keepe them ſtill as they were deliuered vnto them. So that the firſt man hath *A*, the ſecond *B*, and the third *C*. But if 2 remaine, then the firſt man hath *B*, your ſecond man *A*, and your third man *C*. The reſt of the worke and the order thereof are

here apparant by the Table following.

	1	A			1	B
1	2	B		5	2	C
	3	C			3	A
	1	B			1	C
2	2	A		6	2	A
	3	C			3	B
	1	A			1	C
3	2	C		7	2	B
	3	B			3	A

Another diuination of a number vpon the casting of two Dice.

First let the Caster cast both the Dice, and marke well the number: then let him take vp one of them, it maketh no matter which, and looke what number it hath in the bottome, and adde all together: then cast the Dye againe, and keepe in his minde what all together maketh: then let the Dice stand, and bring seuen with you, and thereunto adde the rest of the pits that you see vpon the vpper side of the Dice, and so many did the caster cast in all.

FINIS.

An Appendix concerning the Resolution of the *Square* and *Cube* in numbers, to the finding of their sides

By Ro. Hartwell.

A Figurate number is a number made by the multiplication of one number or more by another. A figurate number, what.

The sides of a figurate number, are the numbers by whose multiplication it is made. The sides of a figurate number, what.

A Figurate number is two-fold, a { *Plaine.* / *Solid.*

And it is { *Of one multiplication,* / *Or consequently of many,* } *as a* { *Plaine.* / *Solid.*

And in each { *Both Equilater.* / *And Inequilater.*

A figurate number made of one multiplication, by two sides or numbers multiplied together, is called a plaine figurate number.

For euery number made by the mutuall multiplication of two numbers, may bee called a *Plaine*, because it bringeth forth a right-angled parallelogramme, according to his vnites disposed in *length* and *breadth*, the *sides* whereof are the two multiplying numbers. As the number 20, made by the mutuall multiplication of 4 and 5 A plaine figurate number.

is

is called a *Plaine*, and the *sides* thereof are 4 and 5 as heere.

Becauſe the vnities thereof diſpoſed in *length* and *breadth*, as the *ſides* doe expreſſe, doe bring forth an *inæquilater Parallelogramme*, for that the numbers, or *ſides* are inequall.

By like reaſon 36 made by multiplication of 6 by 6, is called an *æquilater plaine*, for the *ſides* thereof 6 and 6 are equall.

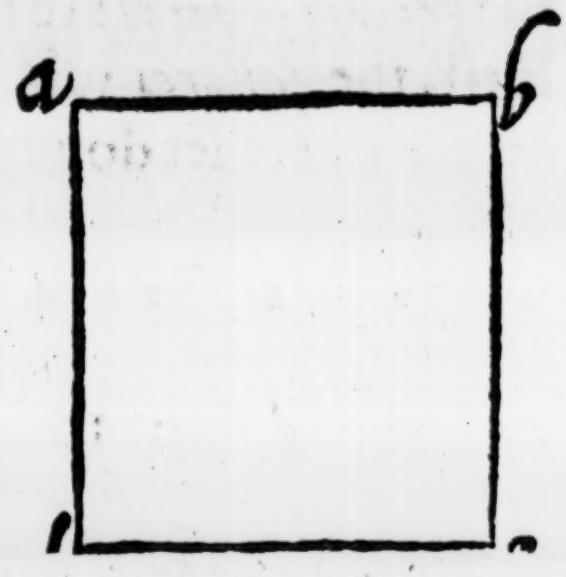
Moreouer, one and the ſame *plaine number* may haue many *ſides*, as the *plaine number* 24, hath ſides 4 and ſixe: 3 and eight; 2 and 12 For it is produced from the mutuall multiplication of theſe numbers: whereupon for the inuention of the ſides, to wit in *inæquilater Plaines*, it is needfull to giue one of the *ſides*, by which the *plaine* it ſelfe diuided, the other *ſide* is made knowne. As the *plaine* 48 being diuided by the *ſide* 8, the *quotient* 6 is the remaining *ſide*. Notwithſtanding another reſolution and inquiſition doth happen in the *ſides* of the *æquilater plaines*.

An æquilater plaine is a number made by two equall ſides, or by any number multiplyed by it ſelfe. It is vulgarly called a ſquare or quadrat; by the Arabians Zenſus, *it is commonly expreſſed by this note* Z; *by vs q.*

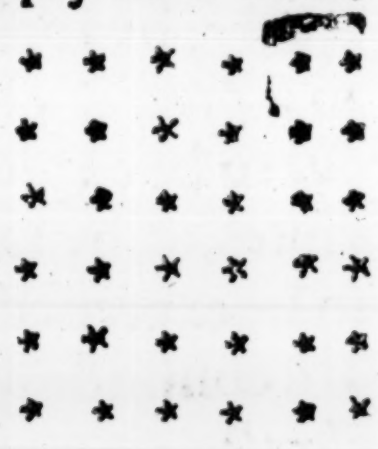
An æquilater Plaine or quadrat what.

A *quadrat* or *ſquare* in *Geometrie* is called a right lined *plaine figure*, made by foure equall right lines, and ſo many right angles, & euery one of the lines is called the *ſide* of the *quadrat*

as this *Figure a b c d*, whose *side* is *a b*, or *b c*, as also *c d*, and *a d*.



To the similitude hereof, that number is called a *Quadrat*, which is made, by the multiplication of two equall numbers, or of one in it selfe of which manner 36 is made, by 6 multiply in it selfe, or by the mutuall multiplication of 6 and 6. For if 36 vnites bee placed in plaine forme, it bringeth forth a perfect *Geometricall Quadrat* hauing in euery side sixe vnities as heere.



The number whereof the Quadrat is produced by multiplication in it selfe, is called the side or root, of the Quadrat.

The side or root of a number what.

Concerning the extraction of the *quadrat or square Roote.*

THerefore to finde the *quadrat root*, or the *side* of any *quadrat number*, is to search a number, which brought or multiplyed in it selfe, maketh the number propounded; concerning the finding wherof as it is requisite that the *sides* (being lesser then 10) of the *squares* vnder an hundred should be gathered by the Table of Multiplication; so the *sides* of

the

the *greater ſquares* are to be ſought out by *Art* Firſt, the *ſquares* whoſe *ſides* are ſimple numbers, are here ſet downe as you ſee.

The Rootes.	1	2	3	4	5	6	7	8	9
The Squares.	1	4	9	16	25	36	49	64	81

The knowledge of a ſquare is by finding out his ſide expreſſed by a whole number.

Although the finding out of the *ſide* of a *ſquare*, be applyed to each number giuen, as to a *ſquare*, yet *ſquare numbers* onely haue a *ſide* to be expreſſed by a certaine number of vnites, or by rationall numbers, the other are to be expreſſed but onely in power. The *ſides* are commonly called *Rootes by a metaphoricall phraſe.*

The roote or ſide of a ſquare is to be found by the Theorem following.

1 If the odde degrees of a *ſquare number* being marked from the right toward the left hand with points, you ſubduct from the number giuen, the particular *ſquare* of the laſt *period*, ſetting the *ſide* thereof alone by it ſelfe.

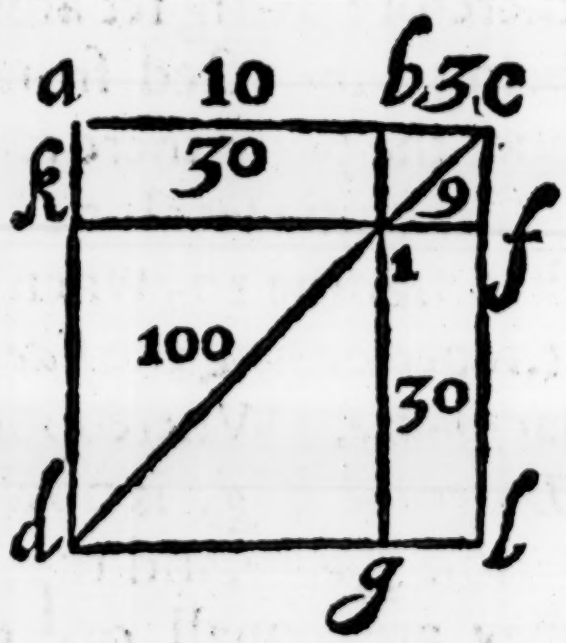
2 Then going on, if you diuide the remainder (if there be any) with the figure going before it by the double of the *ſide* ſet alone by it ſelfe.

3 And multiply the *quotient*, found out (being placed by the *ſide*, which was firſt ſet alone by it ſelfe, and alſo before the doubled number

on

on the right hand) by both the numbers (namely by the double number, & the Figure ſet by it ſelfe) being counted as one *diuiſor*, ſubducting the products from the giuen number, & then renue this laſt worke of diuiſion ſo many times as there are pricks remaining, the *ſide* of the *ſquare* ſhall be found out.

This artificiall deuice is taken out of the 4 *P.* 2 *of* Euclide. *Where by demonſtration it is proued, that if a right line be cut into two* ſegments, *howſoeuer, the* ſquare *of the whole line is equall to the* ſquares *of the* ſegments, *and to the two* right angled figures *made of the* ſegments *as in the figure annexed, the two* diagonals *k g, & b f, are the* ſquares *of the* ſegments *a b*, and *b c*. *Alſo the* complements *b k, and f g, are the* right angled figures *made by multiplying the line a b*, by *b c*.



To extract the ſquare roote.

The ſelfe-ſame parts are to be found in any *ſquare number*. As for *example*, let the *number*, be 169, whoſe *ſide* is 13. This *ſide* being diuided into two pieces 10. and 3. multiplying each peece by it ſelfe once, namely, 10 by 10, and 3 by 3, then multiply one by another, as 10 by 3, and 3 by 10, ſo ſhall you haue foure *plaine numbers*, whereof 2 are *ſquare* as heere you ſee. Therefore

The firſt example.

Therefore as the *ſquare* 169 is made, by adding together of theſe foure *plaine numbers*, ſo by ſubducting them ſeuerally it is reſolued.

10	3
10	3
100	
30	
30	
9	
169	

Firſt therefore I marke each odde place with points, becauſe the *particular ſquares* are to be found in the odde places. Then for ſo much as the vnity ſtanding vnder the firſt point next the left hand, and repreſenting the laſt *period*, is both a *ſquare* and the *ſide* of *a ſquare*: that *figure* therefore being ſet alone in the *quotient*, and being ſubducted from the vnitie ſtanding ouer the point, there remaineth nothing.

This vnitie ſet alone by it ſelfe in the *quotient*, ſhall ſignifie 10, when another *figure* is ſet by it, repreſenting the *ſide* of ſome other particular *ſquare*. Whereupon I ſay, that the greater *Diagonall k g*, is now ſubducted from the whole *ſquare*, and the *ſide* of it *k i*, or *a b*, (for they are equall one to another) and alſo the *ſide* of one of the *complements* is found out.

This is the firſt ſtep to this reſolution.

Moreouer, I double the *figure* found out, becauſe being doubled, it is the ſide of both the *complements* taken iointly together, namely, *k l*, and *g i*. Then ſetting 2 the doubled number vnder

vnder 6, I diuide 6 (which in this place is as much as 60, and repreſenteth both the *complements*) by 2, the *quotient* is 3, repreſenting the other *ſide* remaining of the *complement*, namely *i f*, or *b c*, which number I ſet in the *quotient*, and count it for the *ſegment remaining* of the right line giuen. Wherefore becauſe this number 3 is the ſide of the remaining *Diagonall*, that is to ſay, of the leſſer *ſquare b f*: therefore being ſet by the *diuiſor* on the right hand, and multiplyed by it ſelfe, and alſo by the *diuiſor*, it bringeth forth three *plaine numbers*, namely, the *ſquare b f*, and the two *complements a i*, and *i l*, which being ſubducted from the numbers ſtanding ouer them, there remaineth nothing.

The example is thus.

1 69 (13	*Which is all one, as if you had put down the numbers found out in this manner*	1 6 9
1 2 3		1 0 0 *The greater diagonal.*
3		6 0 *The comple: two-fold.*
69		9 *The leſſer diagonall.*
		169

The ſubtilty of this inuention is illuſtrated by many examples. The ſe- example.

Let the *ſquare* giuen be 1764. This number being marked with two points, telleth vs, that the *ſide* thereof is to be written with two Figures.

First

First therefore beginning at the point on the left hand, I seeke the side of the last period, namely 17. But for so much as it is no *square* number, I take 4 the *side* of the next lesser *square*, which I set alone by it selfe in the *quotient*, and then multiply it by it selfe, the *product* is 16, which being subducted from 17, there resteth 1. Moreouer I double the *side* found out, the product is 8, I place this doubled number vnder 6, and by it I diuide 16 standing aboue it, the *quotient* is 2, which must be set by 4. This *quotient* 2 must bee set before the *diuisor* 8 on the right hand vnder the point, and then must it be multiplied both by it selfe, and into 8, the *product* is 164, which being subducted from the figures standing ouer them, there remaineth nothing: whereby I gather that the number giuen is a iust *square*.

The example standeth thus:

		1			
1	7	6	4		(4 2
1	6	8	2		
			2		
		1	6	4	
1	7	6	4		*The Collection.*

The same manner of working is to bee followed in greater square numbers giuen, sauing that the former part of the worke is to bee vsed

vſed but once, but the latter part is to be followed ſo many times as there are points remaining excepting the laſt.

As in 54756, I ſay, that the *ſide* of the *ſquare* next vnto 5 is 2: therefore 2 being ſet in the *quotient*, & multiplyed by it ſelfe, make 4, and taken from 5, the remainder is 1. Moreouer I double the *quotient*, the product is 4, which I ſet vnder the next *Figure* toward the right hand, and thereby diuide 14. the *quotient* is 3, which 3 being ſet both in the *quotient*, and alſo before the *diuiſor* toward the right hand, I multiply both the numbers by it, the product is 129: this being ſubducted from 147 ſtanding aboue it, the remainder is 18. But becauſe there is yet one point remaining, with which I haue not meddled; I therefore againe double all the whole *quotient*, for in this caſe I muſt take 23 for the ſide of one former *ſquare*, & generally in great numbers, when I light vpon more particular *ſquares* then two, I muſt eſteeme them but as two, and take the *ſides* which are firſt found out, but as the *ſides* of one onely *ſquare*. Therefore twice 23 is 46: by this I diuide 185. then number to be ſet in the *quotient* is 4, which number alſo muſt be ſet before the *diuiſor* on the right hand then muſt 464 bee multiplied by 4: the *product* is 1856: this *product* being ſubtracted from the numbers ſtanding ouer it, there remaineth nothing. The example ſtandeth thus. The third example. Note.

x x 8
5 4 7 5 6 (2 3 4
4 4 3 6 4
1 2 9

4
1 8 3 6
5 4 7 5 6 *The Collection.*

See also the example following.

1 0 9 4 2 8 6 4 (3 3 0 8.

Therefore out of this inuention is this *conſectarie.*

4 Example of a ſurd number.

The number whoſe ſide cannot be expreſſed by whole numbers, is not a ſquare number.

Such are all *prime numbers*, and (the *ſquares* themſelues excepted) all other *compound numbers*. For if in them you deſire to finde out the *ſquare ſide*, you ſhall labour in vaine, becauſe they are not *ſquares*, for to the whole numbers ariſing in the *quotient*, there will be ſome *fraction* adioyned, where by it commeth to paſſe, that the number of the *ſide* is not to bee expreſſed by a true number, and it is commonly called a *ſurd number*.

Notwithſtanding if you adioyne to the *ſide* found out, the number remaining, taking his *denomination* from the double of the *ſide aug-*

mented

mented by an vnitie, you ſhall finde the next *ſide* that may be like to the *ſide of a ſquare.*

As if from 40 you take the neareſt *ſquare*, to wit, 36, the remainer is 4. Here therefore the *ſide* ſought for of the *ſquare*, exceedeth not the *ſide* found out by an vnity, but either by one, or more parts of ſome whole number: wherefore I double 6, the *ſide* found out, and adde an vnity to it being doubled, the totall is 13, this number I ſet vnder 4 the remainer, and ſay that the *ſide* of 40 demanded as neere as may be, is $6\frac{4}{13}$: the *Denominator* of the *Fraction* being added to the greateſt *ſquare* in the number giuen, namely, vnto 36, maketh the next greateſt *ſquare* aboue it, namely 49, whoſe *ſide* is 7. But this *ſurd ſide*, to wit $6\frac{4}{13}$, multiplyed by it ſelfe, maketh $39\frac{133}{169}$, which are not iuſt equall to 40 the giuen number.

Iudge the like concerning the reſt which are not *ſquares.*

Thus much concerning plaine figurate numbers: but eſpecially ſuch as are ſquare numbers.

Concerning ſolid figurate Numbers.

A Solid figurate Number is made of two multiplications by three numbers, or ſides, multiplied together, admitting length, breadth, and thickneſſe.

Therefore euery *number* made by the

A ſolid figurate number.

mutual multiplication of three numbers, may be called a *ſolid*, becauſe it bringeth forth *a right angled Parallelipipedon*, diſpoſed according to his vnites in *length*, *breadth*, and *thickneſſe*, the *ſides* whereof are the three multiplying *numbers*. As the *number* 30 made by the mutuall multiplication of 2, 3, and 5, is called an *inequilater ſolid number*, and the *ſides* thereof are 2, 3, and 5, becauſe the vnities thereof diſpoſed by a certaine diſtance one from another, in *length*, *breadth*, and *depth*: as the *ſides* doe expreſſe, doe bring forth in reſemblance an *inequilater Parallelipipedon*, for that the *numbers* or *ſides* are inequall.

By like reaſon 2 1 6 made by multiplication of 6 by 6, and the product thereof by 6, is called an *Equilater ſolid*, for the *ſides* thereof 6 6 and 6 are equall.

An Equilater Solid or Cube.

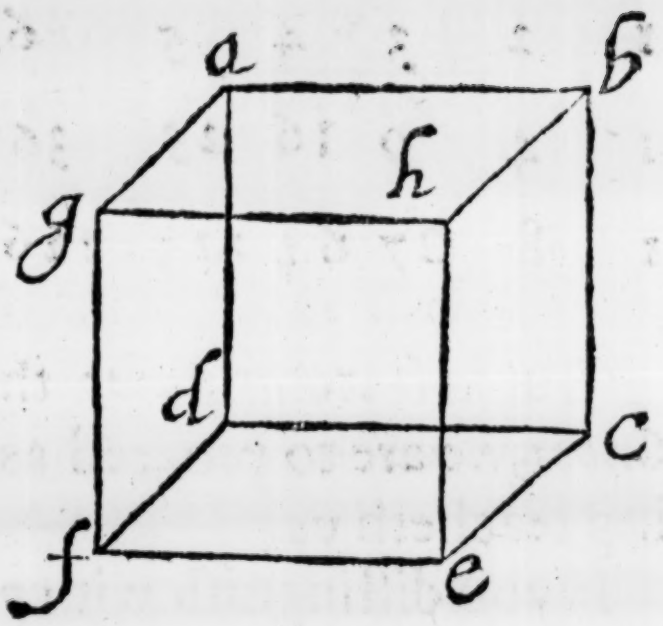
An Equilater, is a number made by three equall ſides, or by any number multiplyed by it ſelf, and that product againe by the foreſaid number. And it is called an Equilater and Equiangled Parallelipipedon or Cube, and is commonly repreſented by this note ☐, *but by vs thus* C.

A Cube in Geometry is a right angled *Parallelipipedon*, having ſix equall *ſurfaces*, and 8

A

ſolid

olid angles, & 12 *ſides*, as this figure *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, whoſe *ſide is a b*, or *a d*, alſo *b c*, or *c d*, either *c e*, or *e f*, likewiſe *e h*, or *h g*, alſo *g f*, or *d f*, or *d a*, and *g a*.



The number where of the Cube is produced by multiplication in it ſelfe twice, is called the ſide or roote of the Cube, which being found out in whole numbers, the Cube is knowne.

The ſide or roote of a Cube.

Concerning the extraction of the *Cubicke roote*.

THerefore euery *Cube* in numbers hath ſuch a *ſide* as may be expreſſed in whole numbers, but in magnitudes it is not alwayes ſo, as indeed in *magnitudes* there are many things not to be expreſſed in *whole number*. Now for as much as the *ſide* of any *Cube* vnder 1000, is a ſimple figure, it is neceſſarie, before we vndertake to finde out the *ſide* of any great number, to know what *Cube* is made of each *ſimple figure*, and what is the *ſide* of any *Cube* leſſer then 1000, as I haue here ſet them downe.

Rootes.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	226	343	512	729

But in ſearching out the *ſides* of greater *Cubes*, we are to proceed as the *theorem* following teacheth vs.

If you diſtinguiſh with *points* as it were into periods, the giuen *Cube*, beginning at the firſt figure on the right hand, and omitting each two figures continually, and firſt of all ſubduct the *particular Cube*, of the laſt *period* from the number giuen, ſetting the *ſide* thereof in the *quotient*: aud then ſet triple of the *quotient* vnder the *figure* next following the former *point* on the right hand, and the *ſquare* of the *quotient* being tripled beneth it one *degree* more toward the left hand: and afterward diuide the number aboue written by the triple of the *ſquare*, ſetting the *quotient* by it ſelfe, and then multiply the *diuiſor* by the *quotient* found out, and the tripled *ſquare* by the *ſquare* of the *quotient*, and the *quotient cubically*, ſubducting the *products* (ſo orderly added together, that each *figure* may anſwere the *numbers* whereof it was multiplied) from the *number* giuen, and renue this laſt manner of diuiſion ſo many times as there are *points* remaining, the *ſide* of the *Cube* ſhall be found out.

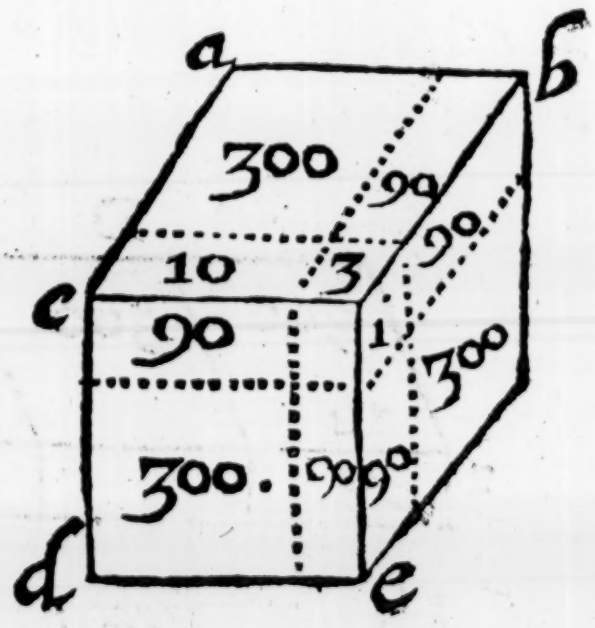
This

This artificiall deuice is drawne out of that *theorem* which *Ramus* made, imitating that of *Euclide* concerning *ſquare numbers* in this manner.

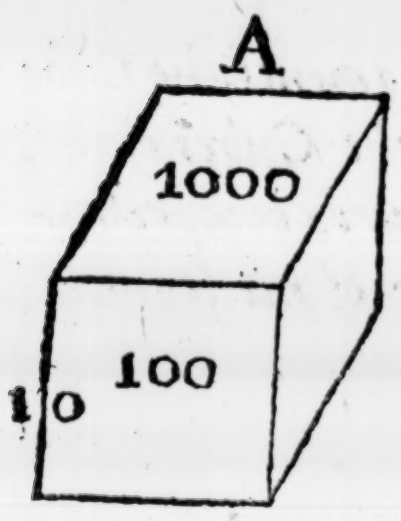
If a right line be cut into two ſegments, the Cube of the whole line ſhall bee equall to the Cubes of the ſegments, and to the two ſolid figures comprehended three times vnder the ſquare of his ſegment, and the ſegment remaining.

The extraction of the cubicke ſide or roote.

As the line *c i*, which is 13, is cut into two *ſegments*, 10 *and* 3 therefor the *Cube* of the whole line, namely 2197, is equall vnto the *Cubes* of the *Segments*, namly vnto 1000, and 27. alſo to the *two-fold ſolids* or *Paralleli-pipedons* thrice taken, whereof three haue like *ſoliditie*, the *ſoliditie* of each of the three leſſer is 90, being made of the *ſquare of the ſegment* 3, that is to ſay of 9, multiplied by the other *ſegment* 10. Theſe three *Parallelipipedons*, iointly taken together, make 270. But of the three greater *Parallelipipedons* each containeth 300, being made of 100, the *ſquare of* the greater *ſegment* 10, multiplyed by the *leſſer ſegment* 3.

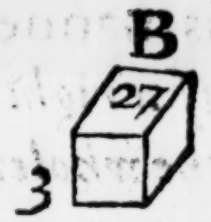


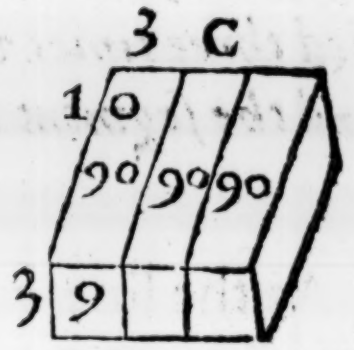
and they beeing taken ioyntly together, make 900.



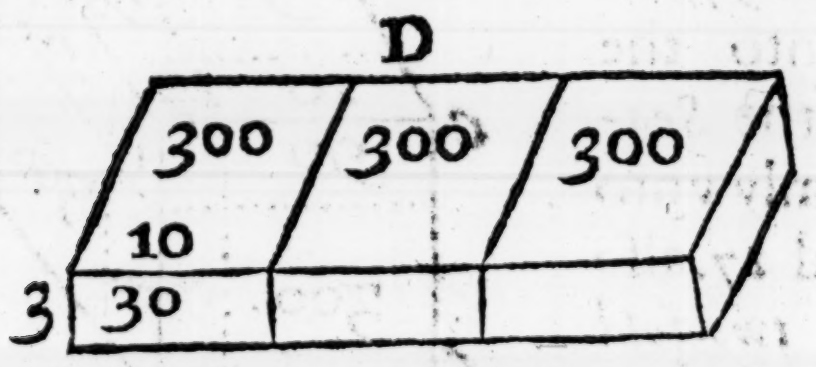
The Cube of the greater ſegment 10.

The Cube of the leſſer ſegment 3.





The 3 leſſer Parallelipipedon,



The three greater Parallelipipedons.

The *Cube* therefore hath eight *particular ſolids in number*, which are made of the parts of the number giuen, namely of 10 and 3, in this maner. Firſt let there be foure *plaine numbers* made, each part being multipled by it ſelfe; and one by another,

10	3
10	3
	9
	30
	30
	100

If againe I multiply the *Plaines* by the ſame parts, there will arile 8 *ſolids*, as you ſee heere.

9	9	
30	30	
30	30	
100	100	
	10	
3		
27	90	*All theſe being added together, are equall to the* Cube *of the whole, to wit,* 2197.
90	300	
90	300	
300	1000	

Therefore the ſame way that is kept in making the Cube, *is alſo to bee followed in reſoluing the* Cube.

The firſt example to ex[illegible] the cubick Roote.

As for example, I marke the *Cube* giuen with points in this manner, 2 1 9 7

Then I ſubduct the particular *Cube* of the number ſet vnder the laſt point: but for ſo much as that number is no *Cube*, I take the neereſt to it, namely, an vnity, which alſo I ſet in the *quotient*. This vnity in the number giuen, is 1000, but in the *quotient* it is but 10, the vnitie ſubducted from 2, the remainder is 1, which muſt bee written ouer the number giuen

giuen. So that the greater *Cube A*, is to be ſuppoſed to bee ſubducted from the number giuen.

This is the firſt ſtep of this worke.

1
2 1 9 7 (1
x

After I triple the *quotient* found out (that is to ſay, I multiply it by 3) this triple repreſenteth the three *ſides* (ioyntly taken together) of the three leſſer *ſolids* marked with *C*, I place the tripled number vnder 9. Againe, I multiply the *quotient* ſquare wiſe, & triple the *product*, which maketh likewiſe 3. This *product* reſembleth the three *ſquare ſides* (taken ioyntly together) of the three *greater ſolids*, marked with *D*, I place the *product* one degree lower toward the left hand vnderneath 1. With it I diuide 11, which is written aboue it, the *quotient* is 3. This *ſegment* or *quotient* 3, being multiplyed by 3 the *Diuiſor*, maketh 9, which in reſpect of the place wherein it ſtandeth, is 900, and repreſenteth the three greater *ſolids*, marked with *D*, taken ioyntly together. Furthermore the ſame *quotient* being multiplyed ſquarewiſe, maketh 9, and multiplyed afterward by the tripled *number* ſtanding vnder 9, it maketh 27, which in reſpect of the place wherein it ſtandeth, is 270, & repreſenteth the three *leſſer ſolids* marked with *C*. Laſt of all, the ſame *quotient* multiplied

plyed *cubically*, breedeth the *leſſer Cube B*. Theſe three *products* therefore being added together, and the totall ſubducted from the *numbers* ſtanding ouer it, there remaineth nothing, which importeth the giuen number is a *Cube*.

The example is as you ſee.

1	2197 (13
2197 (13	1000 *The greater Cube.*
1 3	3
3	3
9 Or thus:	900 *The 3 greater Parellelipipedome.*
27	270 *The leſſer Parallelipipe.*
27	27 *The leſſer Cube.*
2197	2197

The matter may be explained by many examples.

Let the ſide of the giuen *Cube* 16387064, be ſought out, contriue it therefore (as it were) into certaine periods with points. Then firſt of al, ſearch out the *ſide* of the *Cube* next to the left hand: But for as much as 16 is no *Cube*, take 2 the *ſide* of the next *Cube* vnder it, that is to ſay, of 8, and ſet it in the *quotient*, and ſubduct 8 the *Cube* thereof from 16, there remaineth 8. This firſt worke is not to bee renued throughout the whole *number*, but the *rules* following muſt bee repeated as often as there are *points* remaining. The ſecond example of the Cubick root.

The

The first step to finde out the roote, is in this manner:

8
1 6 3 8 7 0 6 4 (2
8

Moreouer triple the *quotient* now found out, and the *product* is 6, which is to be placed vnder 8, namely, vnder the *figure* following the next *pricke* toward the right hand. Then multiply the *quotient* by this tripled *number* (or which is all to one purpose, square the *quotient* and then triple the *product*) it maketh 12, set that number in a lower place one degree neerer the left hand, and make it the *diuisor*: diuide 83 by 12 obseruing this *rule* in choosing your *quotient*, that it be no greater, then that the *numbers* afterward produced by multiplication may not exceed the *numbers* standing ouer it. So that heere you shall take 1 in 8, but 5 times. Afterward by this number 5, multiply the *diuisor* 12, and by the *square* of 5, multiply the tripled *number* 6, and last of all multiply 5 *cubically*: so shall you produce three *numbers*, namely, 60, 150, 125, to be described in such sort as you see. These *numbers* added together, and subducted from 8387, the remainer is 762.

The second step to finde out the root, is in this manner.

```
      8 7 6 2
    1 6 3 8 7 0 6 4 (25
    ---------------
          6
        1 2
      -----
        6 0
      1 5 0
        1 2 5
      -------
      7 6 2 5
```

And because there is yet one point remaining, this last manner of Diuision must be wrought againe.

First therfore I triple the *quotient*, the *product* is 75. which must be so placed, that the *first figure* thereof, namely 5, may stand vnder 6, the *second* vnder the 0. Againe, multiply the *quotient* by this *tripled number* (or which is all one, *square the quotient*, and triple the *product*) it maketh 1875, which must be the *Diuisor*, whose first *figure* namely 5, must be placed vnder 7, the *last figure* of the *tripled number*. Then see that 12 may be contained in 7, many times, but I can make it but 4 times, I set 4 in the *quotient* and multiply the *Diuisor*, by 4, the *product* is 7500. afterward I *square* 4, it maketh 16, which I multiply by the *tripled number* 75, the *product* is 1200. Last of all I multiply 4 *cubically*, it maketh 64, these *products* added all together,

ther, make 762064, which *number* being ſubducted from the *Cube* giuen, there remaineth nothing, whereby I gather that the number giuen is exactly *cubicall.*

The third ſtep to finde out the ſide is in this manner.

```
      7 6 2
   .     .     .
1 6 3 8 7 0 6 4 ( 2 5 4
-----------------------
        7 5
    1 8 7 5
-----------------------
      7 5 0 0
        1 2 0 0
            6 4
-----------------------
      7 6 2 0 6 4
```

Behold alſo the example following.

The third example of the cubick root.

```
  .     .     .     .     .     .
6 1 4 1 2 5 0 0 0 ( 1 8 5 0
```

¶ Another manner of working.

HItherto the Princely high way to find out the ſide of the Cube *hath beene declared.*

But there are moreouer certaine other waies alſo bending thereto, and leaning to the ſame principles, whereof this is one.

The ſecond forme.

Hauing found out in the Table of ſimple *Cubes*, the firſt figure repreſenting the ſide of the *Cube* contained, in the number ſtāding vnder the firſt *point* on the left hand, ſet it in the

quotient

quotient, & ſubduct the particular *Cube* of that figure as you did before: then ſquare that *figure*, and triple that *ſquare* the *product* ſhall be the *Diuiſor*, the firſt *figure* whereof ſhall bee ſet vnder that *figure* which is on the right hand next of all to the *point* (now examined) before going.

2 See how many times the *Diuiſor* is contained in the number written ouer it, and multiply the *Diuiſor* in the *quotient*, and ſubduct the *product* from the *dividend*: yet here you muſt take heed, that you take not a greater *quotient*, then that the *products* made afterward thereby may be ſubducted from the *number* giuen.

3 The ſubduction being done, triple the firſt *figure* which was ſet in the *quotient*, and adde to the triple the laſt *number* which was ſet in the *quotient* on the right hand of the *product*.

4 This totall multiplied by the *ſquare* of the *figure* laſt found out, and ſet downe the *product* ſo, that the firſt *figure* thereof toward the right hand may ſtand vnder the *point* next before going on the ſame hand, and finally ſubduct the ſame from the *number* giuen.

The fourth example of the cubick root.

As in 804357. The particular *Cube*, namely, 729 being taken from the *number* ſtanding vnder the laſt *period* vpon the left hand, there remaineth 75357, the ſide of that particular *Cube* being 9, I ſet in the *quotient*. Then I *ſquare* that *ſide*, it maketh 81, and triple the *ſquare*, the *product* 243 is my *diuiſor*, which I ſet vnder the giuen *number*, ſo that 3 may ſtand vnder 3,

with

with this *diuisor*, diuide the *number* ſtanding ouer it, you ſhall finde 2 to be conteined in 7 three times. Therefore I ſet 3 in the *quotient* & multiply the *diuiſor* by it, the *product* is 729, which being ſubducted from 753, the remainder 24.

The induction is thus:

2
7 5 4
8 0 4 3 5 7 9.8
2 4 3
7 2 9

Moreouer I triple 9, the *product* is 27, by which on the right hand I ſet 3, the *quotient* laſt found out, the totall is 273.

This *number* I multiply by 9 the *ſquare* of the *quotient* laſt found out, the *product* ſhall be 2457, which being ſubducted from the ſuperiour *number*, there remaineth nothing.

The induction is thus:

The fourth example diuerſely.

2 4
8 0 4 3 5 7 9.3
2 7 3
9
2 4 5 7

Another

Another manner.

THe ſelfe-ſame worke may be diſpatched another way a little differing from the former in this manner.

The *figure* in the *quotient* being found out by ſubducting the particular *Cube*, and alſo the ſecond *figure* in the *quotient* being found by diuiſion, let the totall *quotient* be tripled, and let the tripled *number* bee multiplied by the former *figure* in the *quotient*. Then let the *product* be multiplyed againe, be the latter *figure* found out, and let a *cipher* be ſet on the right hand of that *product*. Laſt of all, let the *Cube* of the latter *figure* found out, be added to this *product*, and let the totall ſumme be ſubducted from the *number* giuen. As in 373248. the third forme.

The firſt induction in is this manner.

30

373248 (7

343

Moreouer I *ſquare* the *ſide* found out, it maketh 49, and triple the *ſquare*, the *product* is 147, which ſhall be the *diuiſor*, by this I diuide 302, the number written ouer it, the *quotient* is 2. Now I triple the totall *quotient* 72, it maketh 216, and multiply this triple by 7, the former *figure* in the *quotient*, the *product* is 1512. I multiply this *product* alſo by 2, the latter *figure* of the *quotient*, and ſet a *cypher* on the right hand of it, ſo as it maketh 30240, vnto this number laſt of all I adde 8, the *Cube* The firſt example.

of the latter *figure* found out, the totall is 30248, which being ſubducted from the *figure* aboue it, there remaineth nothing.

The ſecond induction is thus :

30
3 7 3 2 4 8 (72
1 4 7
30248

¶ All the *points* of the *number* giuen being examined, if any thirg remaine, it ſignifieth the *number* giuen is no *Cube* : wherefore the true *ſide* of it cannot be exactly giuen in *numbers*. Yet if it pleaſe you to ſift out the neereſt *ſide* that may be, by the firſt kinde of reduction of *mixt numbers*, you ſhall reduce the *number* giuen vnto a *cubicall* fraction of a greater *denomination*, and afterward ſeeke out the *cubicall ſide* of that *fraction*.

To finde the neereſt Cubicke roote in a ſurd number.

For example ſake, becauſe 120 is no *Cube*, therefore let it be reduced into ſixtie *cubicall* parts, after this manner. Multiply 60 *cubically* in it ſelfe, it maketh 216000, by this being taken for the *denominator* of the *fraction*, multiply 120 the *number* giuen, the *product* is 25920000 whoſe *cubicall ſide* is $\frac{295}{60}$ that is $4\frac{11}{12}$ the neereſt to the true *ſide* that can be.

For the extraction of all ſorts of roots, the table of Logarithmes ſet forth by Mr. *Briggs are moſt excellent, and ready.*

FINIS.

A Table of Board and Timber measure, more perfect then euer hath been made; shewing also the Squares, between 4 and 37 from quarter to quarter, Calculated by *Ro. Hartwell.*

Boarde measure.	Inches and quarters.	Squares.	Timber measure.	Boarde measure.	Inches and quarters.	Squares.	Timber measure.
36. 0. 0	4	16	108. 0. 0	16. 0 0	9	81	21. 3. 3
33. 8. 8	1	18	96. 0. 0	15. 5. 6	1	85	20. 3. 3
32. 0. 0	2	20	86. 4. 0	15. 1. 6	2	90	19. 2. 0
30. 3. 1	3	22	78 5. 4	14. 7. 7	3	95	18 1. 8
28. 8. 0	5	25	69. 1. 2	14. 4. 0	10	100	17. 2. 8
27. 4. 3	1	27	64. 0. 0	14. 0. 5	1	105	16. 4. 6
26. 1. 8	2	30	57. 6. 0	13. 7. 1	2	110	5. 7. 1
25. 0. 4	3	33	52. 3. 6	13. 3. 9	3	115	15. 0. 2
24. 0. 0	6	36	48. 0. 0	13. 0. 9	11	121	14. 2. 8
23. 0. 4	1	39	44. 3. 0	12. 8. 0	1	126	13. 7. 1
22 1. 5	2	42	41. 1. 4	12. 5. 2	2	132	13. 0. 9
21. 3 3	3	45	38. 4 1	12 2 7	3	138	12. 5. 2
20. 5. 7	7	49	35. 2 6	12. 0. 0	12	144	12. 0. 0
19. 8. 6	1	52	33. 2. 3	11. 7. 5	1	150	11. 5. 1
19. 2. 0	2	56	30. 8. 6	11. 5. 2	2	156	11. 0. 7
18. 5. 8	3	60	28. 8. 0	11. 2. 9	3	162	10. 6. 6
18. 0. 0	8	64	27. 0. 0	11. 0. 7	13	169	10 2. 2
17. 4. 6	1	68	25. 4. 1	10 8 7	1	175	9 8. 7
16. 9. 4	2	72	24. 0, 0	10 6 7	2	182	9. 4. 9
16. 4 6	3	76	22. 7. 5	10 4 7	3	189	9. 1. 4

Boarde measure.	Inches and quarters.	Squares.	Timber measure.	Boarde measure	Inches and quarters.	Squares.	Timber measure
10. 2. 8	14	169	8.8.1	6.8.6	21	441	3.9.2
10. 1. 1	1	203	8.5.1	6.7.7	1	451	3.8.2
9. 9. 3	2	210	8.2.3	6.6.9	2	462	3.7.3
9. 7. 6	3	217	7.96	66.1	3	473	3.6.5
9. 6. 0	15	225	7.6.8	6.5.4	22	484	3.5.7
9. 4. 5	1	232	7.4.3	6.4 7	1	495	3.4.9
9. 2. 9	2	240	7.2.0	6.4.0	2	506	3.4.1
9. 1. 4	3	248	6.9.7	6.3.3	3	517	3.3.3
9. 0. 0	16	256	6.7.5	6.2 6	23	529	3.2.7
8. 8. 6	1	267	6.5.4	6.1.9	1	540	3.2.0
8. 7. 3	2	272	6.3.5	6.1.2	2	552	3.1.3
8. 6. 0	3	280	6.1.6	6 0.6	3	564	3.0.6
8. 4. 7	17	289	5.9.8	6 0.0	24	576	3.0 0
8. 3. 5	1	297	5.8.1	5.9.4	1	588	2.9.4
8. 2. 3	9	306	5.6.4	5.8.8	2	600	2.8.8
8. 1. 1	3	315	5.4.8	58.2	3	612	2.8.2
8. 0. 0	18	324	5.3.3	5.7.6	25	625	2.7.6
7. 8. 9	1	333	5 1.8	5.7.0	1	637	2.7.1
7. 7. 8	2	342	5.0 5	5.6.5	2	650	2.6.5
7. 6. 8	3	351	4.9.2	5.5 8	3	862	2.6.6
7. 5. 8	19	361	4 7.8	5.5.4	26	676	2.5.5
7. 4. 8	1	370	4 6.7	5.4 8	1	689	2.5.0
7. 3. 9	2	380	4.5.5	5.4.2	2	702	2 4.7
7. 2. 9	3	390	4.4 3	5. 3.8	3	715	2.4 1
7. 2. 0	20	400	4.3.2	5.3.3	27	729	2.3.7
7. 1. 1	1	410	4.2.1	5.2.8	1	742	2.3.2
7. 0. 2	2	420	4 1.1	5.2.3	2	756	2.2 7
6. 9. 3	3	430	4.0.1	5.1.8	3	767	2.2.4

Boarde measu.	Inches and quarters.	Squares.	Timber measu.	Boarde measu.	Inches and quarters.	Squares.	Timber measu.
5.1.4	28	784	2.2.0	4.3.6	33	1089	1.5.9
5.0.9	1	798	2.1.6	4.3.3	1	1104	1.5.6
5.0.5	2	812	2.1.2	4.3.0	2	1120	1.5.4
5.0.0	3	826	2.0.9	4.2.7	3	1136	1.5.2
4.9.6	29	841	2.0.5	4.2.3	34	1156	1.4.9
4.9.2	1	855	2.0.2	4.2.0	1	1174	1.4.7
4.8.8	2	871	1.9.8	4.1.7	2	1190	1.4.5
4.8.4	3	885	1.9.5	4.1.4	3	1210	1.4.3
4.8.0	30	900	1.9.2	4.1.1	35	1225	1.4.1
4.7.6	1	915	1.8.9	4.0.8	1	1237	1.4.0
4.7.2	2	930	1.8.6	4.0.5	2	1247	1.3.7
4.6.8	3	945	1.8.3	4.0.3	3	1280	1.3.5
4.6.4	31	961	1.7.9	4.0.0	36	1296	1.3.3
4.6.1	1	974	1.7.7	3.9.7	1	1313	1.3.1
4.5.7	2	987	1.7.5	3.9.4	2	1331	1.2.9
4.5.3	3	1000	1.7.2	3.9.1	3	1350	1.2.8
4.5.0	32	1024	1.6.9	3.8.9	36	1369	1.2.6
4.4.6	1	1040	1.6.6	3.8.7	1	1388	1.2.4
4.4.2	2	1056	1.6.3	3.8.4	2	1416	1.2.2
4.4.0	3	1072	1.6.1	3.8.2	3	1425	1.2.0

The vſe of this former Table.

IF vpon a *Scale or Ruler* you diuide one *inch* into 10 *equall parts or primes*, and againe by *diagonals* and *paralell lines*, you ſubdiuide each of them into 10 *equall parts or ſeconds*, with your *compaſſes*, you may take of more exact running meaſure for *board and timber*, then by any other meanes whatſoeuer, and ſo place the ſame, or this *Table* if you will, vpon any *ruler*.

Alſo by meanes of the *columns of ſquares*, your may readily find a *ſquare* equall to any *paralelipipedon* or piece of *timber* which is thicker then it is broad. As for example, ſuppoſe a piece of *timber* to bee ten *inches thicke*, and 29 *inches broad*: if I multiply thoſe *ſides* one by another, they will produce 290, then ſeeking the *columne of ſquares* for 290, which I finde not; but I finde 280 the neereſt *number* to 290, to ſtand againſt 17: therefore I ſay 17 *inches fere*, will make *a ſquare* equall to ſuch an vnlike *ſquared piece*: then looking in the *columne of Timber meaſure* againſt 17, you ſhall finde that 5 *inches*, 9 *prime*, or $\frac{9}{10}$, and 8 *ſeconds*, or $\frac{8}{100}$ of an *inch in length* of that piece will make a *foote of timber*.

Likewiſe for *broad meaſure*, you may finde how

how much in *length* of any *breadth* of *board* must be in one *foote*.

By the like meanes suppose *for example* that a *board*, appointed to be measured, is 15 *inches* $\frac{3}{4}$ *broad*, if I desire to know how much in *length* thereof will make a *foote*. I seeke in the *columnes* that stand vnder *vnites and quarters*, for 15 $\frac{1}{4}$, and also against the same in the *columne* vnder the title of *board measure*, where I find 9 *inches* 1 *prime or tenth* of an *inch*, and 4 *seconds* or *hundreths* of an *inch* will make a *foote* at that *breadth*: the like may be practised for any other *breadth of board* whatsoeuer.

Certaine Tables ſhewing the intereſt of any *ſumme of mony* whatſoeuer vnto 40 yeers; How much *Annuities* reſpited or forborne commeth vnto. And for *buying or ſelling* of *Annuities* for the ſaid time; and alſo the ſame in *reuerſion* after any *number* of *yeeres* vnto 30. what they may be worth in preſent ready money, by *R.C.* and now diligently corrected and amended by *Robert Hartwell.*

Definition of Intereſt.

PRincipall, *is the ſumme from which the* Intereſt *is reckoned.*

2 Intereſt *is the ſumme reckoned for the leading or forbearance of the* Principall *for any termes or time.*

3 Intereſt ſimple, *is that which is counted from the* Principall *onely.*

4 Intereſt compound, *is that which is counted for the* Principall, *together with the* Arerage

5 Intereſt profitable, *is that which is added to the* Principall.

6 Intereſt damagable, *is that which is to be ſubtracted from the* Principall.

The vſe per annum of { 1 li. / 10 ſ. / 5 ſ. / 2 ſ. 6 d. / 12 d. } is { 2 ſ. / 12 d. / 6 d. / 3 d. / 1 d $\frac{1}{2}$ of a penny. }

A Table ſhewing what 1 l. *with intereſt, and intereſt vpon intereſt after* 10 *in the* 100. *comes to euery yeere vnder* 41 *yeeres. As followeth.*

yeeres.	l	s	d		l	s	d	yeeres.
1	1	2	0		7	8	0	21
2	1	4	2		8	2	9	22
3	1	6	7		8	19	1	23
4	1	9	3		9	16	11	24
5	1	12	2		10	16	8	25
6	1	15	5		11	18	4	26
7	1	18	11		13	2	2	27
8	2	2	10		14	8	5	28
9	2	7	1		15	17	3	29
10	2	11	10		17	8	11	30
11	2	17	1		19	3	10	31
12	3	2	9		21	2	3	32
13	3	9	0		23	4	6	33
14	3	15	11		25	10	11	34
15	4	3	6		28	2	0	35
16	4	11	10		30	18	3	36
17	5	1	1		34	0	0	37
18	5	11	2		37	8	1	38
19	6	2	3		41	2	10	39
20	6	14	6		45	5	2	40

By the former Table, if you deſire to know what 1 li. commeth to with *intreſt*, and *intreſt* vpon *intreſt* after 10 in the 100. for any *number* of yeares vnto 40. Looke in the *rowe or margent* (ouer which is written *yeares*) and againſt it on the right hand cloſe vnto it in the the *rowe or Margent of pounds, ſhillings, and pence*, (which is titled thus, li.ſ.d. you ſhall find your deſire.

Example.

I would know what 1 li. *with intreſt, and intreſt vpon intreſt commeth to in* 7 *yeares?*

I looke in the rowe of *yeeres* for the *number* 7. And againſt it on the right hand I finde 1.li. 18.ſ. 11.d. Alſo what it commeth vnto in 13. yeares. I ſeeke among the yeares for 13. and againſt it I finde 3.li.— 9.ſ. Againe for 21. yeares, I looke for 21. among the yeares and I finde 7.li. 8.ſ. 0.d. But if you would know for a greater ſumme then 1.li. Then multiply your ſumme by that ſumme of 1.li. in the Table for any of thoſe yeares, and you ſhall eaſily finde it. As thus, I would know what 10.li. commeth to for 7. yeares with intreſt, &c. I ſee that 1.li. commeth to 1.l. 18.ſ. 11.d. in that time. Then ſay I that 10.li. muſt be 10. times as much in that ſpace, which is 19.li. 9.ſ. 2.d. Alſo of 10.li. in 13. yeares: I ſee that 1.li. in that time commeth vnto 3.li. 9.ſ. Then muſt 10.li. be 10. times as much in that ſpace, which is 34.li. 10.ſ. Alſo what 10.li.

commeth

commeth to in 21. yeares. I finde firſt that 1. li. in that ſpace commeth to in 7.li.8.ſ. Then I ſay 10. muſt be 10. times as much, which is 74.li. Laſtly, I would know what 100 li. commeth to in 7. yeares, I ſee it muſt bee 100. times as much as 1. li. commeth to in that ſpace, which is 194.li.11.ſ.8.d. Hereby you ſee the common ſaying is not true, that 100.li. doth double it ſelfe in 7. yeares, for it wants thereof 5.li.8.ſ.4 d. But in 8. yeares 100.li. commeth to 210.li.8.ſ.4.d. Which you ſee is more then double it ſelf by 10.li.8.ſ.4.d. And in this ſort may any that can but caſt with Counters, or indeede by memory finde the encreaſe of any ſumme whatſoeuer for any of the number of yeares in the foreſaid Table, after they haue found what 1. li. commeth vnto for that time, as before is ſpecified.

A Table ſhewing if 1 l. *Annuity to endure for any number of yeere, vnder* 41. *be all reſpited or forborne, vntill the laſt payment grow due, and then all bee receiued together: with intereſt, and intereſt vpon intereſt after* 10 *in the* 100 per annum, *what they will amount vnto by any of the ſaid number of yeeres. As followeth.* Viz.

yeeres.	l	s	d		l	s	d	yeeres.
1	1	0	0		64	0	0	21
2	2	2	0		71	8	0	22
3	3	6	2		79	10	10	23
4	4	12	10		88	9	11	24
5	6	2	1		98	6	11	25
6	7	14	3		109	3	7	26
7	9	9	8		121	1	11	27
8	11	8	8		134	4	2	28
9	13	11	7		148	12	7	29
10	15	18	8		164	9	10	30
11	18	10	7		181	18	10	31
12	21	7	8		201	2	9	32
13	24	10	5		222	5	0	33
14	27	19	5		245	9	6	34
15	31	15	9		271	0	5	35
16	35	18	11		299	2	6	36
17	40	10	10		330	0	9	37
18	45	11	11		364	0	10	38
19	51	3	2		401	8	11	39
20	57	0	6		442	11	10	40

By this Table, you may know what any Annuitie being reſpited or forborne for any number of yeares vnto 41. with intreſt vpon int after 10. in the 100. will come vnto: firſt ſeeking in the Table what 1.li. will come vnto in that time, and that being found to multiply it by the ſumme you deſire to know.

Example.

Firſt, I would know what 1.li. *Annuity, being forborne or reſpited for* 14. *yeares commeth vnto.*

I looke in this laſt Table (which is for purpoſe) and I find 27.li.19.ſ.5.d.

Againe, what 1.li. *Annuitie* reſpited for 21. *yeares* commeth to. I looke in the ſaid Table for 21 *yeares*, and I finde 64.li. Alſo the like for 1.li. for 30. *yeares* reſpited. I looke, and I finde it to be 164.li.9.ſ.10.d. as by the ſaid Table may appeare. Now for greater *Annuities*, as 30.li. *per annum* reſpited or forborne, what it amounteth to in 16. *yeares*. I ſeeke firſt for 1.li. in this laſt *Table* before for 16. *yeares*, and againſt it I finde 35.li.18.ſ.11.d. Then ſay I, that 30.li. *per annum* being reſpited for that time, will come to 30. times as much, which is 1078.li.7.ſ.6.d. Alſo if there be an *Annuitie* of 45. li. due and vnpaid for 12. *yeares*. I looke in the ſaid Table what 1.li. commeth to 12. yeares being reſpited, and I finde it is 21.li.7.ſ.8.d. Then I conclude that

5.li.

5.li. muſt bee 45. times as much, which is 962.li.5.ſ.

Laſtly, I haue an Annuitie of 50.li. *per annum, which hath beene behinde for* 16. *yeares, and muſt be anſwered vnto me with intreſt, and intreſt vpon intreſt, all at one payment, what ſhall or ought I to receiue in all at the* 16. *yeares end?*

I ſeeke what 1.li comes vnto in that time (as before taught) & I find 35.li.18.ſ.11.d. Then muſt my 50. li. *per Annum* forborne for that time, come to 50. times as much, which is 1797.li.6.ſ.10.d. And thus may you find any other *ſumme* great or ſmall, for any *number* of *years* contained in the foreſaid *Table*, without the helpe of *Arithmetick*, if you can but vſe your *Counters*, or by memorie count well.

A Table ſhewing if 1 l. *Annuity (to endure for any number of yeeres vnto* 41*) bee to bee ſold for preſent ready money. How much ought that ready money to be, reckoning* 10. per 100. per annum *abating intereſt, and intereſt vpon intereſt. As followeth.*

yeeres.	l	s	d		l	s	d	yeeres
1	0	18	2		8	12	11	21
2	1	14	8		8	15	5	22
3	2	9	8		8	17	7	23
4	3	3	4		8	19	8	24
5	3	15	9		9	1	6	25
6	4	7	1		9	3	2	26
7	4	18	4		9	4	8	27
8	5	6	8		9	6	1	28
9	5	15	2		9	7	4	29
10	6	2	10		9	8	6	30
11	6	9	9		9	9	7	31
12	6	16	3		9	10	6	32
13	7	2	0		9	11	4	33
14	7	7	4		9	12	2	34
15	7	12	1		9	12	10	35
16	7	16	5		9	13	6	36
17	8	0	5		9	14	1	37
18	8	4	0		9	14	7	38
19	8	7	3		9	15	1	39
20	8	10	3		9	15	6	40

This Table before laſt ſpecified is very ne ceſſarie and commodious for all *Gentlemen* or others, that ſhall haue cauſe to buy or ſell *Annuities* or ſuch like: for by this they ſhall know what they doe, whether they demaund, or take too little or too much, after the rate of 10. in the 100. by which proportion all theſe *Tables* are ruled.

As for example, I am to buy an Annuitie of 16.li. per annum *for* 12. *yeares, and am demanded for it in ready money* 120.li. *I would know, if I giue this rate, whether I giue too much or too little, according to the proportion of* 10 *in the* 100 *per annum, &c.*

I looke in the *Table* laſt before what 1.li. is worth for 12. *yeares*, and I find againſt 12 this *ſumme* 6.li. 16.ſ.3.d. Now I ſay that 16.l. *Annuity* for that time, and after that proportion commeth to 16 times as much, which is 100. li. So that I ſee the party demaunded of mee 11.li. too much after the rate of 10. in the 100. *per annum*, and therefore I muſt draw him to a lower price, or leaue it.

Againe, I am offered an Annuity of 20.li. *per annum of* 14. *yeares for* 130.li. *I would know if I giue it, whether I giue too much or too little, according to the proportion aforeſaid.*

I ſeeke firſt what 1.li. *Annuitie* is worth for 14. *yeares*, and I finde in the ſaid laſt *Table* 7.li. 7.ſ.4 d. Then ſay that the *Annuity* of 20.li. *per annum*, will come to 20. times as much, and will be worth 147.li.6.ſ.8.d. according to

the

the proportion before mentioned; and is more then his demand by 17.li.6.ſ.8.d. So that I ſee, if I accept of it, I ſhall haue a good *bargaine*. And thus may you know readily by looking in your *Table*, and finding what 1.li. is worth for any time therein contained, how much any greater *ſumme* will come vnto, if you multiply it by that *ſumme* of 1.li. as before is ſufficiently ſhewed.

But ſuppoſe this: I haue 300.li. *ready money & would beſtow the ſame for a valuable Annuity anſwerable thereunto according to the proportion aforeſaid. I would know what Annuity to endure* 21. *yeares this* 300.li. *will buy?*

I looke in the former *Table* what 1.li. *Annuitie* will coſt for that time, and I finde 8.li. 12.ſ 11.d. Then I ſay by the *Rule of Proportion*. If 8.li.-12.ſ.-11.d. will buy. 1.li. *Annuitie* for 21. *yeares*: what *Annuity* ſhall 300.li. buy or be worth for that time. I reduced the *ſummes* to the leaſt *denomination* (which is *pence*) and I finde 34.li.10.ſ.9.d. And after this manner (by the helpe of this *rule*) may you finde all other ſummes for any time contained in the foreſaid laſt *Table*.

A Table ſhewing what 1 *li in reuerſion for a- nynumber of yeeres vnder* 31 *is worth in ready money, the buyer ſtaying vntill the thing be fallen in hand.*

yeeres.	li	ſ	d	li	ſ	d	yeeres.
1	0	18	2	0	4	4	16
2	0	16	6	0	3	11	17
3	0	15	0	0	3	7	18
4	0	13	7	0	3	3	19
5	0	12	5	0	2	11	20
6	0	11	3	0	2	8	21
7	0	10	3	0	2	5	22
8	0	9	3	0	2	2	23
9	0	8	5	0	2	0	24
10	0	7	8	0	1	10	25
11	0	7	0	0	1	8	26
12	0	6	4	0	1	6	27
13	0	5	9	0	1	4	28
14	0	5	3	0	1	3	29
15	0	4	9	0	1	1	30

This laſt *Table* differeth, and is contrary to the other three before mentioned: For whereas the others increaſed more and more according to the *number of yeares* ſpecified: This doth grow and diminiſh leſſe and leſſe, as the *number of yeares* increaſeth. As for *example.*

There is a Tenement the fee ſimple whereof after 7 yeares will bee worth 40.li. *what am I to giue for it in ready money, now ſtaying vntill it fall in hand.*

To know this I looke in this laſt *Table* for *7 yeares*, and againſt it I finde 10.ſ.3.d. So that a thing that after *7 yeares* will bee worth 1.li. is worth now in ready *money* but 10.ſ.3.d. Then ſay I that the foreſaid Tenement (which after *7 yeares* will be worth 40.li.) is now worth 40. times 10.ſ.3 d. which is 20 li.10 ſ.

Againe, there is a Farme which after 9. yeares will be worth the fee-ſimple 420.li. *what is it now worth in ready money ſtaying vntill it fall in hand?*

I looke in the ſaid *Table* what 1 li. is worth in reuerſion after 9 *yeares*, and I finde 8.ſ.5.d. Then ſay I, that the *Farme* of 420.li. ſo long in reuerſion, will bee now worth in ready money, 420. times as much, which is 176 li. 15.ſ.

Laſtly, there is a Lordſhip to be ſold, the Fee-ſimple whereof after 14. *yeares will bee worth* 7500. li. *I would know what the ſame is now worth in ready money for the Reuerſion.*

I looke in this laſt *Table* for 14 yeares, and againſt it I finde 5 ſ.3 d. ſo much 1 li. is worth

in reuerſion after 14 yeares. Then ſay I, that 7500. li. is worth no more in reuerſion for that time then 7500 times 5 ſ. 3 d. which is 1968. li. 15 ſ. And after this manner may you finde out any other *ſumme* whatſoeuer.. And though ſome men of their owne experience can aime (as they thinke) neere enough the marke to ſerue their owne turnes: yet I dare vnder take they ſhall neuer ſo exactly doe it nor iuſtifie what they doe, as if they did it by arte.

FINIS.

New *Tables* of *Intereſt* at 8 *per centum*, *per annum*, exactly calculated for 30 *yeeres* by *Robert Hartwell*, with neceſſary *queſtions* for the vſe of them.

The firſt table expreſſing the increaſe of one pound principall, put out and forborne for any number of yeeres under 31, *at* 8 per centum, per annum.

yeeres	li	ſ	d	q	li	ſ	d	q	yeeres
1	1	1	7	0	3	8	6	0	16
2	1	3	3	3	3	14	0	0	17
3	1	5	2	1	3	19	11	0	18
4	1	7	2	2	4	6	3	3	19
5	1	9	4	2	4	13	2	2	20
6	1	11	8	3	5	0	8	0	21
7	1	14	3	1	5	8	8	3	22
8	1	17	0	0	5	17	5	0	23
9	1	19	11	3	6	6	9	3	24
10	2	3	2	0	6	16	11	2	25
11	2	6	7	2	7	7	11	0	26
12	2	10	4	1	7	19	9	0	27
13	2	14	4	2	8	12	6	2	28
14	2	18	8	3	9	6	4	0	29
15	3	3	5	1	10	1	3	0	30

The deſcription and vſe of the *Tables* of *Intereſt* at 8 *per* 100. *per annum*, being *profitable*.

The firſt of them.

THeſe *Tables* conſiſt of foure *Columnes*, in the *firſt* and *fourth* whereof is written ouer the head *yeeres*, and vnder the *firſt* the number of *yeeres* deſcending from 1 to 15; likewiſe in the *fourth* the number of *yeeres* deſcending from 16 to 30. And againſt euery *yeere* in the *ſecond Columne* toward the right hand, the *pounds*, *ſhillings*, *pence* and *farthings*, which one *pound*, or 20 *ſhillings* principall will amount vnto being put forth and forborne for the number of *yeeres* ſet againſt it; (but the *pounds*, *ſhillings*, *pence*, &c. in the third *Columne* belongeth to the *yeeres* ſet in the laſt *Columne*.

1 Exam-

1 Example.

Let it be required what one pound or 20 *ſhillings, being put forth and forborne for* 12 *yeeres ariſeth to at* 8 per 100, per annum, *intereſt vpon intereſt.*

Seeke in the *firſt Columne* vnder the *title of yeeres*, for 12 the number of *yeeres* propoſed in the queſtion, and right againſt it toward the right hand in the *ſecond Columne* you ſhall finde 2 li — 10 ſ — 4 d — 1 q, which is the *principall* and *increaſe* thereof due for the time required.

2 Example.

If 100 li *be put forth for* 17 *yeeres according to the ſame intereſt, I demand what it will amount to in that time?*

Looke in the *Columne* vnder the *title* of *yeeres* for 17, and right againſt it towards the left hand in the *table* is found 3 l — 14 ſ — 0 d — 0 q, which is the encreaſe of 1 li.

by which you may thus gather the increaſe of 100 li, or any other *ſumme*; a hundreth times 3 li is 300 li: then 100 times 14 *ſhillings* is 70 li, both which added together doe make 370 li—0 s —0 d, which is the *increaſe* of 100 li put forth and forborne 17 *yeeres* the ſolution to the queſtion.

li	ſ	d	q
300	0	0	0
70	0	0	0
370	0	0	0

3 Example.

Suppoſe 60 li be put forth for 19 yeeres according to that rate, what will it increaſe to in that time?

Seeke 19 vnder the *title* of *yeeres*, and againſt it toward the left hand is found 4 li—6 s—3 d—3 q now ſay 60 times 4 li is 240 and 60 times 6 *ſhillings* is 360 *ſhillings* or 18 li. and 60 times 3 d is 180 d or 15 *ſhillings*, and 60 times 3 *farthings* is 3 *ſhillings* 9 d all which added together make 258 li, 18 ſ, 9 d, the increaſe thereof demanded.

l	ſ	d	q
240	0	0	0
18	0	0	0
0	15	0	0
	3	9	0
258	18	9	0

The

The ſecond Table ſhewing what one pound Annuity or yeerly rent is worth at the end of any number of yeeres vnder 31, *being forborne, at* 8 per centum, per annum.

yeeres.	li	ſ	d	q		li	ſ	d	q	yeeres
1	1	0	0	0		30	6	5	3	16
2	2	1	7	0		33	15	0	0	17
3	3	4	11	0		37	9	0	0	18
4	4	10	1	1		41	8	11	0	19
5	5	17	3	3		45	15	2	3	20
6	7	6	8	0		50	8	5	2	21
7	8	18	5	1		55	9	1	2	22
8	10	12	8	3		60	17	10	1	23
9	12	9	9	0		66	15	3	2	24
10	14	9	8	3		73	2	1	1	25
11	16	12	10	3		79	19	1	0	26
12	18	19	6	2		87	7	0	0	27
13	21	9	10	3		95	6	1	2	28
14	24	4	3	2		103	19	3	3	29
15	27	3	0	2		113	5	7	3	30

The vſe of the *ſecond Table* (whoſe diſpoſition is altogether like the former) according to the title thereof, being *profitable:*

1 Example.

There is a leaſe worth 28 li per annum, *to endure* 14 *yeeres, I demand what it will riſe vnto at the end of thoſe yeeres, being all forborne with the intereſt and intereſt vpon intereſt, at the rate preſcribed in this Table.*

Looke in the *third Table* for 14 *yeeres*, againſt which toward the right hand, you ſhall finde 24 li —4 ſ—3 d—2 q. Now multiply 28 li by 24 there ariſeth 672 li, then 28 li by 4 ſ yeeldeth 112 ſ or 5 l–2 ſ. Againe 28 li by 3 d produceth 84 d or 7 ſ; finally, 28 by 2 *farthings* yeeldeth 56 *farthings* or 1 ſ—2 d. All which added together make 678 li—0 ſ—2 d, to be receiued at the end of 14 *yeeres*, the ſame *rent* or *annuity* being reſpited.

li	ſ	d	q
672	0	0	0
5	12	0	0
	7	0	0
	1	2	0
678	0	2	0

2 Exam-

2 Example.

If 60 li *yeerely rent or annuity bee forborne* 20 *yeeres. I demand how much it will increaſe at the end of the ſaid terme?*

In the table I finde that 1 pound in 20 *yeeres* will ariſe to 45 li—15 ſ—2 d—3 q, therefore 60 li in the like terme will yeeld 60 times as much; which I will reckon thus; 60 times 45 li is 2700 li, 60 times 15 ſ is 900 ſ, or 45 l, 60 times 2 d is 120 d or 10 ſ, laſt of all, 60 times 3 q is 180 *farthings*, or 3 ſ —9 d, all which together amount vnto 2745 li —13 ſ —9 d, the value thereof to be receiued at the end of the terme.

li	ſ	d	q
2700	—0	—0	—0
45	—0	—0	—0
	10	—0	—0
	3	—9	—0
2745	—13	—9	—0

3 Example.

The yeerly rent of 6 li—13 ſ—4 d *being behinde and vnpaid the ſpace of* 7 *yeeres, at the end of which terme the tenant is compelled to pay the ſame with the intereſt thereof according to the aboue named rate: I demand what the payment ought to be?*

The

The increaſe of 1 li *yeerely rent* anſwering to 7 *yeeres*, is 8 li — 18 ſ — 5 d — 1 q, which for 6 li *rent* is to bee taken 6 times, which ariſeth to 53 li — 10 ſ — 7 d — 1 q; now becauſe 13 ſ — 4 d is two third parts of 1 li, therefore I take ⅔ of 8 li — 18 ſ — 5 d — 1 q, which is the increaſe of 1 li forborne for 7 *yeeres*, that is 5 li — 18 ſ — 11 d — 2 q, which together make 59 li — 9 ſ — 6 d — 3 q, the *ſumme* to be receiued, as was required.

li	ſ	d	q
35	10	7	q
5	18	11	2
59	9	6	3

The

The third Table declaring what one pound due at the end of any number of yeeres vnder 31 is worth ready money at 8 per centum, per annum.

yeeres.	li	ſ	d	qrs	li	ſ	d	qrs	yeeres.
1	0	18	7	0	0	5	10	0	16
2	0	17	10	3	0	5	4	3	17
3	0	15	1	2	0	5	0	0	18
4	0	14	8	1	0	4	7	2	19
5	0	13	7	1	0	4	3	1	20
6	0	12	7	0	0	3	11	2	21
7	0	11	8	0	0	3	8	0	22
8	0	10	9	2	0	3	4	3	23
9	0	10	0	0	0	3	1	3	24
10	0	9	3	0	0	2	11	0	25
11	0	8	6	3	0	2	8	1	26
12	0	7	11	1	0	2	6	0	27
13	0	7	4	0	0	2	3	3	28
14	0	6	9	2	0	2	1	3	29
15	0	6	3	2	0	1	11	3	30

This *third table* is diſpoſed as the *firſt*, the vſe according to the *title* thereof, being *damageable*.

1 Example.

Suppoſe there is 750 li *due to be paid at the end of* 9 *yeeres, the Creditor would ſell this debt for preſent money, what ought that money to bee at the rate deſcribed in the table.*

Seeke in this *third table* for 9 *yeeres* at the *left ſide* of the *table*, and right againſt it toward the right hand, you ſhall finde 10 *ſhillings*, which multiplied or taken 750 times, yeeldeth 7500 *ſhillings*, which is 375 li, the value of that *debt* in preſent *money*.

2 Example.

There is a leaſe worth 500 li, *after the end of* 7 *yeeres, what is it worth preſent money, according to the rate deſcribed in the table ſtaying till it fall.*

I ſeeke in the *table* for the 7 *yeere*, and right againſt it I finde 11 ſ— 8 d; now I multiply 500 by 11, it yeeldeth 5500 *ſhillings*, or 275 li, then 500 times 8 d maketh 4000 d, which is 16 li—13 ſ—4 d, which added together, is 291 li—13 ſ—4 d, the value of the *leaſe* to be paid before it fall in hand.

li	ſ	d	o
275	—o	—o	—o
16	-13	—4	—o
221	-13	—4	—o

The

The fourth Table expreſſing what one pound yeerely rent or annuity for any number of yeeres not exceeding 30, *is worth ready money at* 8 per centum, per annum.

yeeres.	li	ſ	d	q	li	ſ	d	q	yeeres.
1	0	18	6	0	8	17	0	1	16
2	1	15	7	3	9	2	5	0	17
3	2	11	6	2	9	7	5	1	18
4	3	6	2	3	9	12	0	3	19
5	3	19	10	1	9	16	4	1	20
6	4	12	5	1	10	0	4	0	21
7	5	4	1	2	10	4	0	0	22
8	5	14	11	0	10	7	5	0	23
9	6	4	11	1	10	10	6	3	24
10	6	14	2	1	10	13	5	3	25
11	7	2	9	1	10	16	2	1	26
12	7	10	8	2	10	18	8	1	27
13	7	18	0	3	11	1	0	0	28
14	8	4	10	2	11	3	2	0	29
15	8	11	2	1	11	5	1	3	30

The *fourth Table* is diſpoſed altogether as the former, and the vſe thereof in like ſort being *damageable*.

1 Example.

There is an annuity or rent of 20 ſ per annum *to endure* 25 *yeeres, it is required what it is worth preſent money?*

Looke in the *table* for the 25 *yeere*, and right againſt it you ſhall finde 10 l–13 ſ–5 d–3 q, which is the ſolution.

2 Example.

What is the leaſe of certaine land valued at 140 li per annum *to begin preſently and endure* 18 *yeeres, worth ready money?*

Search in the *table* for 18 *yeeres*, the *terme* named in the *queſtion*, and right againſt it toward the left hand you ſhall finde 9 l——7 ſ 5 d——1 q, which expreſſeth that *one pound rent* to be bought for that *terme* is worth ſo much; therefore that ſumme 140 times is

the value required; Now
140 times 9 li is 1260,
and 140 times 7 ſ is 980 ſ
or 49 li; likewiſe 140
times 5 d is 700 d or 2 li
—18 ſ—4 d, and 140
farthings is 2 ſ— 11 d,
all which added toge-
ther make 1312 li—1 ſ
3 d, for the value of the ſaid *leaſe*, paying no *rent*.

li	ſ	d
1260	—0	—0
49	—0	—0
2	—18	—4
	2	—11
1312	—1	—3

3 Example.

A leaſe taken for 21 *yeeres at* 13 li—6 ſ 8 d per annum, *which after* 5 *yeeres expired, the tenant is deſirous to giue a fyne, and bring the rent downe to* 8 li per annum *for the reſt of the terme: the demand is what fyne is to be paid?*

Subtract 5 *yeeres* from 21 the remaine 16, is the time vnexpired; likewile from the preſent *rent* abate 8 li, the reſt will be 5 li-6 ſ-8 d: now the drift of the *queſtion* is, what 5 li-6 ſ-8 d *yeerly rent or annuity* to endure 16 yeeres is worth preſent money?

The value of 1 li *rent or annuity* anſwering to 16 *yeeres* is 8 li—17 ſ—0 d—1 q: Now

5 times 8 li is 40 li, and 5 times 17 ſ is, 4 li —5 ſ, and 5 times one *farthing*, is 1 d —1 q, and becauſe 6 ſ—8 d is $\frac{1}{3}$ of 1 li, I take $\frac{1}{3}$ of 8 li—17 ſ —0 d—1 q, which is 2 li–19 ſ–0 d, all which added together make 4 li–4 ſ–1 d–0 q, which is the *fyne* that ought to be paid to bring the *rent* to 8 li *per annum*.

li	ſ	d	q
40	0	0	0
4	5	0	0
2	19	1	1
47	4	1	1

The fift Table declaring what yeerely rent or annuity one pound ready money will purchaſe for any number of yeeres under 31, *at* 8 per centum, per annum.

yeeres.	li	ſ	d	q	li	ſ	d	q	yeeres.
1	1	1	7	0		2	9	3	16
2		14	0	0		2	8	3	17
3		9	8	2		2	8	0	18
4		7	6	2		2	7	0	19
5		6	3	0		2	6	2	20
6		5	4	3		2	5	3	21
7		4	9	2		2	5	1	22
8		4	4	0		2	4	3	23
9		4	0	0		2	4	1	24
10		3	8	2		2	4	0	25
11		3	6	0		2	3	3	26
12		3	3	3		2	3	1	27
13		3	1	3		2	3	0	28
14		3	0	1		2	2	3	29
15		2	11	0		2	2	2	30

In the fift *Table* the *numbers* and *columnes* are all diſpoſed as the for-*Tables*, and needeth no further explanation but only *Examples*.

1 Example.

The Table declareth at firſt ſight what yeerely rent or annuity one pound ready money will purchaſe for any terme in the table expreſſed.

But if the *ready money* be aboue one pound, then if any *value or rent* ſet downe in this *table*, be multiplyed by the *number* belonging to the *yeeres in queſtion*, the *product* will ſhew what *yeerly rent or annuity* that *ready money* will purchaſe for the *time propoſed*.

2 Example.

A certaine man hath 750 li to purchaſe an annuity to endure 27 yeeres, ſo as it may yeeld him the like profit, as if it were put out according to the rate in the table expreſſed; it is required what that annuity ought to be?

Becauſe

Becauſe the *annuity* is to endure 27 *yeeres*, ſeeke out the *value or rent* ſet againſt 27 *yeeres* in this *fift table*, which is 2 ſ—3 d—1 q; now this being the *annuity* 20 ſ *ready money* will *purchaſe* for that *terme*, it muſt bee multiplied by 750 li as followeth: becauſe 2 ſ is the tenth part of 20 ſ, therefore take the tenth part of 750 li which is 75 li, which ſet firſt downe, then 750 times 3 d is 9 li—7 ſ—6 d, which ſet vnder the former, laſt of all 750 *farthings* is 15 ſ—7 d—ob: All which added together, produce 85 li—3 ſ—1 d-ob, the *yeerely annuity* required.

li	ſ	d	q
75	0	0	0
9	7	6	0
	15	7	2
85	3	1	2

Deo ſoli laus omnis honor & gloria tribuatur. Amen.

FINIS.

9469136749678125?

ARTS MATHEMATICALL.

In Saint *Bartholomewes* neare the Red Lion in *Cloth-Faire*, is taught by *Robert Hartwell* Practitioner in the Mathematickes, the *Arts*, *Sciences*, and *Faculties* here vnder expressed.

Viz.

- **Arithmeticke.**
  - IN whole Numbers and Fractions.
  - The Extraction of Rootes, viz. { Square, Cube, Squared square, Surfolid. } At one operation.
  - Of Astronomicall or Physicall Fractions.
  - Of proportion.
  - The Rules of Aequation, with Cosse, or Algebra.
- **Geometry**
  - Principles with practise and Demonstration.
  - Measuring of Land, and reducing of Plots or Maps to any proportion.
  - Plotting of Countrys with diuers and sundry instrumen.s.
  - Measuring of any superficiall or solid content. { Boarde, Glasse, &c. / Timber, Stone, &c.
- **The doctrine of Triangles.** { Plaine and Sphericall. } With the vse of the Tab'es of { Sines, Tangents, Secants, and Logarithmes
- **Description, Demonstration, and vse of Instruments, *viz.***
  - Quadrant, Quadrat, Theodelite, plaine Table, Circumferenter, Pantometer. Circular Scale, M Gunters Sector and proportionall Ruler (by him) conuerted into a Crosse Staffe for Sea-mens vse.
- **Nauigation.**
  - Principles thereof, with the making and vse of sundry instruments fitting that Art.
  - The vse and proiection of Maps and Charts.
- **Vse & demonstration of**
  - The Sphere or Globe } both { Terrestiall, and Cœlestiall.
  - Cœlestiall Globe in plaine, by M.D.Hood
  - All sorts of Astrolabes, and M. Blagraues Iewell.
- ***Dialling* of all sorts, *viz.*** { Fixed, and Instrumentall. } aswell { Geometrically, Arithmetically, as Instrumentally.
- **Accompts for**
  - Merchants by order of Debitor & Credit &c.
  - Retayle of all sorts.
  - Perfecting of Accompts in Controuersie.

Fide *sed* *Vide.*

LONDON:
Printed by THO. HARPER,
for *Iohn Hariſon*, 1631.

are open in thy ſight:
10 My heart doth pant, my ſtrength
mine eyes haue loſt their ſight.

11 My louers and my louing friend
ſtand looking on my woe:
And eke my Kinſmen farre away,
are me departed fro.
12 They that did ſeeke my life laid ſn
and they that ſeeke the way
To do me hurt, ſpake lies, and thou
on miſchiefe all the day.

The ſecond part.

13 But as a deafe man I became,
that cannot heare at all:
14 And as one dumbe that opens not
his mouth to ſpeake withall.
15 For all my confidence, O Lord,
is wholly ſet on thee:
O Lord, thou Lord, thou art my Go
thou ſhalt giue eare to me.

16 This did I craue, that they my foe
triumph not ouer mée:
For when my foot did ſlip, then they
did ioy my fall to ſée.
17 And truely I, poore wretch, am ſe
in place a wofull wight:
And eke my grieuous heauineſſe
is euer in my ſight.

18 For while that I my wickedneſſ
in humble wiſe confeſſe:
And whyle I for my ſinfull déedes
my ſorrowes do expreſſe:
19 My foes do ſtill remaine aliue,
and mighty are alſo:
And they that hate me wrongfully,
in number hugely grow.

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TEACHING THE PERFECT worke and practise of Arithmeticke, both in whole Numbers and Fractions, after a more easie and exact forme then in former time hath beene set forth:

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Scientia non habet inimicum nisi ignorantem.

Fide ——— sed ——— Vide.

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the common number, and the other greater then hée: (for two greater or two ſmaller cannot well be linked together:) and the reaſon is this, that one greater and one ſmaller, may be ſo mixed that they will make the meane or common number very well: but two leſſe can neuer make ſo many as the common number, being taken orderly: no more can two ſummes greater then the meane, euer make the meane in due order, as it ſhall appeare better to you hereafter. And, as it is of neceſſity to linke euery ſmaller, (once at the leaſt) with one greater, and euery greater with one ſmaller, ſo it is at liberty to linke them oftner then once: and ſo may there be to one queſtion, many ſolutions. When you haue ſo linked them, then marke how much each of the leſſer numbers is ſmaller then the meane or common number, and that difference ſet againſt the greater numbers which hée linked with thoſe ſmaller, each with his match ſtill on the right hand, and likewiſe the exceſſe of the greater numbers aboue the meane, you ſhall ſet before the leſſer numbers, which hée combined with them. Then ſhall you (by Addition) bring all theſe differences into one ſumme, which ſhall be the firſt number in the Golden rule, and the ſecond number ſhall hée the whole maſſe that you will haue of all thoſe particulars: the third ſumme ſhall hée each difference by it ſelfe, and then by them ſhall be found the fourth number, declaring the iuſt por-

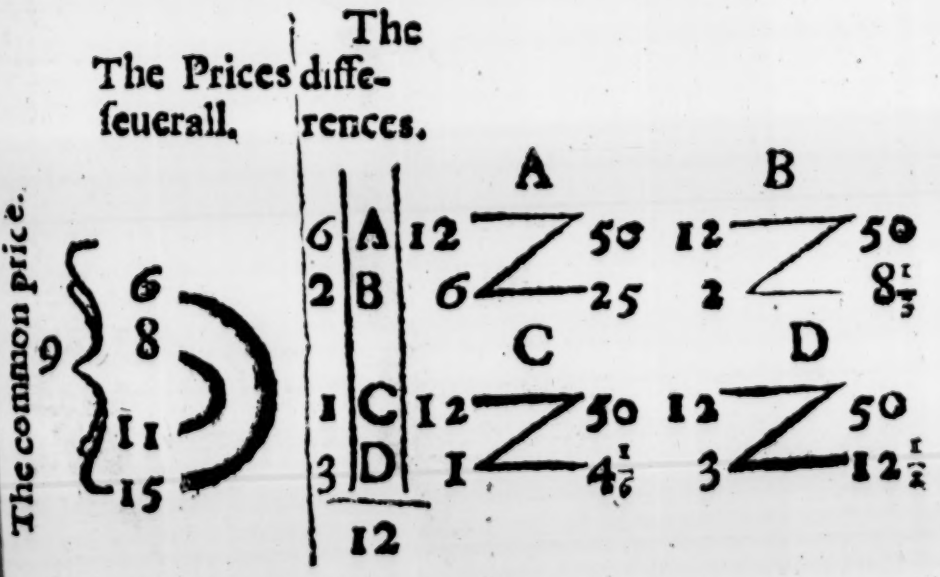
portion of euery particular in that mixture: As now by these examples I will make it plaine,

A question of mixing of wines.

There are foure sorts of Wine, of seuerall prices. one of 6 pence, *a gallon, another of* 8 pence, *the third of* 11 pence, *and the fourth of* 15 pence, *the gallon. Of all these wines would I haue a mixture made, to the summe of fiftie gallons, and so the price of each gallon may be* 9 pence. *Now demaund I, how much must bee taken of euery sort of Wine?*

Scholar. If it shall please you to worke the first example, that I may marke the applying of it to the rule: then I trust I shall bee able, not onely to doe the like but also to see the reason in the order of the worke.

Master. Marke then this forme, and the placing euery kinde of number in it.



Here (you see) I haue set downe the seuerall

rall prices which be 6, 8, 11, 5, and haue linked together 6 with 15, and 8 with 11. The common price 9, I haue set on the left side, and the difference betweene it, and euery particuar price, I haue set on the right hand, not against the summe (whose difference it is) but against the summe that it is linked withall, so the difference of 15 aboue 9, is 6, which I haue set, not against 15 but 6, that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue set against 15. So likewise the difference betweene 8 & 9, is but 1, that I haue set against 11, and the difference of 11 aboue 9 (which is 2) I haue set against 8. Then adde I all those foure differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of Gallons that I should haue, and the third summe is euery particular difference. Now if you worke by the Golden Rule, you shall finde the number of Gallons that shall be taken of each sort of wine: For the better distinction whereof, I haue set these letters, A, B, C, D, both against the numbers for which the works doe serue, and ouer the worke also, which seuerally serue for each of them. And now (if you list to examine the truth of these works,) adde these foure summes together, and they will make 50, that is the totall which I

The proof of this Rule.

would

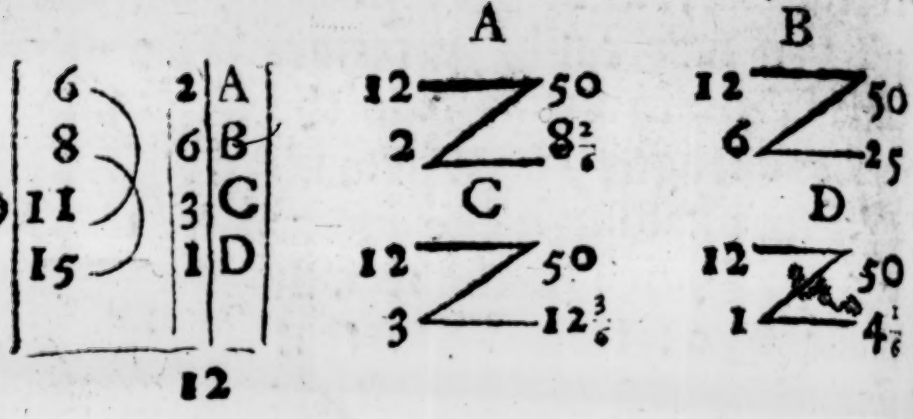
would haue, as by this example you may easily perceiue. And (for to proue how the prices doe agree) doe this: multiply the totall summe 50, by the common price 9, and it will make 450: then keepe that summe by it selfe, and afterward multiply euery seuerall summe of gallons, by the price belonging to the same gallons; & if that summe doe agree with this which you haue kept first, then is your worke well done. As here 25 is the number of gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you shall set downe, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of gallons, and it will make $66\frac{4}{6}$: so againe $4\frac{1}{6}$ multiplied by 11: doth make $45\frac{5}{6}$. And last of all, $12\frac{1}{2}$ multiplied by 15, maketh $187\frac{3}{6}$: and these added together do make 450, as in the example annexed you may see, wherefore seeing it doth agree with the former summe of 50, multiplied by 9, I may iustly affirme this worke to be good, and well done.

25	
8	$\frac{1}{3}$
4	$\frac{1}{6}$
12	$\frac{1}{2}$
50	

150	
66	$\frac{4}{6}$.
45	$\frac{5}{6}$.
187	$\frac{3}{6}$.
450	

And now to proue how you can doe the like I propound the same question, onely willing you to vse some other forme of combining or linking the summes. The variation of this question.

Scholar. That shall I proue with your fauour, & therefore I combine 8 with 15 and 6 with 11, and then the forme will be as followeth.



Whereby amounteth the same summe in totali of the differences as did before: and yet now the differences be altered as the combination is changed, whereof I vnderstand the reason by your former worke. And therefore here appeareth no strange thing, but that now I haue $8\frac{2}{6}$ gallons of 6 pence, and 25 gallons of 8 pence, & 12 gallons and $\frac{3}{6}$ of 11 pence, and so consequently 4 gallons and $\frac{1}{6}$ of 15 pence: so that multiplying $8\frac{3}{6}$ by 6, it maketh 50, and then 25 multiplyed by 8. maketh 200: likewise $12\frac{3}{6}$ multiplied by 11 yeld $137\frac{3}{6}$, and $4\frac{1}{6}$ multiplied by 15, maketh $62\frac{1}{2}$ which 4 summes added into one, will yeld in the total 450, which agreeth with the multiplication of 50, (being the total summe of gallons) by 9 the common or meane price.

	50
	200
	$137\frac{1}{2}$
	$62\frac{1}{2}$
	450.

Master. Seeing you conceiue this worke so well, I will propound another example vnto you of more varietie, in the Alligations or combinings, as thus:

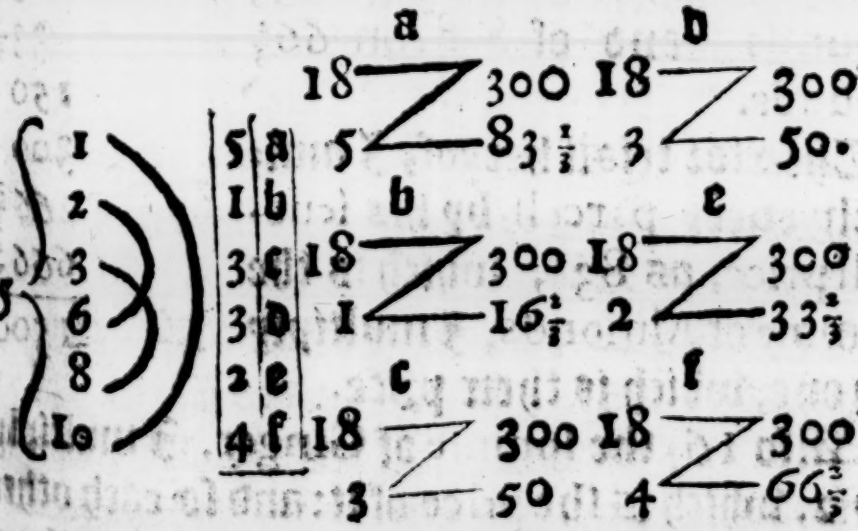
A

A Merchant being minded to make a bargaine for Spices, in a mixt masse (that is to say,) of Cloues, Nutmegs, Saffron, Pepper, Ginger, and Almonds: the Cloues being at 6 shillings, Saffron at 10 shillings, Pepper at 3 shillings, Ginger at 2 shillings, and Almonds at 1 shilling. A queſtion of Spices.

Now would he haue of each ſort ſome, to the value of 300 pound in the whole, and each pound one with another, to beare in price 5 ſhillings: How much ſhall he haue of each ſort?

Scholar. That will I try thus.

Firſt, I ſet downe thoſe ſixe ſeuerall prices, and at the left hand I ſet the common price, 5 ſhillings. Then I linke them thus, 1 with 10, 2 with 6, and 3 with 8: as in the example following.



Maſter. I had minded to haue combined them in more variety: but I am content to ſée your owne worke firſt, and then more varieties in combination may follow anon.

Scholar. Then to continue as I began, I seeke the difference betweene 1 and 5, (which is 4) and that I set against 10; then against 1 I set 5, which is the excesse of 10 aboue 5, so I gather the difference betweene 2 and 5, which is 3: and that I set against 6, because it is combined with 2: and likewise the difference of 6, aboue 5, (which is 1) I set against 2. Then take I the Difference of 3 from 5, which is 2, and that I set against 8: and before that 3, I set the difference of 8 aboue 5, which is 3. Then gather I all these differences by Addition, and they make 18, which I set for my first number in the Golden rule, and so appeareth by those workes, that of Almonds I must take $8\frac{1}{3}$ pound, of Ginger $16\frac{2}{3}$ pound, Pepper 50 pounds, of Cloues, 50 pounds, of Nutmegs $33\frac{1}{3}$ pounds, and of Saffron $66\frac{2}{3}$ pounds.

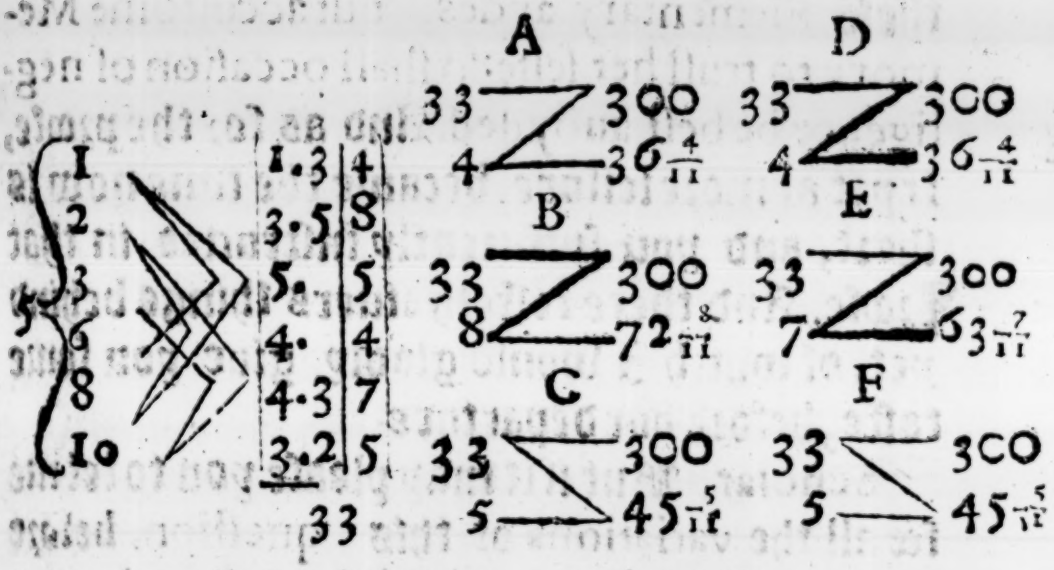
	$83\frac{1}{3}$
	$33\frac{1}{3}$
	150
Then for triall hereof, I mul-	300
tiply euery parcell by his seue-	$266\frac{2}{3}$
rall price, as $8\frac{1}{3}$, which is the	$666\frac{2}{3}$
summe of Almonds, I multiply	1500
by one, which is their price.	

Also $16\frac{2}{3}$ the summe of Ginger, I multiply by 2, which is the price of it: and so each other in his kinde, as this Table annexed doth represent, and then adding them altogether, I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300, by the common price 5, where-

fore

fore it appeareth well wrought.

Master. Now I will make the alligation to proue your cunning somewhat better: but because you shall not thinke your selfe pressed so much, I will also note the differences, as by this example you may see, where I



haue alligate 1 with 6 & 8, and therefore haue I set against 1 both their differences, that is 1 and 3: Likewise, because 2 is combined with 8 and 10, I set before him their differences, 3 & 5. Against 3 I haue set onely 5, which is the difference of 10, with whom 3 is combined onely. Likewise 6 is onely alligate to 1, & therefore is the differences of 1 from 5, which is 4 onely set against it: 8 is linked with 1 and 2, and therefore hath set 4 against him, both their differences, 4 and 3: and 10 is ioyned with 2 and 3, therefore hath hee their differences, 3 and 2. And because of ease for you, in another columne I haue set the differences reduced into one number, for euery seuerall sort,

and haue also added them together, whereby appeareth that they make 33, and so consequently you see the workes of the Golden Rule set forth. For the sixe drugges I haue added the letters, A, B, C, &c. as before.

But I would not wish you to cleaue still to these Elementary aydes, but accustome Memory to trust her selfe: so shall occasion of negligence be best auoyded. And as for the proofe, try it at more leisure, because the time now is short, and you sufficiently instructed in that proofe. And there resteth diuers things behind yet, of which I would gladly giue you some taste, before our departure.

Note.

Scholar. But if it may please you to let me see all the variations of this question, before you goe from it, for me thinketh I could vary it two or three wayes more yet.

Master. I am content to see you make two or three variations, but I would bee loath to stay to see all the variations, for it may be varied aboue 300 wayes, although many of them would not well serue to this purpose.

Scholar. I thought it impossible to make so many variations.

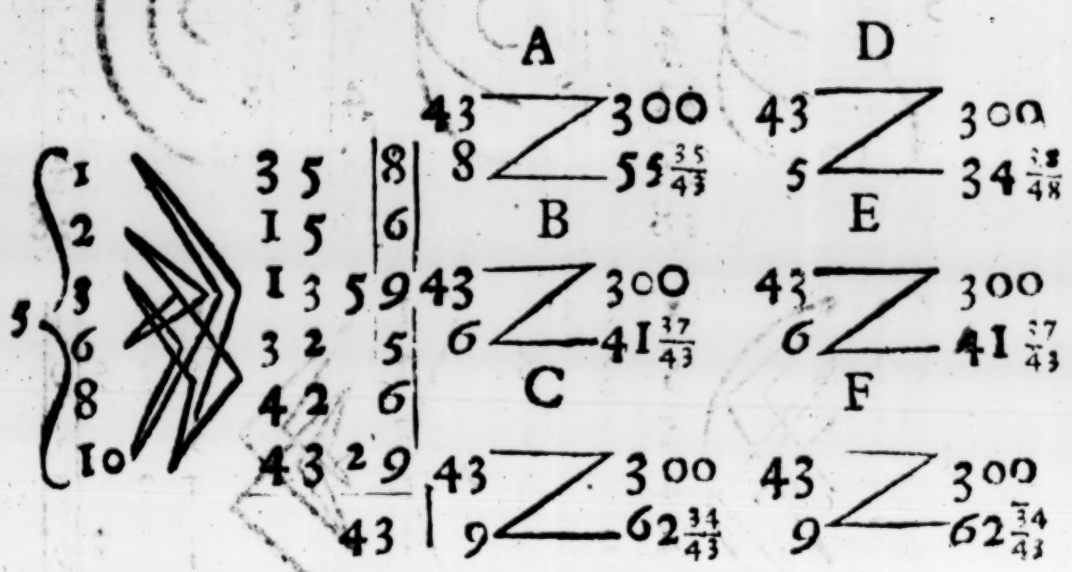
Note.

Master. Marueil not thereat, for some questions of this Rule may be varied aboue 1000 wayes: but I would haue you forget such fantasies, till a time of more leisure. And now goe forward with some variation of this question.

Scholar. For the first variation. I linke

the

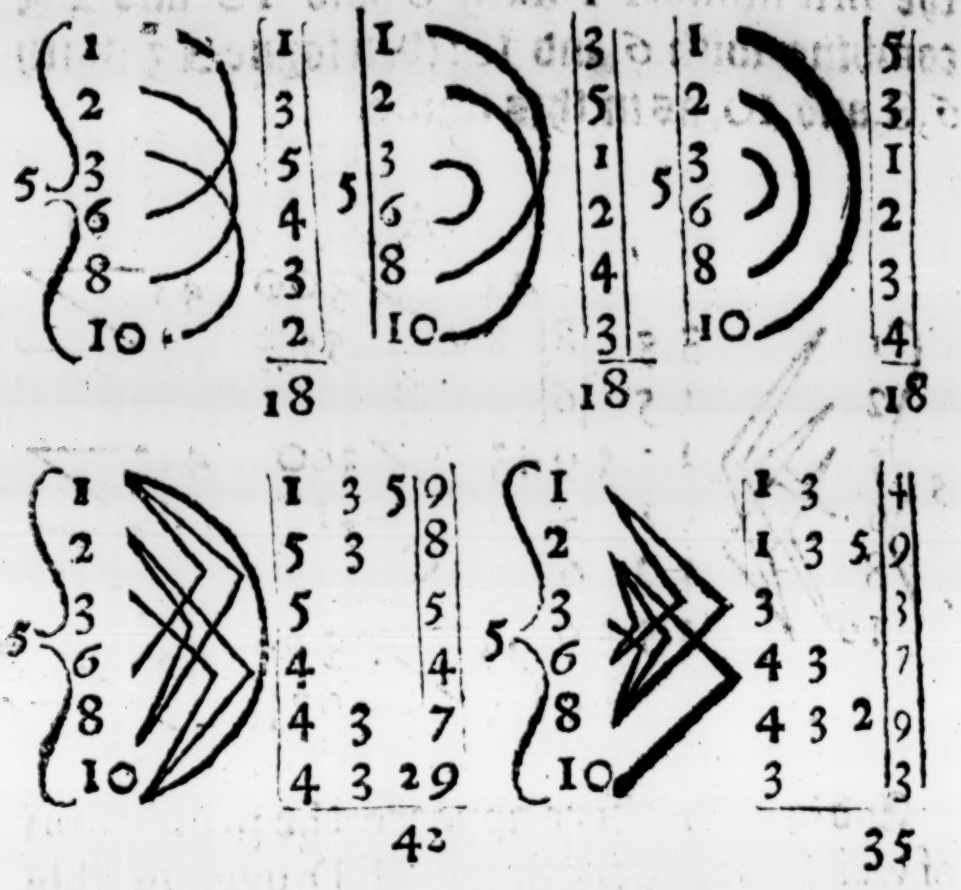
the first number 1 with 8 and 10. and 2 I combine with 6 and 10: then ioyne I 3 with 6,8, and 10, as in this forme.



And so doth there appeare the proportion of weight for euery kinde of Drugge in this mixture. Now for the tryall.

Master. Nay, stay there: you shall not need to make tryall in one example so often, or if you list doe it by your selfe, I am content. But now set forth (for declaration that you conceiue the Rule) two or three examples of seuerall combinations, and then will we passe to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these be the varieties: in which, as the



combinations are ſeuerall, ſo doth it plainely appeare, that the differences by which the proportion of each ſeuerall kinde is taken, are alſo ſeuerall. And yet I ſee in the three firſt of theſe fiue varieties, and in one other before, the totall ſumme of the differences to be one, that is to ſay, 18, whereby I perceiue that the variety of their mixture doth depend on the variety of their differences ſeuerall, and not of the variety of their totall ſumme.

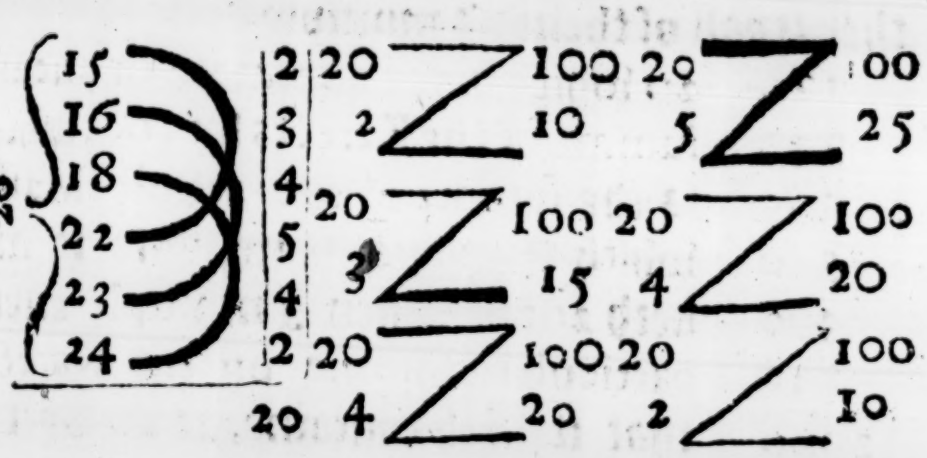
Maſter. So is it. And ſeeing you conceiue it ſo well, I will make an end of this Rule, onely exhibiting vnto you one Queſtion or two of the mixture of Metals, that by it you may deuiſe others like, and exerciſe your ſelfe therein alſo, becauſe the vſe of it ſerueth often

in

in busineſſe of charge, not ſo much for Goldſmiths, as of coynage in Mints. First I demand of you this queſtion; *If a Mynt maſter haue Gold of 22 Karects, and ſome of 23 Karects, ſome of 24: Againe ſome of 15 ſome 16, and ſome of 18 karects, and would mixe them, ſo that he might haue 100 ounces of 20 karects: How much muſt he take of each ſort?*

A queſtion of mixing Gold.

Scholar. To know that, I anſwer in order, thus:



Maſter. You haue wrought the queſtion well: but how chanced you made no doubt of that new name Karect?

Scholar. Becauſe I thought it out of time to demaund ſuch queſtions now, ſeeing you make ſo much haſte to end: and againe in this caſe the proportion of the number, is ſufficient for my purpoſe in this worke: truſting that another time you will inſtruct mee as well of this, as of ſundry other things, which as I haue heard you talke of, ſo I haue a great deſire to them.

Maſter. Your anſwer is reaſonable, and your requeſt and truſt (with Gods helpe)

I intend to satisfie. And now to goe forward with this matter, let me see your examination of this last worke.

Scholar. First for the one part I adde together all the particular summes, as they appeare in the worke, and they make 100, as here by their Addition doth appeare.

And so it fareth that the summes are well gathered: but for the further tryall of them: I multiply first 20 which is the common or meane summe of the Karects by 100, which is the summe of the whole Masse, which I would haue, and it maketh 2000. Then I multiply euery particular summe by the Karects that it doth containe, as 10 by 15, and that maketh 150.

10	
15	
20	
25	
20	
10	
100	

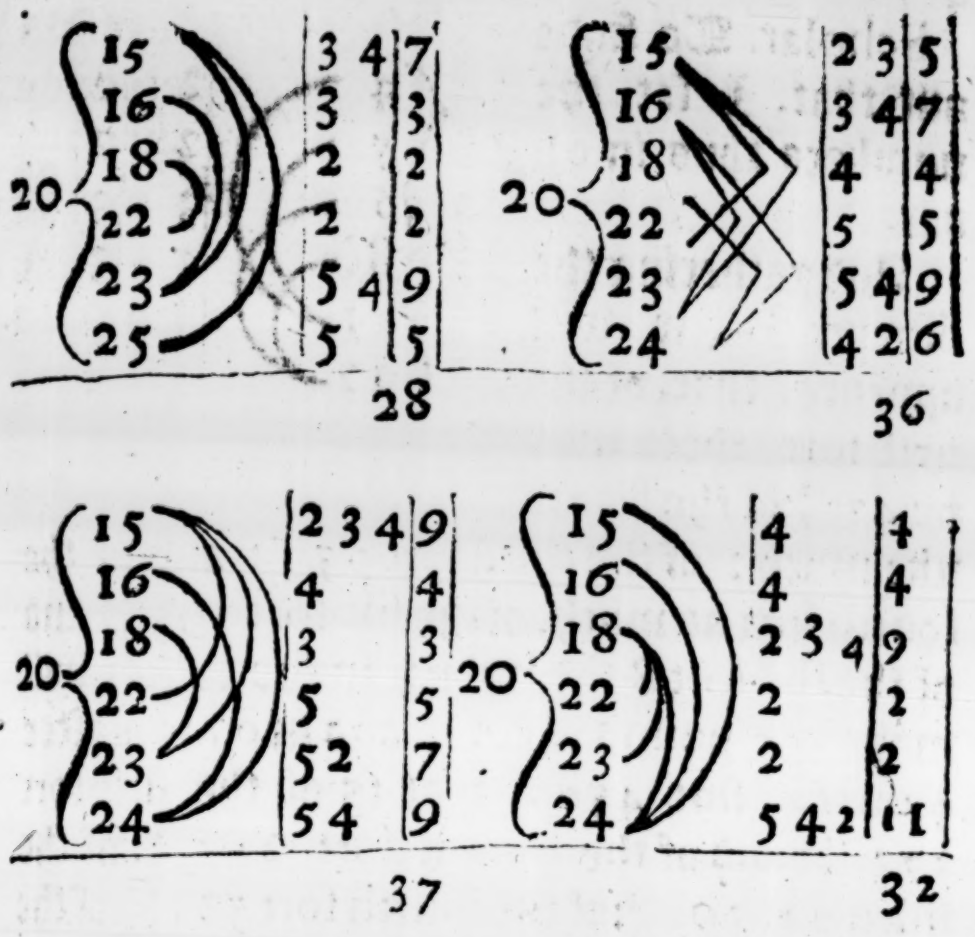
120	
240	
360	
550	
406	
240	
2000	

Likewise I multiply 15 by 16, and it yeeldeth 240: so 20 by 18, maketh 360. And 25 by 22, yeeldeth 550: likewise 20 by 23, bringeth forth 460; and last of all 10 multiplied by 24, yeeldeth 240: which summes all ioyned together make 2000, that doth agree with the like summe before, wherefore I may well say, that the worke is good. And now if (it please you) I would set forth some varieties of this question to proue my wit.

Master. Goe to let me see.

Scholar. Here be foure varieties.

And



And more yet could I make, but not like to the number that you speake of in the variation of the other question.

Master. That will I teach you at more leisure, seeing it is a thing rather of pleasure then of any necessity.

A question of mixing of siluer.

But now for your exercise in this Rule, one other question I will propose. A Mint-master hath sixe Ingots of siluer of sundry fineneſſe, some of foure ounces fine, and some of fiue ounces, some of sixe, and other of eight, some of 11, *and other of* 12, *and his desire is to mixe* 500 *pounds weight, so that in the whole Masse euery pound weight should beare nine ounces of fine siluer: How much shall he take (say you) of euery sort of siluer?*

Scholar.

Scholar. To finde out that, I set the numbers thus in order.

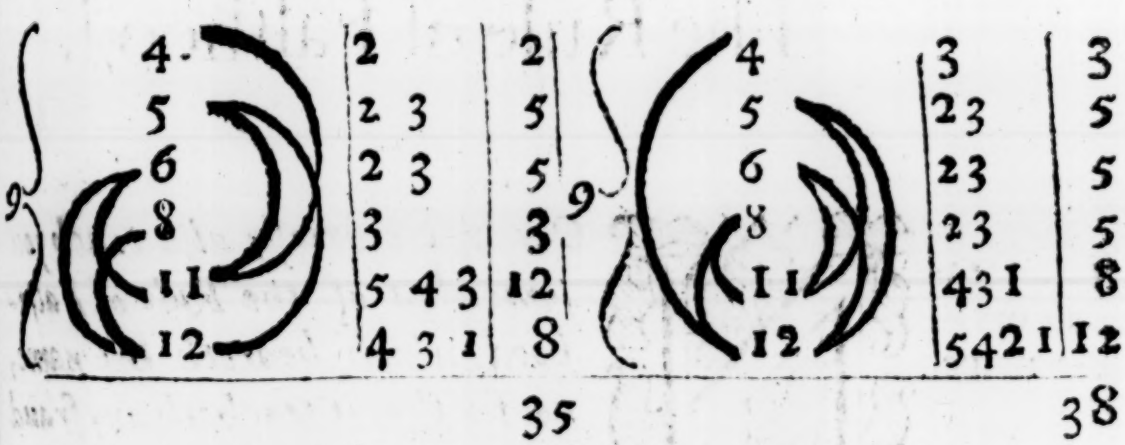
And gathering the differences, it will appeare, that, of the first sort there must be $43\frac{11}{23}$: of the second like much: of the third sort $65\frac{5}{23}$: and of the fourth sort as much: of the fift sort $195\frac{17}{23}$ and of the sixt sort $86\frac{22}{23}$, which in the whole will make 500 pound weight, and in ounces after 9 ounces fine 4500, that is of the first sort $173\frac{21}{23}$: and of the second sort $217\frac{9}{23}$: of the third sort $391\frac{3}{23}$: of the fourth sort $521\frac{17}{23}$, of the fift sort $2152\frac{4}{23}$, and of the sixt sort $1043\frac{11}{23}$, which altogether doe make 4500 ounces, agréeable to the multiplication of 9 by 500.

Master, This is well done of you, therefore now make three or foure varieties, and so an end of this Rule.

Scholar. These 4 varieties I set for example.

Master.

Maſter. And by theſe it appeareth, that you can finde out more, with which I will not now meddle, ſaue onely (for to ſhew you an eaſie helpe in drawing the lines of Combination) I will ſet forth two varieties here.



And this ſhall ſuffice now for the Rule of Alligation or mixture: for by theſe examples may you eaſily coniecture ſuch other as doe appertaine to it, as well for the due working, as for variety of drawing the lines of combination.

Scholar. Sir, albeit it pleaſed you erewhile to put me from my muſing at the many varieties that may fall in theſe combinations and termed them fantaſies; yet my fantaſy

fantaſy giueth me, that the conſideration of this ſhould in many other examples and caſes of importance be very needfull, and the knowledge of it moſt profitable. Therefore ye may well thinke, that at another time conuenient I will requeſt you to aide me herein.

Maſter. Truth it is that this conſideration may fall in practiſe as well Politicke as Philoſophicall, and ſundry wayes in them be applied: Therefore when time ſhall fall fit for the diſcuſſing of this conſideration, you ſhall not want my helping hand.

The Rule of Falſhood.

The occaſion of the name.

NOw will I briefly alſo teach you ſomewhat of the Rule of Falſehood, which beareth his name, not for that it teacheth any fraud or Falſhood, but for that by falſe numbers taken at all aduentures, it teacheth how to finde thoſe true numbers that you ſeeke for.

Scholar. So might any other rule be called the Rule of Falſhood, for they worke by wrong numbers, and by them finde out the right numbers: ſo doth the Rule of Alligation, the Rule of Fellowſhip, and the Golden Rule partly.

Maſter.

Master. In the Golden Rule, the Rule of Fellowship, and the Rule of Alligation, although the numbers that you worke by, bee not the true numbers that you seeke for, yet are they numbers in iust proportion, and are found by orderly worke: whereas in this Rule the numbers are not taken in any proportion, nor found by orderly worke, but taken at all aduentures.

And therefore I sometimes being merry with my friends, and talking of such questions, haue caused them that proponed such questions, to call vnto them such children or Idiots as happened to be in the place, and to take their answere, declaring that I would make them solue those questions, that seemed so doubtfull.

And indeed, I did answer to the question, and worke the triall therof also by those answeres which they happened at all aduentures to make: which numbers seeing they be taken as manifest false, therefore is this Rule called the Rule of false Positions, and for briefenesse, the Rule of Falshood: which Rule for readinesse of remembrance, I haue compriſed in these few verses following, in forme of an obscure Riddle.

Gesse at this worke as hap doth leed,
By chance to truth you may proceed,
And first worke by the question,
Although no truth therein be don.

Such